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Performance of the new enhanced AMLS method for structural dynamics problems

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ABSTRACT

In this paper, we introduce the new enhanced automated multilevel substructuring (EAMLS) method for dynamic analyses of structures. To improve the computational efficiency of the original EAMLS method, an interface subspace reduction and a residual mode correction are applied. Through submatrix level computations, the new EAMLS method considerably reduces the computational cost compared to the original EAMLS method. We present the solution approaches for both eigenvalue and transient response analyses. The improved performance of the new method is shown through illustrative solutions.

1. INTRODUCTION

The automated multilevel substructuring (AMLS) method is effective in dynamic analyses of large finite element models (Kaplan 2001, Bennighof 2004). Compared with the conventional component mode synthesis (CMS) methods (Hurty 1965, Craig 1968), the computational cost is substantially reduced due to multilevel substructuring. However, in the AMLS method, the residual substructural normal modes are simply truncated, which induces loss of accuracy.

Because the reliability of numerical solutions is of great interest, there have been many efforts to improve the approximate solution accuracy of CMS methods (Kim 2017, Boo 2018). In particular, the enhanced AMLS (EAMLS) method (Kim 2015) compensates for the residual substructural normal modes to improve the accuracy of the AMLS method. While this approach provides highly accurate reduced models, the computational efficiency deteriorates rapidly when the number of degrees of freedom

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(DOFs) increases.

Recently, a new EAMLS method has been developed to improve the computational efficiency of the original EAMLS method (Hyun 2020). Unlike the original method, an interface subspace reduction and a new compensation procedure are performed after the transformation of substructures. As a result, required computational resources are significantly reduced with minimal loss of accuracy.

In the following sections, the new EAMLS method is briefly reviewed, and the scheme for dynamic analyses in the reduced model is presented. We also give some illustrative solutions to investigate the performance of the new EAMLS method.

2. NEW ENHANCED AMLS METHOD

In this section, we briefly introduce the new EAMLS method and its applications for the eigenvalue and transient response analyses. The detailed derivations are described in Ref. (Hyun 2020).

2.1 Model order reduction procedure

According to multilevel substructuring (George 1973, Boo 2017, Boo 2017), the global structure is partitioned into many substructures (see Fig. 1). After substructuring, the generalized eigenvalue problem is defined as

$$\mathbf{K}\boldsymbol{\varphi} = \lambda \mathbf{M}\boldsymbol{\varphi}, \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the reordered stiffness and mass matrices, respectively, and λ and $\boldsymbol{\varphi}$ are the eigenvalue and corresponding eigenvector, respectively.

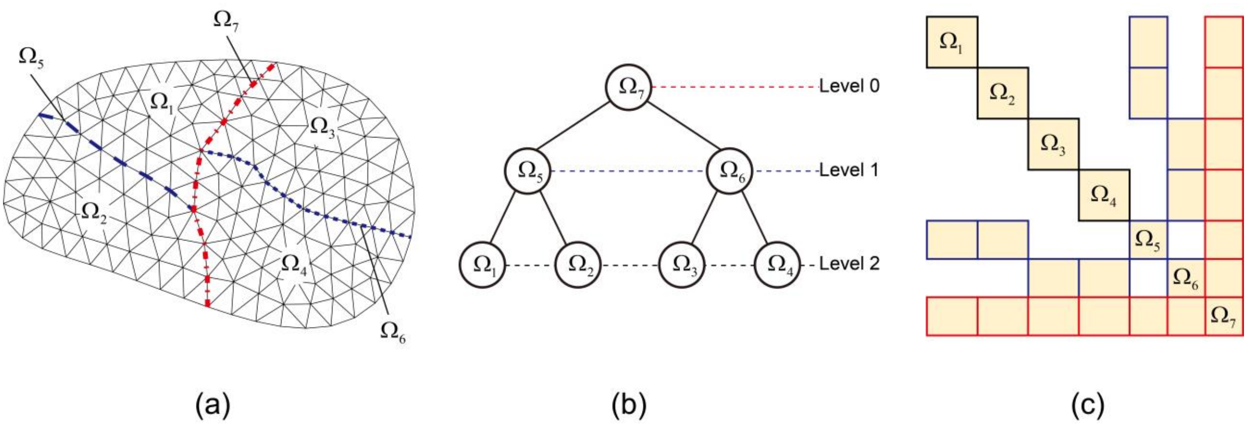

Fig. 1. Multilevel substructuring of a generic structure: (a) partitioned structure, (b) substructure tree, and (c) block matrix pattern for the reordered matrix (Hyun 2020).

For n substructures, the dominant transformation matrix is defined as

$$\mathbf{T} = \hat{\mathbf{\Psi}}\mathbf{\Phi}, \tag{2}$$

where $\hat{\mathbf{\Psi}}$ and $\mathbf{\Phi}$ is the multilevel constraint mode matrix (Kaplan 2001, Kim 2015) and dominant substructural normal mode matrix, respectively.

The reduced matrices using the dominant transformation matrix $\mathbf{T}$ are given by

$$\bar{\mathbf{K}} = \mathbf{T}^T\mathbf{K}\mathbf{T} = \begin{bmatrix} \bar{\mathbf{K}}_b & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{K}}_\Gamma \end{bmatrix}, \quad \bar{\mathbf{M}} = \mathbf{T}^T\mathbf{M}\mathbf{T} = \begin{bmatrix} \bar{\mathbf{M}}_b & \bar{\mathbf{M}}_{b,\Gamma} \\ \bar{\mathbf{M}}_{b,\Gamma}^T & \bar{\mathbf{M}}_\Gamma \end{bmatrix}, \tag{3}$$

where the subscripts b and Γ denote the quantities for the bottom-level substructures and interface, respectively.

From Eq. (3), the eigenvalue problem for the interface subspace is of the form

$$\bar{\mathbf{K}}_\Gamma\mathbf{\Xi} = \bar{\mathbf{M}}_\Gamma\mathbf{\Xi}\mathbf{\Theta}, \tag{4}$$

where $\mathbf{\Theta}$ and $\mathbf{\Xi}$ are the dominant eigenvalue and eigenvector matrices for the interface subspace, respectively.

Using the eigensolutions in Eq. (4), the interface subspace reduction is performed on the dominant transformation matrix $\mathbf{T}$ as follows:

$$\tilde{\mathbf{T}} = \hat{\mathbf{\Psi}}\tilde{\mathbf{\Phi}} = \begin{bmatrix} \mathbf{\Phi}_b & \tilde{\mathbf{T}}_c \\ \mathbf{0} & \tilde{\mathbf{T}}_\Gamma \end{bmatrix} \text{ with } \tilde{\mathbf{\Phi}} = \begin{bmatrix} \mathbf{\Phi}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_\Gamma\mathbf{\Xi} \end{bmatrix}. \tag{5}$$

For the residual mode correction, the additional transformation $\tilde{\mathbf{T}}_a$ is defined as

$$\tilde{\mathbf{T}}_a = \begin{bmatrix} \mathbf{0} & \mathbf{F}_b(\mathbf{M}_b\tilde{\mathbf{T}}_c + \mathbf{M}_{b,\Gamma}\tilde{\mathbf{T}}_\Gamma) \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{6}$$

where $\mathbf{F}_b$ is the residual flexibility matrices for bottom-level substructures (Hyun 2020). Note that, unlike the original EAMLS method (Kim 2015), the residual flexibility for higher-level substructures is not considered.

Finally, the reduced matrices for the new EAMLS method are defined as

$$\tilde{\mathbf{K}} = \tilde{\mathbf{T}}^T\mathbf{K}\tilde{\mathbf{T}}, \quad \tilde{\mathbf{M}} = \tilde{\mathbf{T}}^T\mathbf{M}\tilde{\mathbf{T}} + \tilde{\mathbf{T}}^T\mathbf{M}\tilde{\mathbf{T}}_a\tilde{\mathbf{H}} \text{ with } \tilde{\mathbf{H}} = (\tilde{\mathbf{T}}^T\mathbf{M}\tilde{\mathbf{T}})^{-1}(\tilde{\mathbf{T}}^T\mathbf{K}\tilde{\mathbf{T}}) \tag{7}$$

2.2 dynamic analyses using the reduced model

We here give numerical procedures for the eigenvalue and transient analyses using the reduced model.

In order to obtain approximate eigensolutions, the reduced eigenvalue problem is defined as

$$\tilde{\mathbf{K}}\tilde{\mathbf{x}} = \tilde{\lambda}\tilde{\mathbf{M}}\tilde{\mathbf{x}}, \tag{8}$$

where $\tilde{\lambda}$ and $\tilde{\mathbf{x}}$ are approximate eigenvalue and corresponding reduced eigenvector, respectively.

Then, based on the Rayleigh-Ritz analysis, the approximate eigenvector $\tilde{\boldsymbol{\varphi}}$ is easily obtained by

$$\boldsymbol{\varphi} \approx \tilde{\boldsymbol{\varphi}} = \tilde{\mathbf{T}}_e\tilde{\mathbf{x}} \quad \text{with} \quad \tilde{\mathbf{T}}_e = \tilde{\mathbf{T}} + \tilde{\mathbf{T}}_a\mathbf{H}. \tag{9}$$

Next, we consider the transient response analysis using the reduced model. The structural dynamics equation without damping matrix is expressed by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}, \tag{10}$$

where $\mathbf{u}$ and $\mathbf{f}$ are the displacement and force vectors, respectively, and the dot $(\cdot)$ over the displacement $\mathbf{u}$ denotes the time differentiation.

In a similar way to the eigenvalue analysis, the reduced dynamics equation is defined by

$$\tilde{\mathbf{M}}\ddot{\tilde{\mathbf{u}}} + \tilde{\mathbf{K}}\tilde{\mathbf{u}} = \tilde{\mathbf{f}} \quad \text{with} \quad \tilde{\mathbf{f}} = \tilde{\mathbf{T}}_e^T\mathbf{f}. \tag{11}$$

Employing a time integration scheme, the reduced responses $\tilde{\mathbf{u}}$, $\dot{\tilde{\mathbf{u}}}$, and $\ddot{\tilde{\mathbf{u}}}$ are computed. Then, using the reduced responses instead of $\tilde{\mathbf{x}}$ in Eq. (9), approximate responses are obtained.

3. ILLUSTRATIVE SOLUTIONS

In order to evaluate the performance of the new EAMLS method, we consider a cantilever plate subjected to a shear load as shown in Fig. 2, where the eigenvalue and transient analyses are performed. A sinusoidal load is given with a frequency of 70 rad/s and an amplitude of 100 N; the response for 200 steps is calculated by the Newmark method with time step $\Delta t = 0.005s$. Length L, width B, and thickness H are 10 m, 1 m, and 0.01 m, respectively. The plate is modeled by 2000 four-node shell finite elements (Lee 2014, Lee 2015, Ko 2017, Lee 2019) (10500 DOFs) and partitioned into 33 substructures by using the unstructured graph partition program METIS (Karypis 1998). The dominant substructural modes and interface subspace are selected by the frequency cutoff method (Hyun 2020). All numerical computations are performed using MATLAB 2016b under a PC with Intel i7 7700 3.60 GHz and 32 GB RAM.

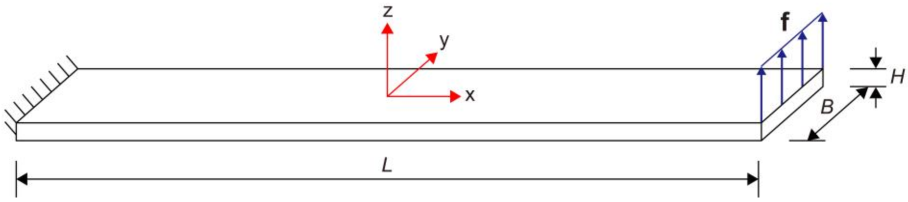

Fig. 2. Cantilever plate subjected to a shear load.

We consider the same size of the reduced models with 156 DOFs obtained by the AMLS, EAMLS, and new EAMLS methods. Table 1 lists the elapsed times for the reduction procedure, including the construction of the transformation matrix. The result shows that the new EAMLS method only requires 1.07 times more computation time than the AMLS method and is 1.9 times faster than the original EAMLS method.

Table 1. Elapsed times for the cantilever plate.

Method	Computation time (s)	Ratio (%)
AMLS	1.22	100.00
Original EAMLS	2.49	204.10
New EAMLS	1.31	107.38

To investigate the accuracy, we consider the relative eigenvalue and total energy errors and deflection at the free end. The reference solutions are obtained by the non-reduced model. The relative eigenvalue errors corresponding to 1^{st}~20^{th} modes are shown in Fig. 3, and the deflection at the free end and the relative total energy errors are shown in Fig. 4. These results show that the new EAMLS method has a similar accuracy to that of the original EAMLS method.

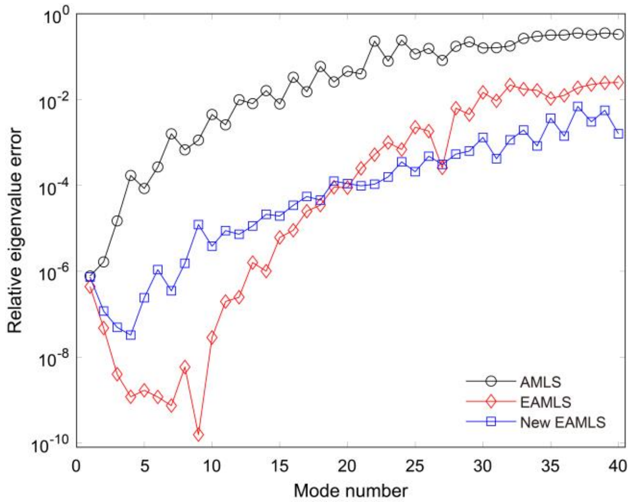

Fig. 3. Relative eigenvalue errors for the cantilever plate.

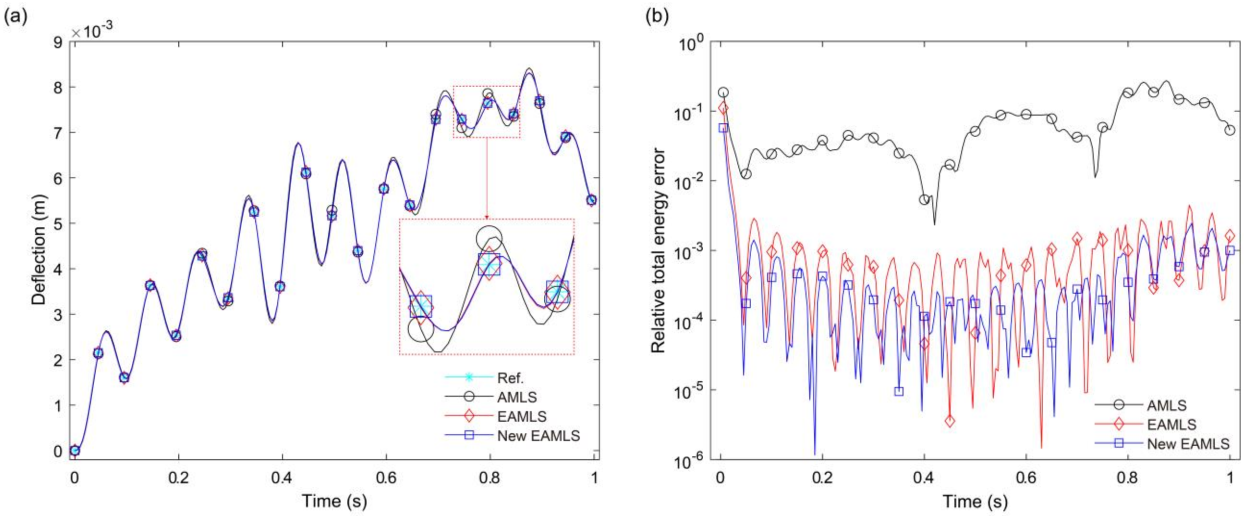

Fig. 4. Transient response for the cantilever plate: (a) deflection at the free end and (b) relative total energy errors.

4. CONCLUSIONS

In this paper, we introduced the new enhanced automated multilevel substructuring (EAMLS) method and its applications for dynamic analyses of structures. The performance of the new method was demonstrated by illustrative solutions, including eigenvalue and transient response analyses. The new method efficiently constructed a reduced model with minimal loss of accuracy compared to that of the original EAMLS method. For engineering practice, future efforts to develop a parallel algorithm for the proposed method would be valuable.

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