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Size effect analysis on quasi-brittle fracture of parallel strand bamboo lumber under bending load

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ABSTRACT

Due to its quasi-brittle fracture behavior and material defects, parallel strand bamboo lumber (PSBL) has obvious size effect phenomenon in its material strength. This work introduces a simple fracture model to obtain the theoretical plastic tensile strength of PSBL. According to the results of the three-point-bending (3p-b) fracture test on notched PSBL specimens, the calculated plastic tensile strength by this fracture model is insensitive to size effect for both geometrical similar and dissimilar samples, which is the intrinsic parameter dependent on the material microstructure. With the determined plastic tensile strength, the defect size is determined and related with sample height by a semi-logarithmic fitting on un-notched 3-p-b samples. Therefore, the influence result of size effect on the tensile strength of PSBL can be decided by the theoretical plastic tensile strength and the sample size.

1. INTRODUCTION

Parallel strand bamboo lumber (PSBL) is a biomass unidirectional fiber reinforced composite material made from the raw bamboo, by cutting, defibering, drying, gum dipping and hot or cold pressing (Yu 2015). Due to the compact and reinforced microstructure, this material has better and more stable mechanical properties than raw bamboo (Huang 2015). It has a raw material utilization rate higher than 80%. Moreover, it has higher strengths than other engineered bamboo products made from the same raw bamboo (Sharma 2015). It also has comparable or even better mechanical properties than many structural biomaterials or bio-composites, such as timber, glulam and laminated veneer lumber (Verma 2014). Furthermore, this material can overcome the size limitation of raw bamboo. Obviously, PSBL has a broad prospect in construction engineering.

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From previous investigations on the fracture properties of heterogeneous materials, like concrete and rock, it is known that such materials exhibit obvious size effect phenomenon (Bažant 1995). The cause of size effect is that the fracture process zone (FPZ) passives the crack tip, and takes softening behavior to material strength. The FPZ size, which is related to the scale of the inherent material heterogeneity, has much influence on the size effect result. If the FPZ length is large enough such that it cannot be neglected in the calculations of the load bearing capacity, the material's strength behavior is named quasi-brittle and the mentioned size effect can be observed (Blank 2017). This size effect is usually denoted as fracture-related behavior and analysed by non-linear fracture mechanics (non-LEFM). Up till now, some frequently-used models have been constructed to determine the size effect behavior of heterogeneous materials, such as fictitious crack model (FCM) (Hillerborg 1976), size effect law (SEL) (Bažant 1984), universal size effect law (USEL) (Bažant 2009), boundary effect model (BEM) (Hu 2017).

PSBL is not only heterogeneous but also inhomogeneous, with apparent quasi-brittle properties in both its interlaminar and transverse fracture (Huang 2018, Liu 2019). In theory, non-LEFM would provide an appropriate method to analyse the tensile strength of PSBL under bending or tensile action. However, in practice, this approach is rarely used. Like timber, brittle strength theory is applied conventionally to describe the tensile strength of PSBL under bending load, as shown by Eq. (1), where f represents the nominal tensile strength which is a system parameter derived from the ultimate resistance of the structure but not an actual material parameter (Blank 2017), M_{max} is the ultimate bending moment of the specimen, and b and d are its net width and height.

$$f = \frac{6M_{\max}}{bh^2} \qquad (1)$$

This study focuses on the fracture behavior of PSBL and the resulting size effect on its longitudinal tensile strength subjected to bending action. The longitudinal tensile strength was obtained by the three-point-bending (3-p-b) fracture test in the transverse direction. 3-p-b notched and un-notched samples of geometric similarity and dissimilarity were tested and used to analyse the size effect behavior of PSBL under bending load.

2. MATERIAL AND EXPERIMENT

2.1 Material and test method

The PSBL material used in this study was made from Moso bamboo (*Phyllostachys pubescens*) (Fujian province in China) by hot press. The ration between raw bamboo and PF resin was 4:1, and the PF resin had 50% solid content. The oven dry density of PSBL was 1101.6 kg/m^3. During the test, its water content was 7.5%. 3-p-b fracture test method was applied in its transverse direction. To investigate the size effect, ten groups of geometrically similar and dissimilar samples with different heights *W*, span-height ratios *S*/*W*, and *α*-ratios were prepared. Geometrically similar samples with the largest and smallest heights as150 mm and 10 mm, a height ratio as big as 15, were prepared. And geometrically dissimilar samples of four *α*-ratios ranging from 0.1 to 0.4 were fabricated. The initial notch was prepared by a handsaw with a width of approximately 1 mm. Also *S*/*W* = 2.5 and 4 were adopted. Moreover, in order to show the size effect phenomenon of the tensile strength, some un-notched specimens were also prepared, with the height *W* varying from 10 mm to 50 mm. The size conditions of all the sample groups are listed in Table 1.

The test was carried out using a 10t mechanical testing machine, and displacement control with a loading rate of 0.5-2.0 mm/min was adopted, corresponding to the loading time before P_{max} was set as around 3 minutes.

Table 1 Test groups of 3-p-b test samples with different *W*, *S*/*W* or *α*-ratios.

Sample type	Group No.	*W* /mm	*B* /mm	*S* /mm	*S*/*W*	a_0 /mm	*α*
Notched samples: Geometric similarity	S-10-2.5-0.3	10	21	25	2.5	3	0.3
	S-20-2.5-0.3	20	21	50	2.5	6	0.3
	S-30-2.5-0.3	30	21	75	2.5	9	0.3
	S-50-2.5-0.3	50	21	125	2.5	15	0.3
	S-100-2.5-0.3	100	21	250	2.5	30	0.3
	S-150-2.5-0.3	150	21	375	2.5	45	0.3
Notched samples: Geometric dissimilarity	S-50-4-0.1	50	21	200	4	5	0.1
	S-50-4-0.2	50	21	200	4	10	0.2
	S-50-4-0.3	50	21	200	4	15	0.3
	S-50-4-0.4	50	21	200	4	20	0.4
Un-notched samples	S-10-2.5-0	10	21	25	2.5	0	0
	S-20-2.5-0	20	21	50	2.5	0	0
	S-30-2.5-0	30	21	75	2.5	0	0
	S-50-2.5-0	50	21	125	2.5	0	0

Note: The three numbers in the Group No. refer to *W*, ***S***/*W*, and *α*-ratio, respectively, for example, S-50-2.5-0.3 means the sample with the size of *W*, *S*/*W* and *α*-ratio as 50 mm, 2.5 and 0.3.

2.2 Size effect of brittle tensile strength

According to brittle strength theory shown in Eq.(1), the nominal tensile strength *f* of notched and un-notched samples with *S*/*W*=2.5 were calculated, and the results are shown in Fig. 1. The numbers above the sample group box represent the average calculated *f* values. It is seen that size effect phenomenon in brittle tensile strength is apparent for 3-p-b bending specimens, in that *f* decreases obviously with the increase of specimen size. Moreover, even with the same net height *h*, *f* also differs for notched and un-notched specimens, for example, groups S-30-2.5-0.3 and S-20-2.5-0 have similar net cross section height, but their average *f* values are rather different. So, this size effect problem brings difficulty for engineering design with the tensile strength by lab test.

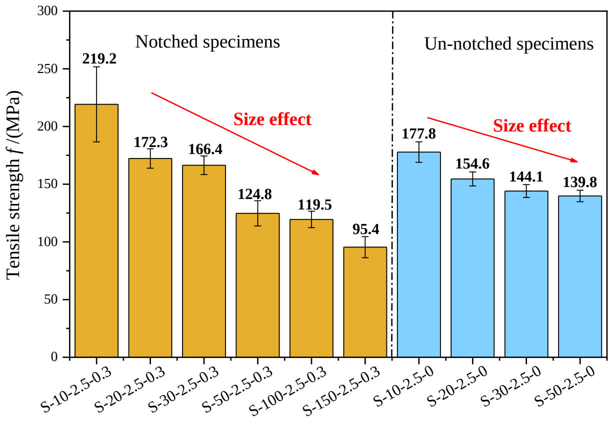

Fig. 1 Size effect phenomenon in brittle tensile strength under bending load

3. FRACTURE MODEL OF TENSILE STRENGTH

3.1 Fracture-related size effect mechanism of PSBL

According to Bažant's SEL for geometric similar samples, in the frame work of LEFM, the size dependency of the nominal fracture strength σ_N for brittle materials can be expressed as follows,

$$\sigma_N(D) = \frac{c}{\sqrt{D}} \tag{2}$$

Where, *D* is the geometrical measure related size of the specimen, for example the height, width or thickness, and *c* is a size independent function of the shape, loads and support conditions of the specimen (Bažant 1984).

In the small scale limit, Eq. (2) predicts $\sigma_N \to \infty$ for $D \to 0$, which is obviously only a theoretical result. In fact, the tensile strength of small structures is affected by FPZ size (Bažant 1998). For PSBL, the material defects, such as microcracks, voids and debonding et ac., would produces FPZ in front of the crack tip, which induces softening behavior. The behavior of PSBL with such a non-negligible size of the FPZ is denoted as quasi-brittle, which can be analyzed within the framework of non-LEFM of un-notched specimen (Liu 2020). Therefore, in a limit consideration for $D \to 0$, σ_N will approach the theoretical plastic limit state associated with a perfectly plastic tensile behavior, denoted as f_{tp}, as is shown by the left side of the curved dash dot line in Fig. 2. This small limit would be small enough where the material microstructure should be account for. However, it is difficult to approach the theoretical small-scale limit.

According to Eq. (2), LEFM predicts that, for material with a sharp crack, $\sigma_N \rightarrow 0$ for $D \rightarrow \infty$. For large enough structures whose FPZ size is negligible, brittle fracture happens, and the stress intensity factor K increases monotonically with the crack growth, until K_{IC} it reaches the fracture toughness of the material. In this case, LFEM applies. Eq. (2) describes the well-known size effect predicted by LEFM for brittle materials with a sharp crack, as illustrated by the straight das line in Fig. 2.

Considering the quasi-brittle fracture behavior of small structure and brittle fracture behavior of big structure, an asymptotic method is used to decide its size effect law. Fig. 2 illustrates this quasi-brittle fracture criterion by the curved solid line, which shows the non-LEFM notched behavior for $D \rightarrow 0$ and LEFM notched behavior for $D \rightarrow \infty$. In this line, the quasi-brittle behavior is bridging the perfectly quasi-brittle and perfectly brittle behavior (Bažant 1984).

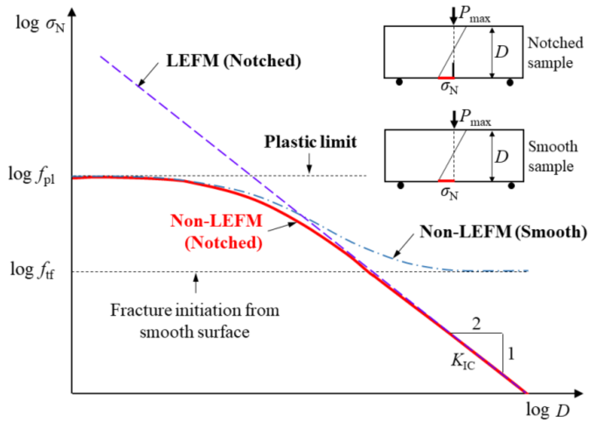

Fig. 2 Schematic diagram regarding the size dependency in a double-logarithmic scale

Based on the size effect explanation in Fig. 2, boundary effect model (BEM) is applied to analyse the size effect mechanism of PSBL, which uses the crack-size ratio to determine the size effect dependency of quasi-brittle material. As shown in Fig. 3, the equivalent crack length a_e and the characteristic crack length a^*_{ch} are used to reflect the size effect result (Guan 2016). When a_e is sufficiently large, i.e. $a_e >> a^*_{ch}$, brittle fracture occurs and the failure can be decided by LEFM, because that the crack size is much larger than the microstructure, inducing to an approximate brittle fracture. On another hand, when $a_e << a^*_{ch}$, the fracture failure is controlled by the theoretical plastic strength f_{tp}, which is the ideal tensile strength f_t of PSBL. However, for the structures with general size, like a_e generally between 0.1 and $10a^*_{ch}$, the fracture failure behavior is controlled by quasi-brittle fracture. Strength failure and quasi-brittle fracture in Fig. 3 should be analysed by non-LEFM.

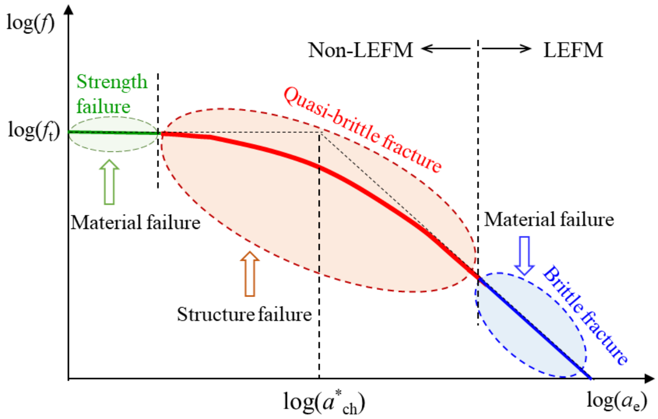

Fig. 3 Size effect analysis of PSBL by equivalent crack a_e

3.2 Fracture model of tensile strength of PSBL

According to the above analysis, the tensile strength f_t is of key importance to determine the size effect behavior of quasi-brittle materials. From the authors' previous study, for a single-edge-notched PSBL specimen under bending load, the equivalent crack a_e is defined by Eq. (3). Moreover, the characteristic crack a^*_{ch} is a material constant which is the intersection point of tensile strength f_t and fracture toughness K_{IC}, as shown in Fig. 4. It is explicitly linked to the material microstructure unit d_{av}, as illustrated in Eq. (3), where d_{av} is the average diameter of average fiber bundle size [24].

$$a_e = \left[\frac{(1-\alpha)^2 \cdot Y(\alpha)}{1.12}\right]^2 \cdot a_0 \tag{3}$$

$$a^*_{ch} = 0.25 \cdot \left(\frac{K_{IC}}{f_t}\right)^2 \approx 3d_{av} \tag{4}$$

By linking the fictitious crack zone Δa_{fic} at the peak load P_{max} to d_{av}, a linear fracture model between P_{max} and an equivalent area $A_e(W, a_0, d_{av})$ is obtained with f_t as the line slope, as shown in Eq. (5) (Liu 2020). It is clear that the model solution has the advantage that the P_{max} - A_e curve is a straight line passing through the origin point. This enables the tensile strength f_t to be easily obtained by a single 3-p-b fracture test.

$$f_t = P_{max} / A_e \tag{5a}$$

$$A_e = \frac{W^2(1-\alpha)\left(1-\alpha+{3\beta d_{av}}/{W}\right)}{1.5\left({S}/{B}\right)\sqrt{1+\frac{a_e}{3d_{av}}}} \tag{5b}$$

4. SIZE EFFECT ANALYSIS

4.1 Size effect analysis of notched samples

According to (5), the P_{max} - A_e line of all PSBL notched specimens is fitted with normal analysis method, as shown in Fig. 4. The average tensile strength f_t is obtained as the mean values of all specimens by the normal distribution method, $f_t = \mu_f$ = 216.4 MPa. 96% reliability is applied on the P_{max} - A_e line. It is seen that the upper and lower bounds cover almost all of the test data, meaning that the tensile strength obtained by the fracture model of all the notched samples with geometric similarity and dissimilarity have good consistence with each other.

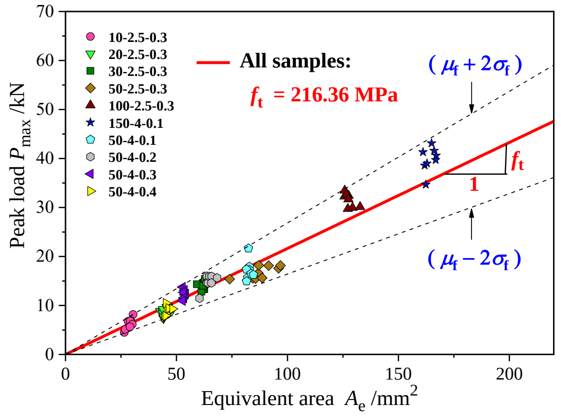

Fig. 4 Linear P_{max} - A_e relation fitting of all notched samples

The individual tensile strength f_t of geometrical similar samples with the heights varying from 10 mm to 150 mm are calculated, as shown in Fig. 5. It is seen that most data of samples with 10 ~ 50 mm height have good consistency with the f_t value of all notched specimens, but the data of samples with 100 mm and 150 mm height are almost all above f_t value. This is because that the same Δa_{fic} value is used for different sample heights. Indeed, in quasi-brittle fracture, larger specimen means longer Δa_{fic}, which would produce a larger A_e and of course produce a smaller tensile strengh. However, although applying the same Δa_{fic} value, the sample scatters with different heights are located inside the 96% reliability interval, indicating that the tensile strength calculated by Eq. (5) can be regarded as the theoretical plastic strength of PSBL, which is the ideal tensile strength independent from size effect.

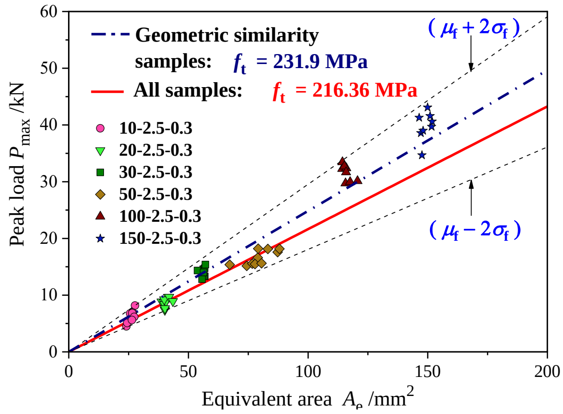

Fig. 5 Tensile strength of geometrical similar samples with different heights

One obvious advantage of this model is that it can be applied on geometrical dissimilar specimens (Hu 2017). Fig. 6 shows the data of geometrical dissimilar samples with the same size and *S*/*W* but different *α*-ratios. It is seen that the calculated f_t reaches the maximum value when $\alpha \approx 0.25$, and then decreases about 15% from $\alpha = 0.3$ to $\alpha = 0.4$. The fitting tensile strength of samples with different *α* is 216.0 MPa, which is perfectly coincided with the f_t value of all notched specimens, suggesting that the fracture model by Eq. (5) shows insensitive to size effect of different *α*-ratios. Fig. 7 shows the data of samples with the same height and *α*-ratio but two different *S*/*W* values. It is seen that the calculated f_t of samples with *S*/*W*=2.5 is a little smaller than that of *S*/*W*=4. However, all of the data are located within the 96% reliability interval of all notched specimens, and their fitting tensile strength is very close, also verifying that the tensile strength calculated by Eq. (5) shows no significant size effect for specimens with *S*/*W* changes.

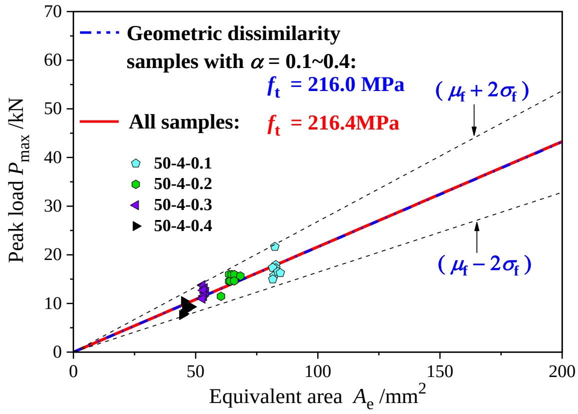

Fig. 6 Tensile strength of geometrical dissimilar samples with different *α*-ratios

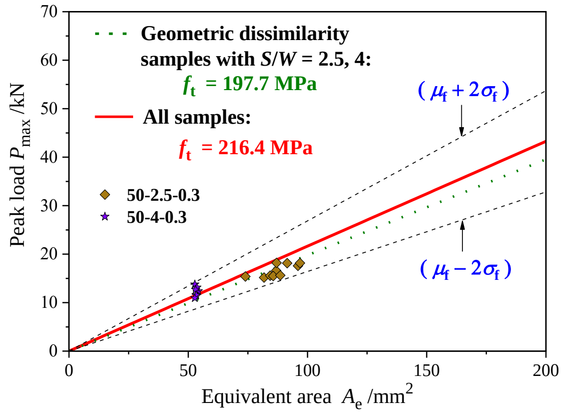

Fig.7 Tensile strength of geometrical dissimilar samples with *S*/*W* as 2.5 and 4

4.2 Size effect analysis of un-notched samples

As explained in section 3.1, because of the material defects, un-notched samples of PSBL also have quasi-brittle fracture behavior, which produces obvious size effect result of the nominal tensile strength according to brittle strength theory. If the defect size on the sample surface is considered as the pre-cut notch length a_0, it can be determined by the plastic tensile strength which is obtained by the notched samples. Here, a_0 of un-notched samples is calculated based on Eq. (5) with f_t decided as 216.7 MPa of notched samples. As shown in Fig. 8, Semi-logarithmic curves are used to fit the relation between surface defect size and specimen height, as $a_0 = C_1 + C_2 \log W$ and $a_e = C_3 + C_4 \log W$. C_1, C_2, C_3 and C_4 are the coefficients.

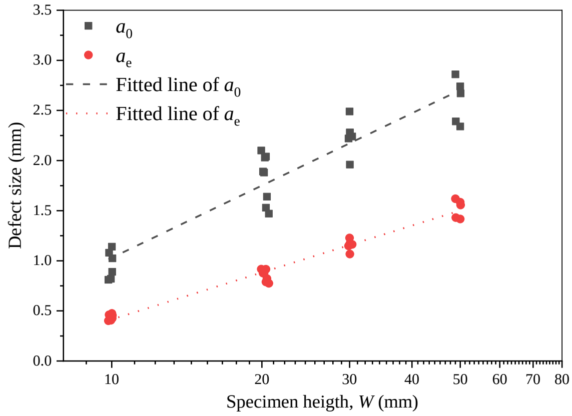

Fig. 8 Semi-logarithmic fitting between material defect size with sample height

With these fitting curves, the surface defect size of un-notched specimens are decided, which is considered as initial crack length a_0. And then, the tensile strength of these specimens is calculated according to Eq. (5). The results are shown in Fig. 9. It is seen that all the data of notched and un-notched samples distribute around the plastic tensile strength f_t. So the brittle tensile strength of PSBL with different size can be decided by the size-independent plastic strength and the material defect size curve.

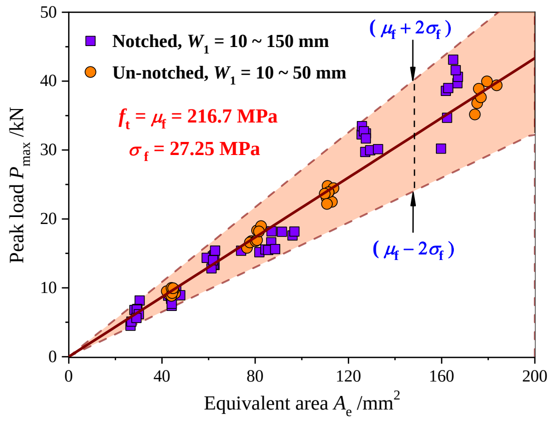

Fig. 9 Semi-logarithmic fitting between material defect size with sample height

5. CONCLUSION

The longitudinal tensile strength of PSBL has obvious size effect behavior according to the brittle strength theory, in that the test tensile strength decreases apparently with the increase of sample size, no matter notched or un-notched bending specimens. The reason for this size effect phenomenon is that the PSBL has quasi-brittle fracture character and the defects in front of the crack tip produce FPZ, which passives the crack tip and the causes strain-softening phenomenon during the fracture process. Because that FPZ size increases with structure size, the tested mechanical strength decreases with the increase of structure size

A simple model of tensile strength based on non-LEFM is introduced to analyse the size effect problem. With this model, the idea plastic tensile strength of PSBL is obtained, which is size-independent and suitable for both geometrical similar and dissimilar samples with different heights, span-to-height ratios and *α*-ratios. Furthermore, the defect size is determined by the calculated plastic strength, and it is fitted by a semi-logarithmic relation with sample size. By this way, the solution for size effect phenomenon can be solved by such size-independent plastic strength and the material defect size curve.

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