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Damage Locating Indices in RC Beams Using Mode Shape Derivatives

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Abstract

Application of two algorithms for locating of damage in reinforced concrete beams are presented in this paper. The Curvature Damage Factor (CDF) computes the change in mode shape curvature between two sets of mode vectors i.e. undamaged and damaged conditions and summates it for all the considered mode shapes. The second algorithm investigated referred to as Local Stiffness Indicator (LSI), is based on the fourth derivative of the mode shape. By computing the values of LSI for both undamaged and damage conditions, a comparison can be made wherein the values vary due to the occurrence of damage while that in the undamaged state the values are constant. In order to verify the suitability of the algorithms, both Eigen value coupled with non-linear static analyses on a finite element model of a reinforced concrete beam were carried out and the eigenvectors for two different damage locations and intensity of load were obtained. The values of the damage localization indices derived using both algorithms were compared. Generally, the CDF suffers from insensitivity to detect damage location which also applies for the LSI at lower modes. However, at higher modes the LSI gave acceptable indication of damage location. Both the algorithms showed inconsistencies and anomalies at the supports.

Keywords: damage detection; damage location; mode shape curvature; mode shape fourth derivatives; RC beams

1. Introduction

Development of damage detection techniques based on modal parameters has attracted significant attention with regards to civil engineering applications. Numerous research works have been done in the field of damage detection and a variety of methods has been developed and proposed. These methods are mainly based on the relationship between the dynamic characteristics and the damage parameters like crack depth and its location. In general, cracks will cause a reduction in stiffness and correspondingly cause a change in the dynamic parameters like mode shape and its derivatives. Therefore, it is possible to detect the damage location by measuring the change in the mode shape derivatives. Mode shape curvatures are more sensitive to damage and the concept of curvature mode shape was introduced by Pandey et. al. (1991). This approach

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was extended by Ratcliffe (1997) using both analytical and experimental results of the curvature of a damaged beam without need of a priori knowledge of the undamaged state. The proposed method applies the Laplace operator on the discrete mode shape and the presence of severe damage was detectable in the form of a jump in the Laplace. Stubbs et. al. (1995) developed a damage index method to locate the damage which utilizes the characteristics of the mode shape curvature for a beam as the main variable in the derived damage localization algorithm based on the relative differences in modal strain energy before and after damage. Wahab and De Roeck (1999) utilized the change in curvature of mode shape to detect the damage location which further reinforces that the change in curvature is more sensitive compared to the mode shape itself. Dutta and Talukdar (2004) carried out Eigen value analysis using Lanczos algorithm in an adaptive h-version finite element environment in order to control the discretization error for accurate evaluation of the modal parameters. It was found that there was better localization of damage by considering curvature of mode shapes. Ismail and Abdul Razak (2006) used mode shape derivatives to detect the location of damage due to single crack and honeycombs in reinforced concrete beams where Local Stiffness Indicator (LSI) was proposed as a damage location indicator and was obtained by rearranging the equation of free vibration for a uniform beam and applying the fourth order centered finite difference formula to the regressed mode shape data. Fayyadh and Razak (2011) and Fayyadh and Razak (2012) studied the effect of support condition and differential support condition on modal parameters and found that different flexural modes react differently to change in support condition or structural element stiffness. Razak and Fayyadh (2013) studied the effect of RC elements composite action and concluded that the first bending mode can be good indicator for bond action between reinforcement steel and concrete, Mode 2 is govern by the amount of steel reinforcement, while Modes 3 and above can be indicators for concrete softening. Fayyadh and Razak (2013) reviewed the use of natural frequencies and mode shape for damage detection in RC structures and concluded that more work is required in the area of damage detection and source identification in RC structures. Pholdee and Bureerat (2016) conducted comparative performance of various meta heuristics for use in structural damage detection based on change in modal data. The study concluded that teaching-learning-based optimisation method is outstanding for a large-scale problem, and differential evaluation is the overall best method. Frans et. al. (2017) conducted comparative study of mode shape curvature and damage location vector methods for damage detection in structures and concluded that damage location vector method can predict the damage member, while the mode shape curvature method can only predict the nodes in the vicinity of the damaged members. Shokrani et. al. (2018) introduce a new method using mode shape for damage localization under varying environmental conditions. The propose method was applied on numerical case studies and found to be effective, however further investigations deem required for further validation.

The objective of this study is to examine the effectiveness of two damage locating indices, one based on the change in curvature of mode shape and the other based on the fourth derivative of mode shape, for detecting damage location in RC beam with different damage locations and severity.

2. Case Study and Finite Element modelling

The case study presented herein is a simply supported beam design according to ACI-318-08 with a span of 2.2 m, with dimensions 150mm wide and 250mm deep. The beam is reinforced with 2 Nos. of 12mm diameter high yield bars as main longitudinal reinforcement and 8mm shear links at 100mm spacing. The details of the beam are shown in Fig 1. A finite element model of the beam was created using 20-node brick elements to represent the concrete while the reinforcement was modelled with 2-node embedded bar elements inside 3-D brick elements, Fig. 2. Initially Eigen value analysis of the undamaged beam was performed, and the mode shape vectors were obtained. Subsequently non-linear static analysis was carried out for two different applied load conditions namely up to 50% and 70% of the ultimate load in order to induce damage in the beam. For the case when the load was applied at 0.5L, ultimate load was 45 kN, such that 50% and 70% are 23kN and 32.2kN, respectively. Correspondingly when applied load was at 0.25L, the ultimate load was 56kN, giving the 50% and 70% values of 28kN and 39.2kN. After each of the applied load condition, the load was released, and Eigen value analysis was again carried out to obtain the mode shape vectors for the damaged beam. Different locations of damage in the beam were achieved by applying the concentrated point load at mid-span and quarter span points along the beam. Fig. 3 shows the crack pattern in the RC beam after application of the damage load.

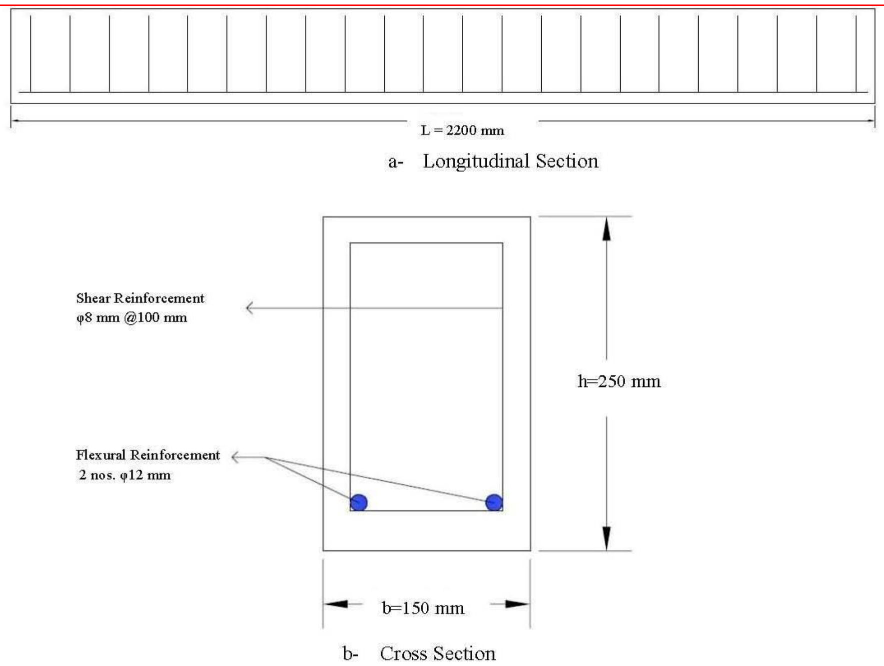


Fig. 1 Reinforced concrete beam dimensions and details

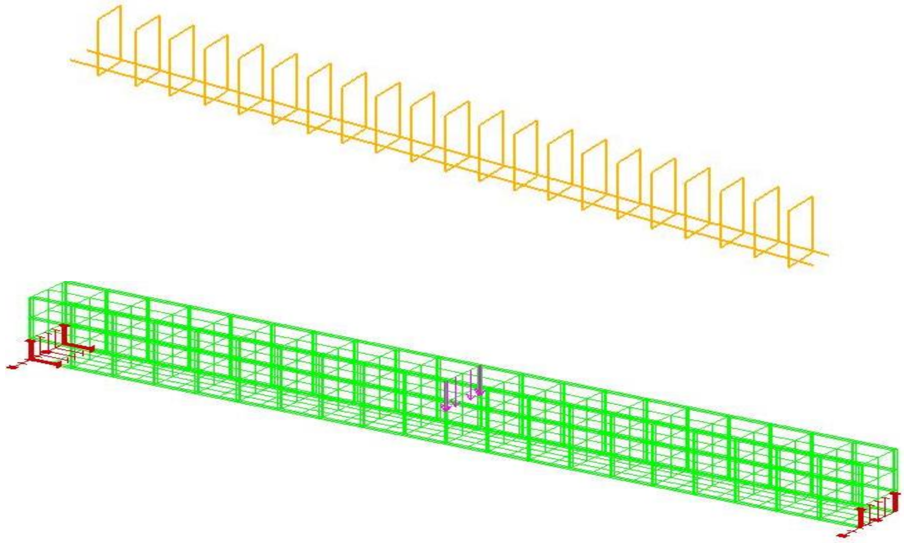
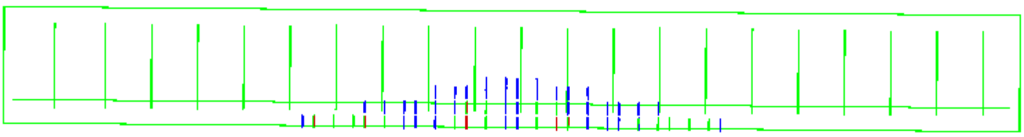
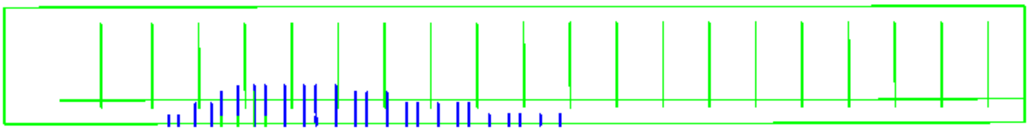


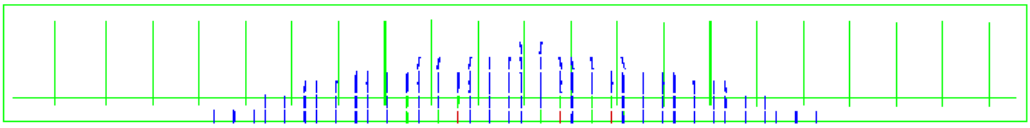
Fig. 2 Finite element model of reinforced concrete beam (top: reinforced bars modelling; bottom: concrete beam modelling)



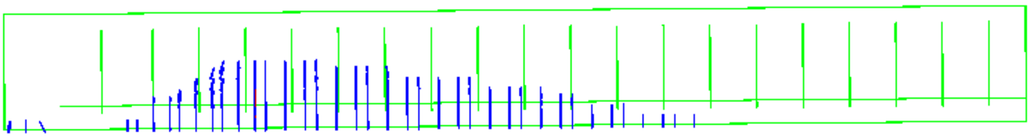
a-under 50% (23kN) at 0.5L



b-under 50% (28kN) at 0.25 L



c-under 70% (32.2 kN) at 0.5 L



d-under 70% (39.2 kN) at 0.25 L

Fig. 3 Crack pattern of RC beam under different damage load located at different distance from left support (Cracks are shown in blue and red)

3. Damage detection algorithms

The results obtained from the finite element analysis were subsequently utilized to verify and compare the sensitivity and accuracy to detect and locate the damage positions, respectively in this study. The eigenvectors were substituted into the equations for the damage algorithms namely the Curvature Damage Factor (CDF) and Local Stiffness Indicator (LSI).

3.1 Curvature Damage Factor CDF

Proposed by Wahab and De Roeck (1999), the mode shape curvature at each point is computed from central difference approximation using mode displacement as given in Eq. (1) below.

$$Ci = (y_{i+1} - 2y_i + y_{i-1})/h^2 \tag{1}$$

where C is the curvature, i is node number, y is the Eigen vector at ith node, and h is the distance between each sequenced nodes.
Subsequently the change in curvature between two sets of mode vectors i.e. the control and damaged cases is as shown in Eq. (2);

$$\text{CDF} = \frac{1}{N}\sum_{j=1}^{N} |C_{ci} - C_{di}| \tag{2}$$

where, CDF is the Curvature Damage Factor, N is the total number of modes, ‘c’ indicates control case when no load was applied and ‘d’ indicates damage case when damage load was applied and released and C is the curvature at ith node.
Figs. 4 and 5 show the results of CDF according to finite element modelling results with 50% of ultimate load (UL) applied at 550 mm and 1100 mm from the left support. Correspondingly the values of CDF with 70% of ultimate load applied to the beam at the same locations are illustrated in Figs. 6 and 7. The results were summated for the first six bending modes.
The results show that the CDF correlated well when the damage location was at 550 mm from left support for different degree of damage and it is apparent in Figs. 5 and 7 and match up to the crack pattern shown in Figure 3(b) and (d). However, when the damage was at mid-span it was less sensitive. Furthermore, values of CDF in all the cases considered, returned high values at the supports which is an anomaly and indicates a flaw in the algorithm. Thus, CDF in its original form is rather unreliable near the supports.

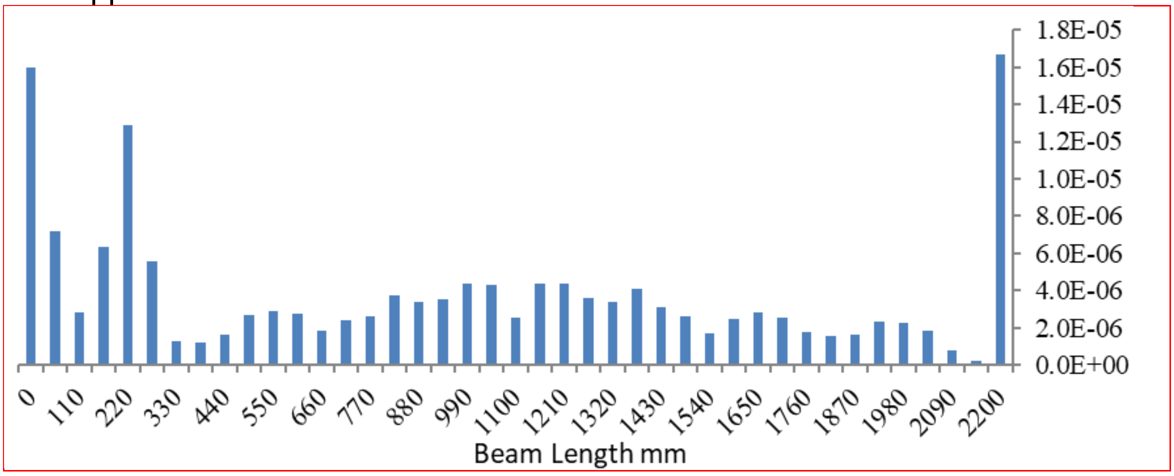


Fig. 4 CDF for beam under 50% UL (23 kN) located 1100mm from left support

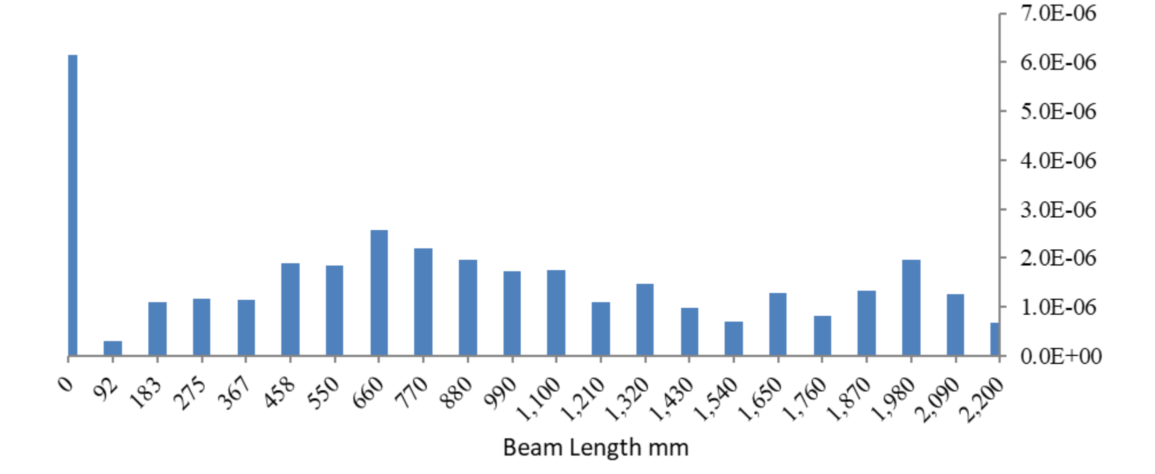


Fig. 5 CDF for beam under 50% UL (28 kN) located 550 mm from left support

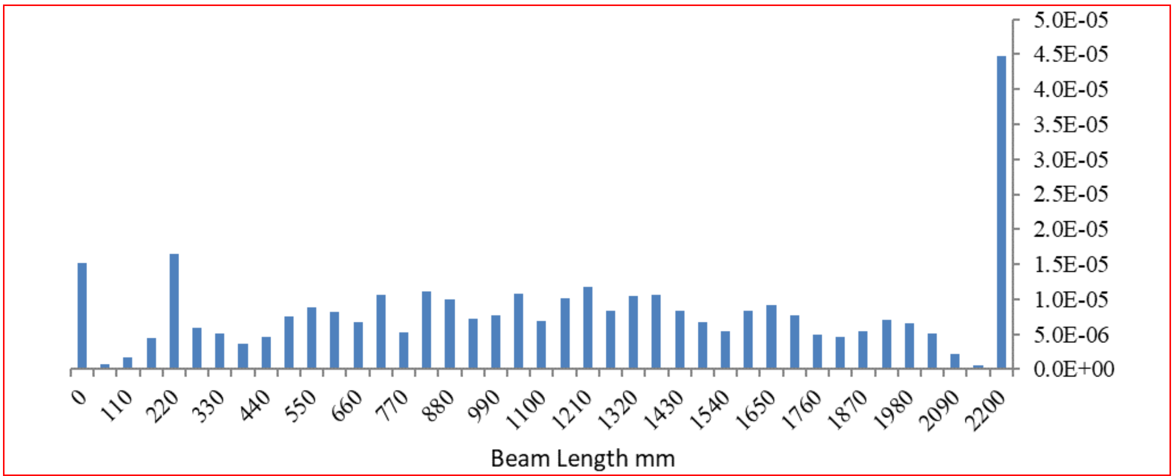


Fig. 6 CDF for beam under 70% UL (32.2 kN) located 1100 mm from left support

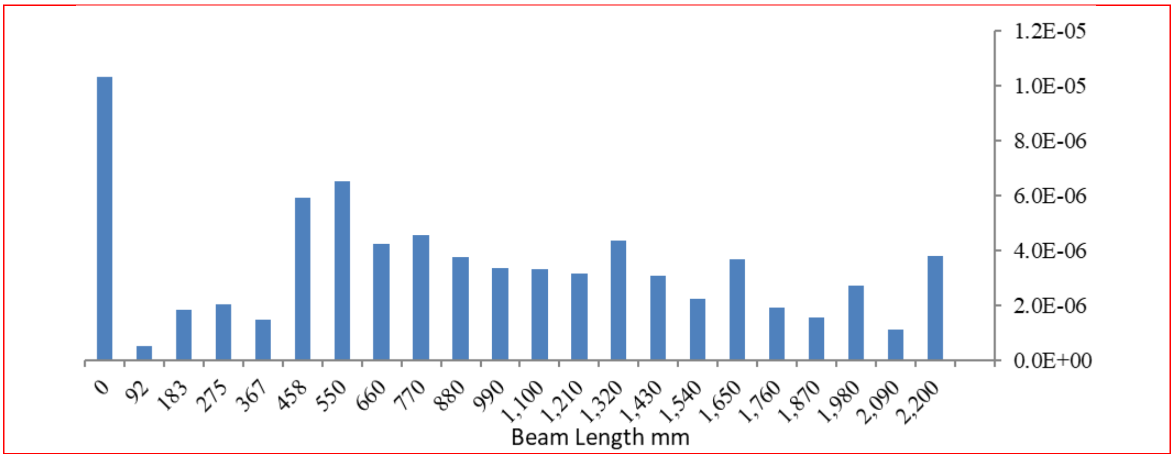


Fig. 7 CDF for beam under 70% UL (39.2 kN) located 550 mm from left support

3.2 Local Stiffness Indicator LSI

Proposed by Ismail and Abdul Razak (2006), it is based on the equation for free vibration of the Euler beam as shown below.

$$\frac{d^4y}{dx^2} - \lambda^4 \ y = 0 \quad (3)$$

Re-arranging the above equation in the following form;

$$\lambda^4 = |\frac{y^4}{y}| \quad (4)$$

In addition, applying the fourth order centered finite difference,

$$y^4 = (y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2})/h^4 \quad (5)$$

where y^4 is the fourth derivative.

Thus, the Local Stiffness Indicator is defined as,

$$LSI = \lambda^4 \quad (6)$$

The solutions to Eqs. 3 and 6 when the node is located at the supports require special consideration. In this case, for a simply supported beam there is angle of rotation at the support which implies the line of elastic deformation which has the shape of the jth mode shape will pass through the support node to the next span and has the same angle of slope. Thus, the elastic deformation line will have the same shape but having opposite curvature. Fig. 8 shows the assumption of for the support node case. Subsequently, yi-1 = - yi+1 and yi-2 = - yi+2. The same assumption can be used at the other support,

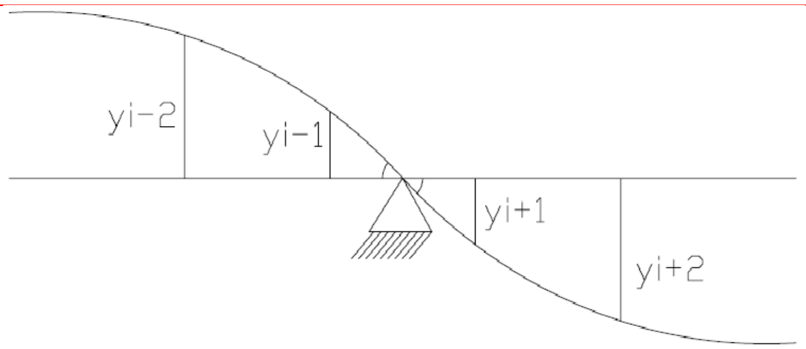


Fig. 8 Special consideration at support

The eigenvectors from the finite element model were extracted and substituted into the above equations to determine the LSI at each node. The occurrence of damage in the beam will cause a change in the LSI value at the damage location as compared to the undamaged beam where the values should remain constant throughout its length. Figs. 9 &10 depict the values of LSI for the finite element RC beam model with 50% of ultimate load applied at two different positions i.e. 550 mm and 1100 mm from left support. Figs. 11 &12 show the value of LSI for the finite element RC beam model with 70% of ultimate load applied at two different positions i.e. 550 mm and 1100 mm from the left support.

Apparently, the LSI is a less sensitive damage indicator compared to the CDF for damage location in the regions considered. However, the sensitivity is better at higher modes, and this can be observed when the damage is located at 0.25L by eliminating the support anomalies for modes 1,4,5 and 6 and subsequently reflects the ability to detect damage as shown in Figs. 10 and12. For regions at the support the values appear as anomalies and this is the major drawback of LSI indicator in its original form.

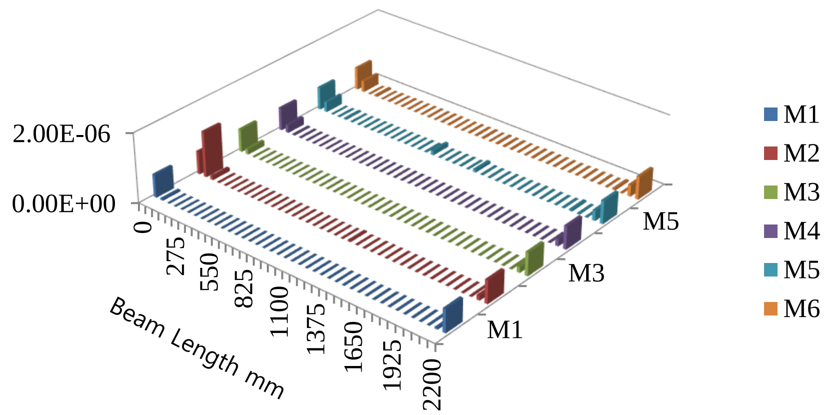


Fig. 9 LSI for beam under 50% UL (23kN) located at 1100 mm from left support, for mode shapes M1 to M6

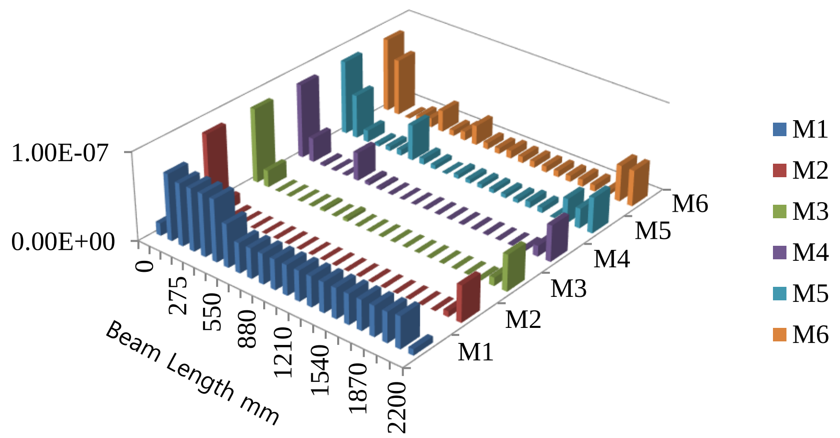


Fig. 10 LSI for beam under 50% UL (28 kN) located at 550 mm from left support, for mode shapes M1 to M6

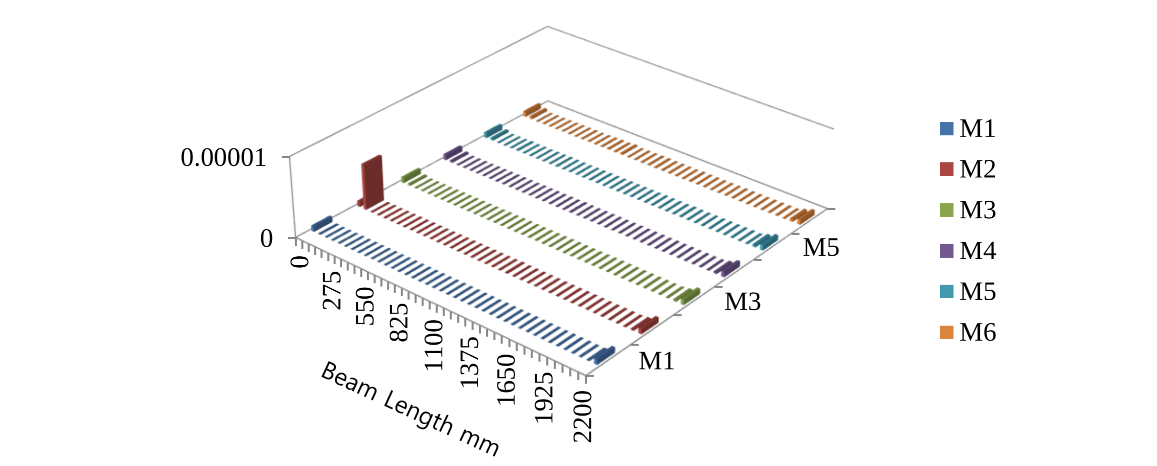


Fig. 11 LSI for beam under 70% UL (32.2 kN) located at 1100 mm from left support, for mode shapes M1 to M6

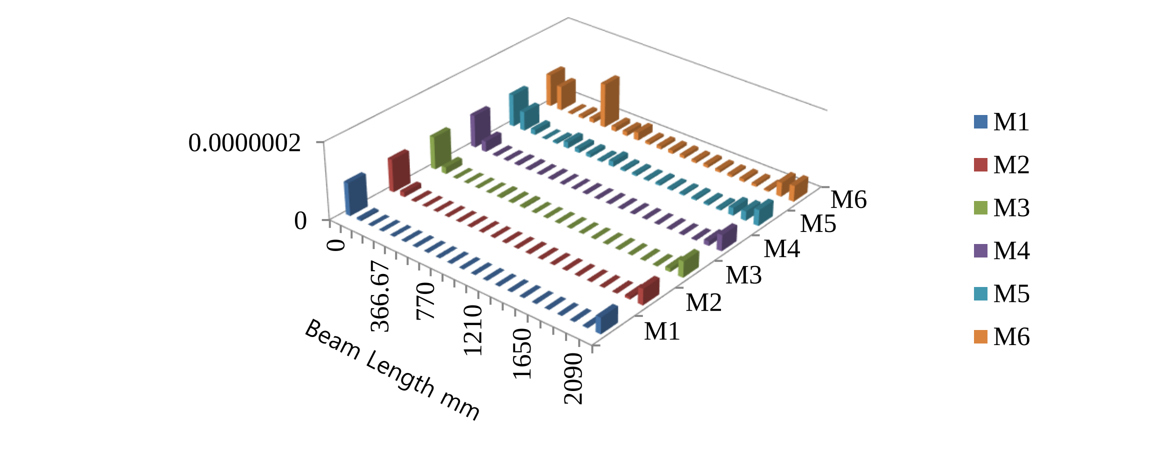


Fig. 12 LSI for beam under 70% UL (39.2 kN) located at 550 mm from left support, for mode shapes M1 to M6

4. Conclusions

From this study, the following conclusions can be drawn:

- Change in mode shape curvature correlated well when the damage location was at 550 mm from left support for different degree of damage. However, when the damage was at mid-span it was less sensitive.
- Values of mode shape curvature-based index in all considered cases returned high values at the supports which is an anomaly and indicates a flaw in the algorithm.
- The fourth derivative based index is a less sensitive damage indicator compared to the curvature-based index for damage location in the regions considered.
- The sensitivity of the fourth derivative based index is better at higher modes, and this can be observed when the damage is located at 0.25L by eliminating the support anomalies for modes 1,4,5 and 6.

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