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Maximum torsional reinforcement and failure mode in reinforced concrete beams

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ABSTRACT

The maximum torsional reinforcement plays an important role to guarantee the ductile torsional behavior of a reinforced concrete (RC) beam failed in torsion. However, current design codes provide various equations to estimate the maximum limit of torsional reinforcements, and those code models also result in about three times differences each other. In this study, a new equation for estimating the maximum torsional reinforcement ratio was derived based on the equilibrium and compatibility conditions in the so-called truss model approach, in which several key influencing factors were considered. A large number of test results were collected to verify the proposed equation from existing studies, and it appeared that the proposed equation for the maximum torsional reinforcement ratio well identified the failure modes of torsion-critical RC beams.

1. INTRODUCTION

Nowadays, various irregular structures have members requiring consideration of the torsion due to various type of loads (Ju et al. 2013). With superposition of various loads, the member can be over-reinforced to resist each load type such as shear force and torsional moment. To prevent over-reinforcement the codes for concrete structures (ACI318 2014; CSA 2014; EC2 2004) provide the maximum amount of rebar in shear

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and torsional design as shown in Fig. 1. Each code specifies the maximum torsional reinforcement, which differs by up to four times, according to the compressive strength of concrete (f_c'). When only the maximum amount of transverse reinforcement is presented, it is difficult to define the failure mode of the member. Therefore, this study presents the maximum transverse and longitudinal torsional reinforcement with theoretical backgrounds, which are derived in a simple equation.

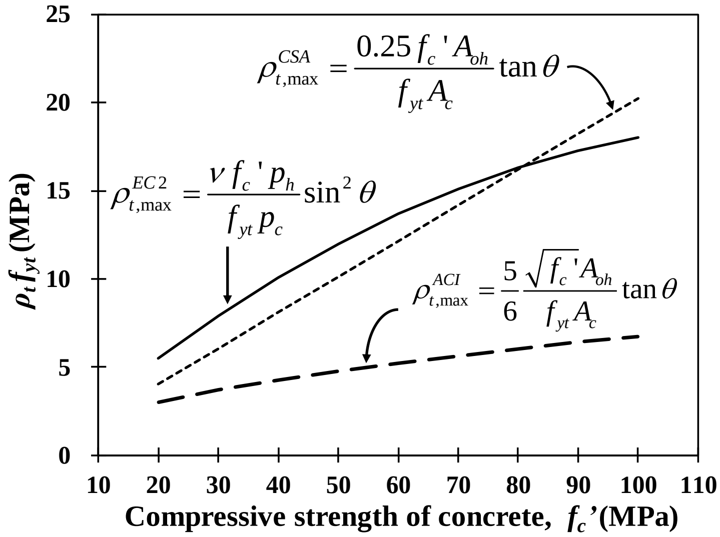

Fig. 1 Maximum torsional reinforcement ratios in design codes

2. PROPOSED MODEL

2.1 Derivation of the basic form

The thin-walled tube (Bredt 1896) theory explains that the torsional moment (T) is resisted by shear flow (q) within the effective thickness (t_d), as shown in Fig. 2. The equilibrium of the forces in the transverse and longitudinal directions in the shear element of the thin-walled tubes can be expressed, respectively, as follows:

$$A_t f_{st} = (\sigma_2 \sin^2\theta - \sigma_1 \cos^2\theta) s t_d \tag{1}$$

$$A_l f_{sl} = (\sigma_2 \cos^2\theta - \sigma_1 \sin^2\theta) p_o t_d \tag{2}$$

where σ_2 and σ_1 are average compressive and tensile stresses, respectively, A_t and f_{st} are the cross-sectional area and stress of the transverse reinforcement, respectively, A_l and f_{sl} are the cross-sectional area and stress of the longitudinal reinforcement. In addition, p_o and A_o are the perimeter length and area surrounded by the centerline of the shear flow (q) in the cross-section of the torsional member, respectively, as shown in Fig. 1. Eqs. (1) and (2) can be expressed in reinforcement ratios, as follows:

$$\rho_t = \frac{(\sigma_2 \sin^2\theta - \sigma_1 \cos^2\theta) t_d p_{oh}}{f_{st} A_c} \quad (3)$$

$$\rho_l = \frac{(\sigma_2 \cos^2\theta - \sigma_1 \sin^2\theta) t_d p_o}{f_{st} A_c} \quad (4)$$

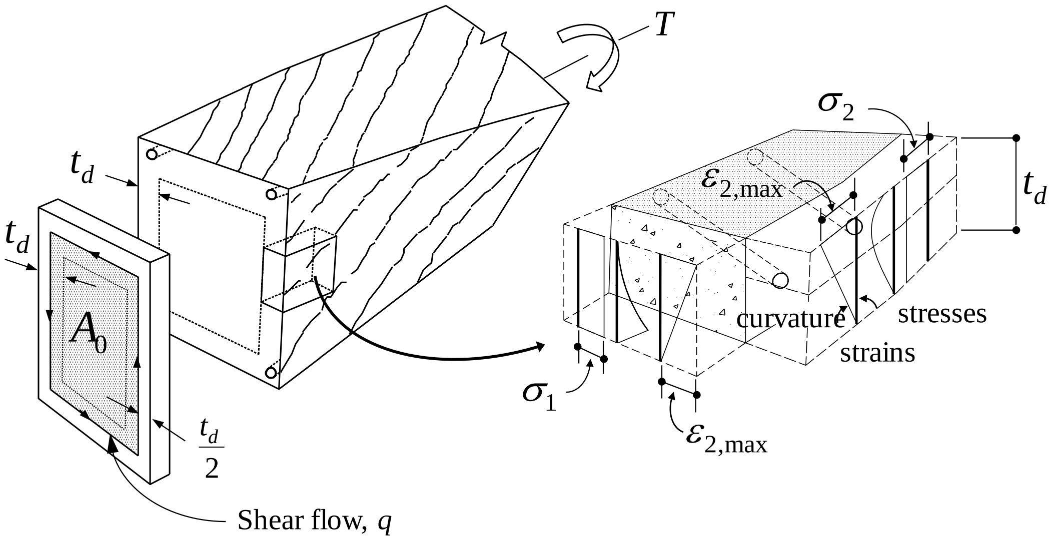

Fig. 2 Torsional member and shear element within the effective thickness

With the relationship between the average shear strain of the thin-walled shear element (γ), the torsional angle per unit length (ϕ), the curvature of the strut (ψ), and effective thickness (t_d), the Eqs. (3) and (4) are derived as follows:

$$\rho_t = \frac{A_o \varepsilon_2}{A_c(\varepsilon_t - \varepsilon_2) f_{st}} \left(\sigma_2 - \sigma_1 \cot^2\theta\right) \frac{p_{oh}}{p_o} \quad (5)$$

$$\rho_l = \frac{A_o \varepsilon_2}{A_c(\varepsilon_l - \varepsilon_2) f_{sl}} \left(\sigma_2 - \sigma_1 \tan^2\theta\right) \quad (6)$$

2.2 Development of a simple form

The Eqs. (3) and (4) can be simplified with the assumption of reinforcement yielding and compressive crushing strain of concrete, however, the average stresses (σ_2 and σ_1) are not easy to be determined, because they are affected by several influencing factors such as f_c' and reinforcement ratio (ρ). In this study, a parametric study of the members with various variables was performed using the softened membrane model for torsion (Jeng and Hsu 2009). Base on the analysis results of 5,184 reinforced concrete members under pure torsion, the variables were determined as a function of f_c', and the maximum torsional reinforcement ratios were derived as follows (Ju et al. 2020):

$$\rho_{t,\max} = 1.215 \frac{A_{oh}}{A_c} \frac{\sqrt{f_c'}}{f_{yt}} \tag{7}$$

$$\rho_{l,\max} = 1.215 \frac{A_{oh}}{A_c} \frac{\sqrt{f_c'}}{f_{yl}} \tag{8}$$

3. CONCLUSIONS

A total of 98 beam members (Chakraborty 1977; Chiu et al. 2007; Hsu 1968; Koutchoukali and Belarbi 2001; Lee and Kim 2010; McMullen and Rangan 1979) under pure torsion were used to validate the suggested equations for maximum torsional reinforcement. The failure modes are distinguished according to the yielding of the transverse and longitudinal reinforcement at the torsional strength, which are UR (under-reinforced) section, where both the reinforcement yield; the POR (partially over-reinforced) section, where either the transverse rebar or the longitudinal reinforcement yields; or the OR (over-reinforced) section, where neither of the reinforcement yields. The maximum torsional reinforcement ratio presented in Eqs. (7) and (8) accurately estimated the failure modes of the members, except for only 7 specimens as shown in Fig. 3.

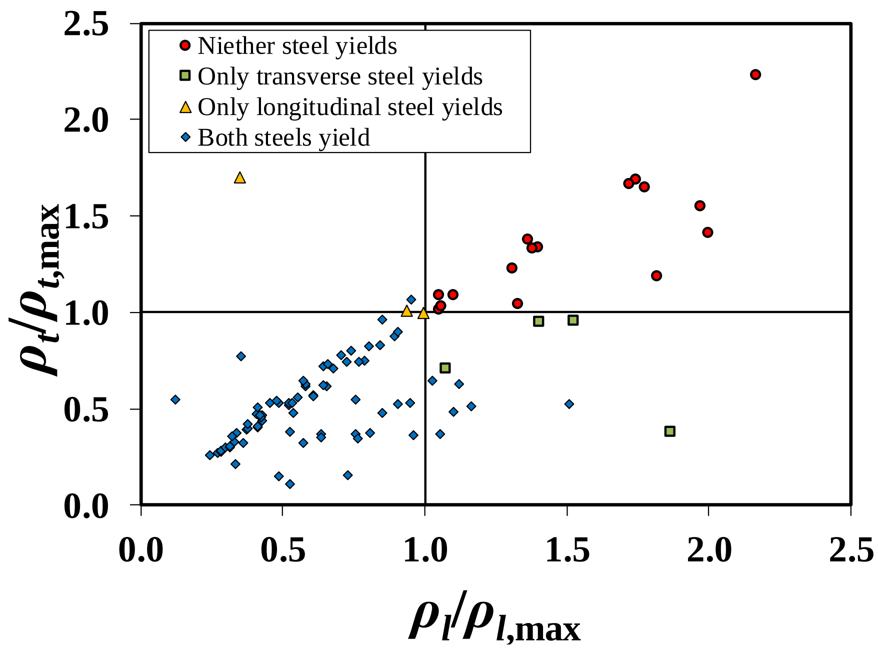

Fig. 3 Validation of the proposed model

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