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General Parametric Glubam Spherical Grid Shell Design Approach

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ABSTRACT

Comparing with conventional steel and timber grid shell which are often shaped as plane or column, spherical grid shell has an advantage to be artistically unique. Despite its unique property of being rotational symmetric and theoretically favorable to structural design, spherical shell remains uncommon due to its complexity. This paper introduces parametric design method for grid shells composed of glued laminated bamboo (glubam) rods and steel joints. Various geometry solutions to a spherical grid shell under common design objectives are concluded first, with regard to economic efficiency, artistry, extensibility, etc. Specific solutions of design interest are then further investigated so that approaches for a generalized parametric design are proposed.

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1. GLUBAM SPHERICAL SHELL OVERVIEW

Bio-based material like wood are widely used in large-span shell construction. The unique property of timber material, i.e. lightweight and high strength, makes it a preferable solution than others. Moreover, the potential to store carbon inside the structural volume rather than releasing it makes wood an attractive eco-friendly construction solution under the current global warming trend. On the flipside, overlogging and deforestation may speed up the global warming process.

In China, timber resource is limited due to the strict government restriction resulted from decades of overlogging. Considering the relative ample resource of bamboo in China, it is reasonable to use bamboo as an alternative. Glubam, as short for glued laminated bamboo, has then been developed under such background (Xiao, et al. 2008).

Currently there are mainly two types of grid shell building method. The traditional building method is to simply assemble rigid elements into a predefined shape, while the deformation method is to deform a plane timber grid shell into a curved shape through boundary conditions. Due to the limitation of the deformation method, it is impossible to bend a plane directly into a spherical shape. Therefore, and also to take advantage of the relatively high bending strength of glubam material, the traditional building method is adopted in this paper.

2. DESIGN RULES

Several objectives are proposed for the spherical grid shell as a set of basic geometry rules for parametric design. These rules may not be compatible with each other, so compromise shall be made to have maximum compliance in general.

2.1 Basic Geometry

The general shape of the shell should resemble a hemisphere or part of a hemisphere. There is no reason for this rule to be compromised as it is the designing objective.

<u>RULE 1 All joints need to be on the sphere.</u>

2.2 Economic Efficiency

To maximize productivity, it is more efficient to use as small as possible the types of construction elements (rod, connector etc.). The best solution would be just one type of rod (lines of equal length) and one type of connector (isogonal), while compromise can be made.

When compromising, there is still a requirement for the lines to be of similar length so with same cross-section the rods can be of similar slenderness for optimal structural design.

<u>RULE 2 Have rods of same cross-section with same (or at least similar) length</u>

2.3 Stability and Extensibility

As the triangle is the only stable shape geometry when pinned, the tessellation should always be made of triangles. When considering extensibility, the parametric design should be easy to scale. Considering the restriction on slenderness the grids should be easily subdivided into denser ones. In this scope, triangle also works perfectly.

RULE 3 Triangle tessellation only.

2.4 Reasonably symmetric

A reasonably symmetric solution is preferred for structural purpose.

RULE 4 Reasonably symmetric.

3. BASE GEOMETRY

Starting from RULE 1 and 2, to divide a sphere into an isogonal convex shell with equal length of rods, only platonic solids (5 types in total) and Archimedean solids (which allows different regular polygonal faces, 13 types in total) meet the requirements.

It is worth noting that RULE 3 requires triangle tessellation but any regular polygon can be divided into isosceles triangle by connecting its vertices with centroid. Considering the spherical bulge will lead to an extension of legs, squares, pentagons, and hexagons are all qualified for RULE 2. However, for the square it will be too much bulge for the isosceles to become regular, i.e. octahedron is way too simple to simulate a sphere, so that only pentagons and hexagons will be considered.

Apparently, geometries that are composed of only triangles, pentagons, hexagons, and are reasonably symmetric meet the requirements. Of all the 18 types of geometry, regular icosahedron, regular dodecahedron, truncated icosahedron, and icosidodecahedron are selected as the only base geometries to satisfy these requirements, see Fig. 1.

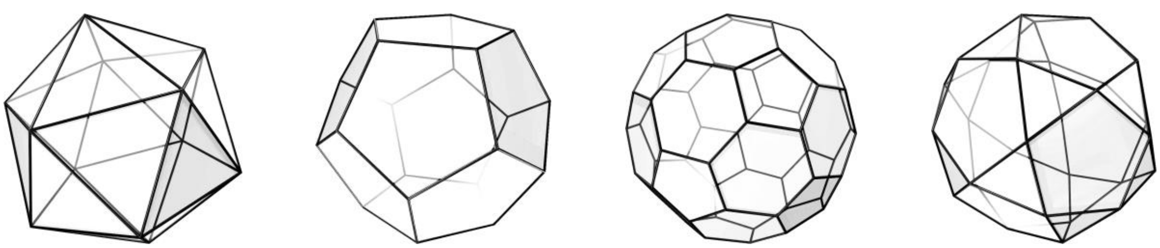

Fig. 1 (From left) regular icosahedron, regular dodecahedron, truncated icosahedron, and icosidodecahedron

The 4 selected geometries are literally the only perfect shapes to fully comply RULE 1, 2 and 4. Still, they are too simple, i.e. rods will be too long if scaled up without subdivision. Besides, regular dodecahedron, truncated icosahedron, and icosidodecahedron need to be triangle-divided first, i.e. to turn pentagons and hexagons into triangles first, for stability purpose.

It will be proven later that the triangle-divided version of the later 3 types of geometry are either exactly same as, or very similar to specific subdivisions of regular icosahedron. Therefore, the only real base geometry to be discussed in this paper is regular icosahedron.

It is also worth noting that from now on, RULE 1, 3, 4 are fully compliant with subdivision, while RULE 2 will be the optimization goal in the subdivision part.

4. SUBDIVISION

Subdivision of a spherical polyhedron can be grouped into 3 typical classes (Wenninger 1979). Tessellation grids are parallel or perpendicular to the base polyhedron's edge in Class I or II subdivision respectively, and neither parallel nor perpendicular in Class III. Triacon subdivision (Stuart 1952) , one of the Class II subdivision, is claimed to have the fewest number of different parts of any subdivision technique (Popko 2012). Still, authors found out that Class I subdivision can still have some advantages when the subdivision number is low.

With some effort it can be proven that triangle-divided truncated icosahedron has the exactly same typology as 1/3 Class I subdivided regular icosahedron, that triangle-divided icosidodecahedron is exactly the same as 1/2 Class I subdivided regular icosahedron, and that triangle-divided regular dodecahedron is exactly the same as 1/2 triacon subdivided regular icosahedron.

In terms of symmetry, Class III is inferior to Class I and II. With the goal to minimize the rod and connector types, Class I and II subdivisions of regular icosahedron are discussed below.

4.1 Class I subdivided regular icosahedron

A regular icosahedron is composed of 20 exactly same equilateral triangle, or 120 exactly same right triangles by slicing the equilateral triangles along their three axes of symmetry. The projections of these right triangles are referred to as least common denominators (LCD). A shell has the same numbers of rod and connector types as its LCD.

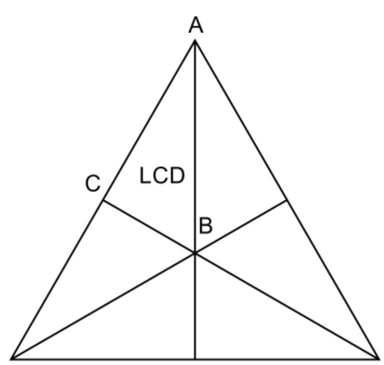

Fig. 2 Regular Icosahedron face and LCD

Consequently, a way to minimize the number of rod types is described as below:

(1) Equally subdivide AC to get initial points.
(2) Fill in the rest of the points by intersecting 2 perpendicular bisectors of 3 nearby known points with the sphere.

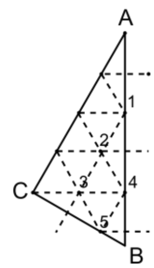

Fig. 3 Regular icosahedron LCD class I subdivision with point generation order $(n = 8)$

For a semi-spherical shell, the rod type number t_{rod} and connector type number t_{con} with subdivision number n are calculated as Tab. 1.

Tab. 1 Class I subdivided regular icosahedron semi-sphere[1)]

n	1	2	3	4	5	6	7	8	9	10	…
t_{rod}	1	2	3	4	5	7	8	9	12	12	…
t_{con}	2	4	6	9	10	14	16	21	25	27	…

4.2 Class II subdivided regular icosahedron

Considering our goal to minimize the types of rods and connector, the Class II subdivision will be limited to triacon in this paper. A way to generate a triacon subdivision is described as below:

(1) Equally subdivide AC to get auxiliary points.
(2) Draw perpendicular arcs of AC at auxiliary points from last step and get points on AB.
(3) Divide arcs from last step in such a way that longer arcs always contain shorter arcs as sub-arc.
(4) Connect relevant points.

[1)] Calculated at the precision of 1/100000 shell radius (1mm over shell radius of 100m) and 0.001 rad, same with the rest of the paper

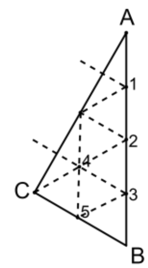

Fig. 4 Regular icosahedron LCD class II subdivision with point generation order ($n = 8$)

Triacon subdivision only works when the division number n is even. For a semi-spherical shell, the rod type number t_{rod} and connector type number t_{con} with subdivision number n are calculated as Tab. 2Tab. .

Tab. 2 Class II subdivided regular icosahedron semi-sphere

n	2	4	6	8	10	12	14	16	18	20	…
t_{rod}	2	4	6	8	10	12	14	16	18	20	…
t_{con}	4	8	12	16	20	22	28	30	36	38	…

Authors managed to program aforementioned two types of subdivision method under Rhino-Grasshopper platform using C#.

At this point, the modeling process is fully automatic for any given subdivision number n, with detailed information of type of rod and connector marked ready for blueprint. Full examples are shown in next chapter.

5. TYPICAL SOLUTION

Structural analysis is performed parametrically within the Rhino-Grasshopper platform for best efficiency, using material property acquired from previous tests (Ma and Xiao 2019). Simple supports are deployed at or near the equator of the sphere. Design loads listed in Tab. 3 are later combined according to Chinese load code (GB-50009 2012).

Tab. 3 Design loads

Load effect	Value
Dead load	$0.6\ kN/m^2$
Wind load	$-1.36\ kN/m^2$
Snow load	$0.65\ kN/m^2$
Live load	$0.5\ kN/m^2$
Temperature Effect	$+27℃;\ -16\ ℃$

5.1 Class I subdivided regular icosahedron ($n = 2$)

With the help of parametric modeling it is now easy to efficiently generate spherical shells for comparison. Class I subdivision shows advantage when n = 2 as the boundary for a semi-sphere is a perfect plane circle at equator, which makes the shell easy to be supported.

Due to the low number of subdivision ($n = 2$), a relatively small radius is preferred, e.g. 4 meters.

A standard glubam rod with the cross section of $5.6 \times 5.6\ cm$ is calculated to be appropriate for such a design, with maximum glubam utilization of 89.8%.

A total of 2 types of rods and 4 types of connectors are used to assemble the shell, as shown in Fig. 5 and Tab. 4.

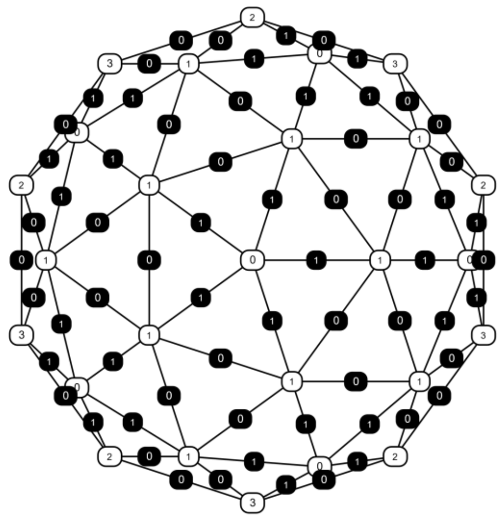

Fig. 5 Class I subdivided regular icosahedron $(\mathrm{n} = 2)$

Tab. 4 Class I subdivided regular icosahedron $(\mathrm{n} = 2)$[1]

Type	Specification	Number
Rod 0	2.472m	35
Rod 1	2.186m	30
Joint 0	{-0.623, 0.782, 0.000} {-0.954, -0.299, 0.000} {-0.087, -0.930, -0.356} { 0.781, -0.239, -0.577} { 0.450, 0.819, -0.356}	6
Joint 1	{-1.000, 0.000, 0.000} {-0.565, -0.825, 0.000} { 0.397, -0.878, -0.268}	10

[1] Rods are specified by length in a 4-meter-radius-shell, joints are specified by unit direction vectors of connected rods

	{ 0.851, -0.102, -0.516} { 0.397, 0.709, -0.583} {-0.565, 0.762, -0.315}	
Joint 2	{ 1.000, 0.000, 0.000} { 0.500, 0.866, 0.000} {-0.397, 0.882, -0.255} {-0.809, 0.110, -0.577}	5
Joint 3	{-1.000, 0.000, 0.000} {-0.565, -0.825, 0.000} { 0.309, -0.897, -0.315} { 0.809, -0.074, -0.583}	5

5.2 Class II subdivided regular icosahedron ($n = 6$)

For a large span spherical shell, e.g. shell with a radius of 12 meters, a larger number of subdivision is required, thus a Class II triacon subdivision is preferred. Using subdivision number $n = 6$, a standard glubam rod with the cross section of $11.2 \times 11.2\ cm$ is calculated to be appropriate for such a design, with maximum glubam utilization of 85.8%.

A total of 6 types of rods and 12 types of connectors are used to assemble the shell. Detailed specification omitted.

6. CONCLUSION

(1) Regular icosahedron, regular dodecahedron, truncated icosahedron, and icosidodecahedron are the only 4 designs that use only 1 type of rod and 2 types of connector. The later 3 require a rigid connection at joints.
(2) Class II subdivided regular icosahedron (Triacon) has advantage over Class I in most of the cases.
(3) Class I subdivided regular icosahedron has advantage over Class II when subdivision number $n = 2$ due to a much more regular boundary, and when $n = 3$ because odd numbers are not available to Class II.
(4) The rational of using glubam for spherical grid shell is validated via a whole set of standardized design procedure.
(5) A fully automatic design process for optimized spherical grid shell is accomplished, including modeling, analysis, and manufacturing detail output.

7. REFERENCES

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