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Prestress-force monitoring for prestressed concrete beam using genetic algorithm-based structural identification

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ABSTRACT

In this study, a genetic algorithm (GA)-based structural identification technique for prestressed concrete (PSC) beam using vibration test results is proposed. The objective of the structural identification is to identify the structural parameters by minimizing the difference between the numerical and experimental natural frequencies. In order to achieve the objective, the following approaches are implemented. Firstly, the numerical frequencies of first two bending modes are determined using a 3D finite element model which is established for the PSC beam. Secondly, the sensitivities of the initial model's stiffness parameters are analyzed in order to choose the proper parameters for model update. Thirdly, from forced vibration tests, the corresponding experimental frequencies are also obtained. Fourthly, the GA is employed for performing the structural identification. Based on updated results, the influence of prestress-loss on the stiffness parameters is evaluated. Finally, the prestress-force in PSC beam is assessed by measuring the change in natural frequencies.

Keywords: prestress-force; prestressed concrete beam; structural identification; structural health monitoring; genetic algorithm.

1. INTRODUCTION

Recently, the development of new methods for accurate and reliable structural identification of civil structures has become increasingly important. In structural engineering analysis and design, an accurate finite element (FE) model is necessary for many civil engineering applications such as damage detection, health monitoring, structural control, and structural assessment. However, it is not easy to generate accurate FE models for complex structures, because material properties and boundary

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conditions of those structures are not completely known. For this reason, it is usual to make simplifying assumptions when modeling the structure. As a result, the initial FE model may not truly represent all the physical aspects of an actual structure. On the other hand, it is generally agreed that experimental results can be considered as more reliable than numerical ones due to the advances made in instrumentation and measurement techniques. Therefore, it presents an important issue that how to update the FE model using experimental results so that the predicted modal properties match the measured ones. During the two last decades, a number of model-update methods in structural dynamics have been proposed (Friswell et al. 1995, Brownjohn et al. 2001, Zhang et al. 2001, Jaishi and Ren 2005).

In addition, prestressed concrete (PSC) beams have been widely used in the field of civil engineering. Therefore, the interest on the safety assessment of existing PSC beams has been increasing. For a PSC beam, the prestress-force is one of the important monitoring targets for the serviceability and safety of PSC beams against external loads and environmental conditions. Kim et al. (2004) proposed a method to identify the change in prestress-forces by measuring dynamic responses of prestressed beams. Ho et al. (2012) presented a multi-phase system identification approach to identify the relationship between the prestress-loss and the change in geometric, material properties and boundary conditions of the PSC beams.

The main objective of this study is to identify a baseline model of a PSC beam which is needed to estimate the prestress-force by using limited modal information. Based on the previous works, in this study, a genetic algorithm (GA)-based model-update method for structural identification of PSC beams for which vibration tests are performed on a set of prestress-force scenarios is presented. In order to achieve the objective of the model-update, which is to minimize the difference between the numerical and experimental natural frequencies, the following approaches are implemented. Firstly, the natural frequencies of first two bending modes are numerically calculated from a 3D FE model. Secondly, the eigenvalue sensitivity of potential model-update parameters is analyzed. Thirdly, structural subsystems are identified by GA-based structural identification technique. Finally, the effect of prestress-loss on the parameters is evaluated and the prestress-force in PSC beam is predicted by measuring the change in natural frequencies.

2. FREQUENCY-BASED PRESTRESS-LOSS ESTIMATION METHOD FOR PSC BEAM

Generally, damage occurrence may be detected using frequency responses of the structure. The basic idea is that frequency responses are functions of the structural properties such as mass, damping and stiffness. Damage causes the change in structural properties which results in the change in frequency responses of the structure. For a simply supported, uniform cross-sectional, PSC beam with a straight concentric tendon, Kim et al. (2004) proposed an equivalent flexural rigidity model to evaluate the prestress-loss in PSC beams.

$$\frac{\delta T_n}{T_n} = \frac{\delta f_n^2}{f_n^2 - f_{o,n}^2} \tag{1}$$

where, the relative change, estimated by the n^{th} mode, in prestress-force between a reference prestress state ($T_{n,ref}$) and a prestress-loss state ($T_{n,los}$) can be estimated as $\delta T_n / T_n = (T_{n,ref} - T_{n,los}) / T_{n,ref}$ by measuring the corresponding n^{th} natural frequencies $f_{n,ref}$ and $f_{n,los}$, from which the reference eigenvalue is defined as $f_n^2 = f_{n,ref}^2$ and the eigenvalue changes is computed as $\delta f_n^2 = f_{n,ref}^2 - f_{n,los}^2$. In order to apply Eq. (1) for prestress-loss estimation, natural frequencies of PSC beam with zero prestress-force, $f_{o,n}$, are needed to be determined. There are two alternative ways to obtain $f_{o,n}$: (1) measure experimentally at as-built state; (2) estimate numerically from system identification process. However, the first way is not realistic because $f_{o,n}$ may not be available in existing in-service structures. Therefore, $f_{o,n}$ should be estimated from well-established system identification process.

3. STRUCTURAL IDENTIFICATION TECHNIQUE USING GENETIC ALGORITHM

In general, there are two alternative ways for updating the FE model of a structure, non-iterative way and iterative way, depending on whether the system matrices or the structural parameters are selected for updating. Non-iterative methods that directly update the elements of stiffness and mass matrices are one-step procedures. As a result, the updated matrices reproduce the measured structural modal properties exactly but do not generally maintain structural connectivity and the corrections are not always physically meaningful. In addition, this method cannot handle the situation whereby the changes in mass and stiffness matrices are coupled together. Meanwhile, iterative methods is to select the geometric, material properties and boundary conditions of the FE model as model-update parameters; then modify them through iterations to minimize the differences in dynamic characteristics between numerical analysis and experimental measurements. The modification can be performed on individual or selected groups of elements. Among iterative methods, GA has become one of the effective techniques for optimization problems.

The main process of GA is as follows initialization operator, selection operator, genetic operators (i.e., crossover operator and mutation operator), and termination operator. The population size depends on the nature of the problem; the larger of population size, the more accurate in solution. The initial population is usually generated randomly, allowing the entire range of possible solutions. In this study, for the selection operator, Roulette wheel method is employed. For the genetic operators, intermediate crossover method is used for crossover operator; meanwhile, adapt feasible mutation method is used for mutation operator. A mutation rate which is too high may lead to loss of good solutions; therefore, elitist selection need to be employed. After selection, crossover, and mutation operators, a new population, or new generation, is created. This process is repeated until a termination condition has been reach. In this study, the objective function is defined as

$$J_f = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{f_{m,i} - f_{a,i}}{f_{m,i}}\right| \quad (2)$$

where n is the number of modes, f_m and f_a are the measured and analytical natural frequencies, respectively.

4. VIBRATION TEST ON PSC BEAM

A dynamic test of the lab-scale post-tension PSC beam was performed to determine the experimental natural frequencies. The schematic of the test structure is shown in Fig. 1. The details of the test can be found in Ho et al. (2012). The prestress-force was applied to the test structure up to 5 different prestress-cases (i.e., T1-T5). Table 1 presents natural frequencies of the first two bending modes with respect to 5 prestress-cases. The experimental results show that for the PSC beam, the natural frequencies are reduced as a result of the loss of prestress-force. Figure 2 shows the frequency responses and the first two bending modes collecting from the measurement system.

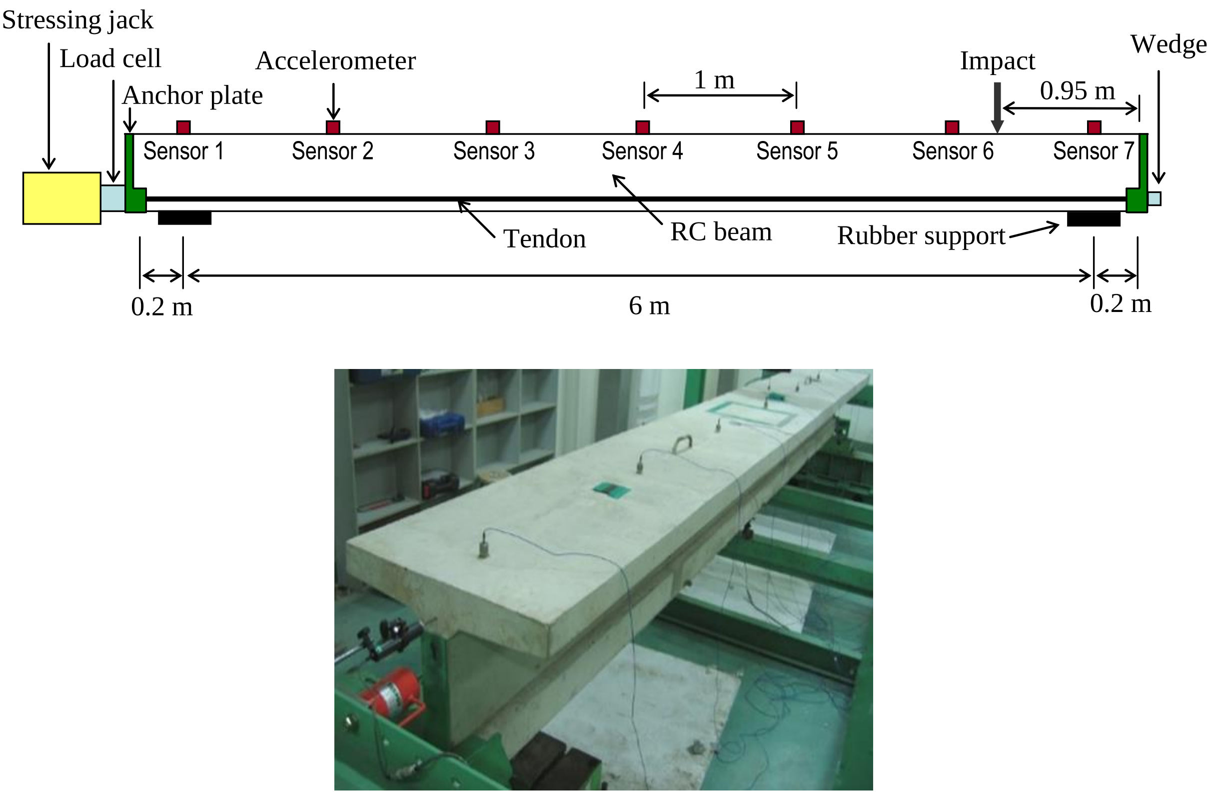
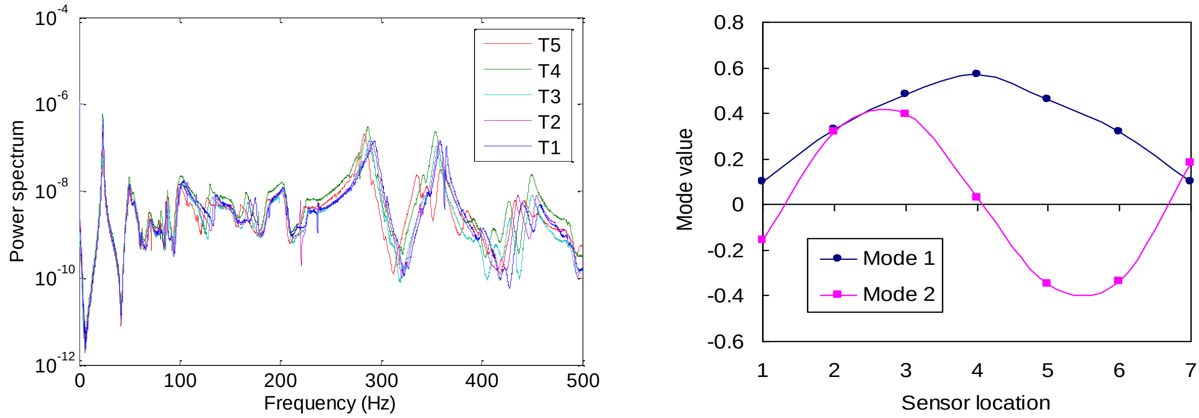


Fig. 1. Experimental setup



(a) Frequency responses (b) Bending mode shapes

Fig. 2. Frequency responses and mode shapes

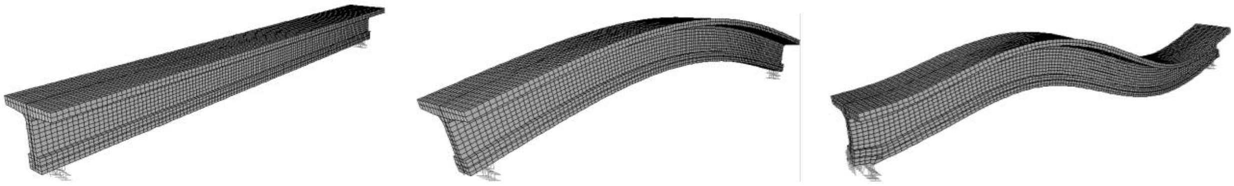
Table 1. Experimental natural frequencies for five prestress-cases (Ho et al. 2012)

Prestress-case	Prestress-force (kN)	Natural frequency (Hz)		Variation of frequency (%)	
		Mode 1	Mode 2	Mode 1	Mode 2
T1	117.7	23.72	102.54	-	-
T2	98.1	23.60	101.70	0.51	0.82
T3	78.5	23.39	101.65	1.39	0.87
T4	58.9	23.23	101.39	2.07	1.12
T5	39.2	23.08	98.73	2.70	3.72

5. STRUCTURAL IDENTIFICATION OF PSC BEAM

5.1. Initial FE model and model-updating parameters

A structural analysis and design software, SAP2000, was used to model the PSC beam. The beam is established by a 3D FE model using solid elements for concrete, tendon elements for tendon, and link elements for links between concrete and tendon. The dimensions of the model are the same as the real structure. For the boundary conditions, spring restraints are assigned at the supports, as shown in Fig. 3(a). Initial values of material, geometric properties and boundary conditions of the FE model are assumed as follows: (1) for the concrete bam, elastic modulus $E_c = 2\times10^{10}$ N/m^2, the second moment of area $I_c = 4.93\times10^{-3}$ m^4, mass density $\rho_c = 2500$ kg/m^3, and Poisson's ratio $\nu_c = 0.2$; (2) for the steel tendon, elastic modulus $E_p = 3\times10^{11}$ N/m^2, the second moment of area $I_p = 1.46\times10^{-5}$ m^4, mass density $\rho_p = 7850$kg/m^3, and Poisson's ratio $\nu_p = 0.3$; and (3) the stiffness of vertical springs $k_v = 10^9$ N/m and horizontal springs $k_h = 10^{12}$ N/m. The structural dynamic analysis was performed with the initial FE model. The first two bending frequencies are 22.59 Hz for mode 1 and 96.00 Hz for mode 2. Figures 3(b) and 3(c) shows the first two bending modes of the PSC beam obtained from the FE analysis.



(a) Initial FE model (b) Mode 1 (c) Mode 2

Fig. 3. FE analysis of PSC beam

Choosing appropriate structural parameters is an important step in the FE model-updating procedure. All parameters related to structural geometries, material properties, and boundary conditions can be potential choices for adjustment in the model-updating procedure. For the PSC beam, therefore, structural parameters which were relatively uncertain in the FE model due to the lack of knowledge on their properties were selected as model update parameters. Also, structural parameters which are relatively sensitive to vibration responses were considered as priori choices. As shown in Fig. 4, for the present PSC beam, six model update parameters were selected as follows: (1) flexural rigidity of concrete beam ($E_c I_c$) in the simple-span domain, (2) flexural rigidity of steel tendon ($E_p I_p$) in the overall structure, (3) flexural rigidity of the left overhang zone ($E_l I_l$), (4) flexural rigidity of the right overhang zone ($E_r I_r$), (5) vertical spring stiffness (k_v) at the left and right supports, and (6) horizontal spring stiffness (k_h) at the left support. Note that the left overhang zone includes stressing-jack, load-cell, tendon anchor, and 0.2 m beam section at the left edge, as shown in Fig. 1. Also, the right overhang zone includes tendon anchor and 0.2 m beam section at the right edge. Both overhang zones were selected due to the uncertainty in the stiffness due to the effect of tendon anchors and concrete sections on dynamic responses under varying prestress-forces.

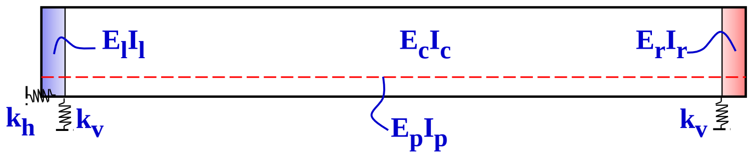


Fig. 4. Six model-update parameters for PSC beam

In order to select the proper parameters for FE model-update, we must first calculate the sensitivity of the modal parameter with respect to the main structural parameters and then identify the most sensitive and insensitive parameters to the responses. Based on the initial FE model, the eigenvalue sensitivity analysis was carried out for six potential model-update parameters. The analytical results are given in Table 2. It can be seen that the flexural rigidity of concrete is the most sensitive parameter for both mode 1 and mode 2. Meanwhile, the other parameters have low influence on two vibration modes.

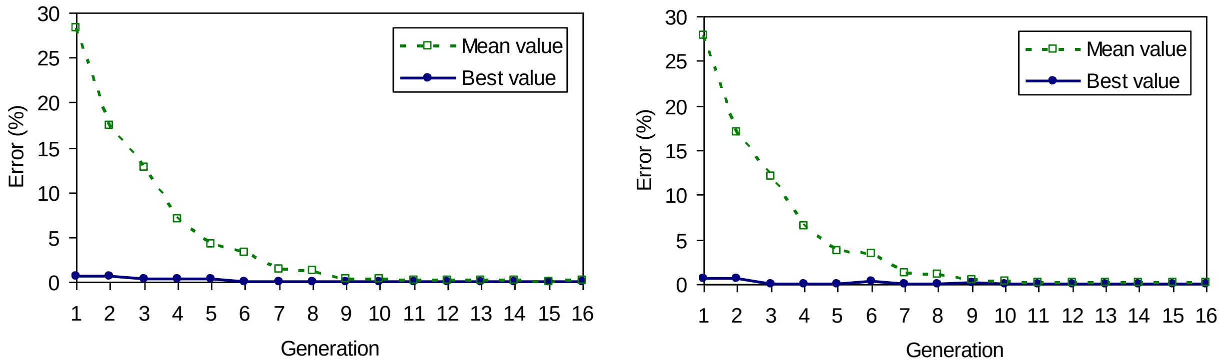
Table 2. Eigenvalue sensitivities of six model-update parameters

Mode	Eigenvalue sensitivities					
	E_cI_c	E_pI_p	E_lI_l	E_rI_r	k_v	k_h
1	0.9468	0.0409	0.0054	0.0029	0.0023	0.00002
2	0.9296	0.0213	0.0239	0.0086	0.0119	0.00005

5.2. Structural identification results for various prestress-forces

After selection of vibration modes and model-updating parameters, an iterative procedure using GA-based structural identification technique was carried out for model update. In this study, MATLAB software was employed for solving the optimization problem using GA. The initial population size was chosen of 40 individuals. Roulette wheel selection with 8 individuals, intermediate crossover with probability of 0.8, and elitist selection with 4 individuals were used. As a result, the problem has 29 individuals for crossover and 7 individuals for mutation. Consequently, the analytical natural frequencies determined at the end of generations gradually approached those experimental values.

The identification results for prestress-case T1, using the first two bending frequencies and 16 generations, are shown in Fig. 5. The convergence errors of updated natural frequencies with compared to target natural frequencies which were experimentally measured at the prestress-force of 117.7 kN. In order to evaluate the effectiveness of GA, two values for each generation which are best value and mean value were calculated. In there, the best value is adaptive value of the best individual; and the mean value is average of adaptive values of all individuals for each generation. After only 16 generations, the identified frequencies converge to the values for the target structure. For the best value, the errors were 0% for both mode 1 and mode 2. For the mean value, the errors were 0.12% for mode 1 and 0.15% for mode 2.



(a) Mode 1 (b) Mode 2

Fig. 5. Convergence errors of natural frequencies for prestress-case T1

The structural identification results for 5 prestress-cases, T1-T5, are summarized in Table 3 and Table 4. For all five prestress-cases, a good agreement was obtained between the measured and updated frequencies. As listed in Table 4, the updated model parameters were changed as the prestress-forces were changed from T1 (117.7 kN) to T5 (39.2 kN). To estimate the relationships between the prestress-forces and the six structural parameters, six empirical equations based on the linear regression analysis were established, Eqs. (3)-(8).

$$E_c I_c = 6.03\times10^4 T + 1.01\times10^8 \quad \text{Nm}^2 \tag{3}$$

$$E_p I_p = 1.77\times10^4 T + 2.78\times10^6 \quad \text{Nm}^2 \tag{4}$$

$$E_l I_l = 4.63\times10^5 T + 2.88\times10^8 \quad \text{Nm}^2 \tag{5}$$

$$E_r I_r = 7.34\times10^4 T + 1.11\times10^8 \quad \text{Nm}^2 \tag{6}$$

$$k_v = 1.64\times10^7 T + 7.95\times10^8 \quad \text{N/m} \tag{7}$$

$$k_h = 7.56\times10^6 T + 1.45\times10^{12} \quad \text{N/m} \tag{8}$$

Table 3. Natural frequencies of updated FE models and target structures for five prestress-cases

Prestress-case	Prestress-force (kN)	First frequency (Hz)			Second frequency (Hz)		
		Exp.	FEM	Error (%)	Exp.	FEM	Error (%)
T1	117.7	23.72	23.72	0.00	102.54	102.54	0.00
T2	98.1	23.60	23.60	0.01	101.70	101.71	0.01
T3	78.5	23.39	23.39	0.00	101.65	101.66	0.01
T4	58.9	23.23	23.23	0.00	101.39	101.39	0.00
T5	39.2	23.08	23.07	0.04	98.73	98.74	0.01

Table 4. Identified values of model update parameters for five prestress-cases

Prestress-case	Updated model parameter					
	$E_c I_c$ (Nm^2)	$E_p I_p$ (Nm^2)	$E_l I_l$ (Nm^2)	$E_r I_r$ (Nm^2)	k_v (N/m)	k_h (N/m)
T1	1.08E+08	5.19E+06	3.31E+08	1.36E+08	2.10E+09	1.84E+12
T2	1.07E+08	5.46E+06	3.20E+08	5.22E+07	2.93E+09	3.21E+11
T3	1.07E+08	4.05E+06	3.11E+08	1.27E+08	2.31E+09	1.95E+12
T4	1.05E+08	3.58E+06	4.09E+08	2.53E+08	3.14E+09	2.06E+12
T5	1.02E+08	4.85E+06	2.38E+08	7.67E+07	7.68E+08	1.21E+12

6. PRESTRESS-FORCE ASSESSMENT OF PSC BEAM

This section describes the application of structural identification results to assess the loss of prestress-force on PSC beam. The prestress-loss in the PSC beam is calculated by Eq. (1). By using Eqs. (3)-(8), the first two bending natural frequencies of PSC beam with zero prestress-force were estimated as 22.73 Hz for mode 1 and 98.17 Hz for mode 2. By setting the maximum prestress-case, T1, as the reference state, the prestress-loss between the reference, T1, and four other cases were calculated (Table 5). The average error is 7% for mode 1 and 33% for mode 2. As shown in Fig. 6, the predicted results are relatively good correlation with the actual ones.

Table 5. Prestress-loss prediction

Prestress-case	Experiment				Prediction		Error (%)	
	T (kN)	$\frac{\delta T}{T_{ref}}$	f_1 (Hz)	f_2 (Hz)	Mode 1	Mode 2	Mode 1	Mode 2
T1	117.7	0.0	23.72	102.54	-	-	-	-
T2	98.1	0.17	23.60	101.70	0.12	0.20	26	17
T3	78.5	0.33	23.39	101.65	0.34	0.21	2	38
T4	58.9	0.50	23.23	101.39	0.50	0.27	0	47
T5	39.2	0.67	23.73	98.17	0.65	0.87	2	31

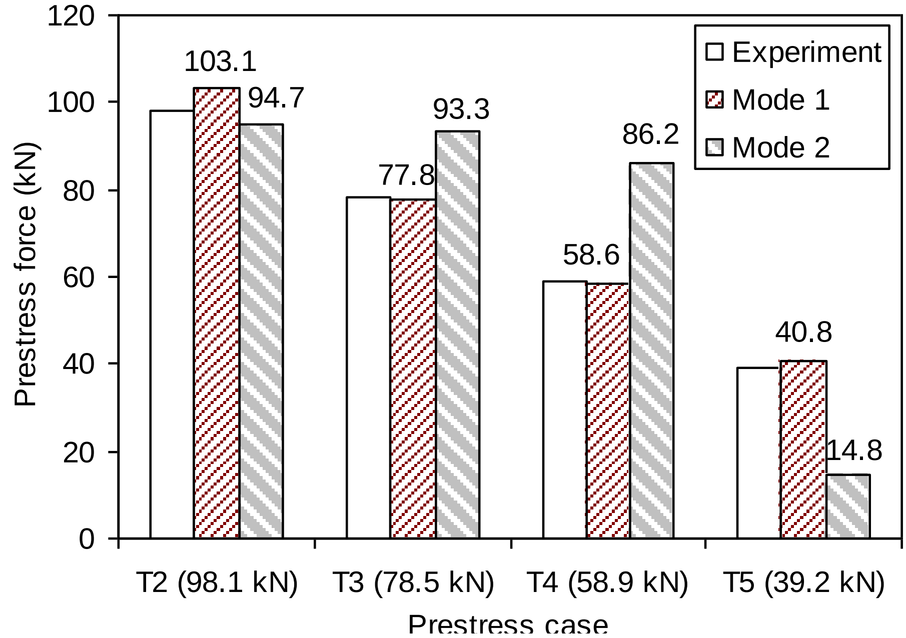


Fig. 6. Prestress-forces assessment

7. CONCLUSIONS

In this study, a genetic algorithm-based structural identification technique was successfully developed for prestress-loss assessment of PSC beam using vibration test results. Good correlations of natural frequencies between updated FE models and the target PSC beam were obtained for the various prestress-forces. From linear regression analysis of the results, the linear relationships between updated model parameters and prestress-forces were well established to estimate the influence of prestress-forces on the performance of structural subsystems. Natural frequencies of the PSC beam under the various prestress-forces were estimated by FE models, from which zero-prestress state models of the PSC beam were identified. As a result, prestress-forces were accurately predicted by using the measured natural frequencies and the identified zero-prestress state models.

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