The 2018 World Congress on
Advances in Civil, Environmental, & Materials Research (ACEM18)
Songdo Convensia, Incheon, Korea, August 27 - 31, 2018

Keynote Paper

Computational development of design aids for anticlastic membrane tension structures

*Thomas Kang[1)] and Marta Gil Pérez[2)]

[1),2)] *Department of Architecture and Architectural Engineering, Seoul National University, Seoul 08826, Korea*
[1)] *tkang@snu.ac.kr*

ABSTRACT

This paper introduces the computational development process of design aids for anticlastic membrane tension structures attached to edge beams on all four sides of the membrane. First, the design of regular anticlastic membrane structures supported between symmetric arches is dealt with. The three parameters are employed: the arch curvature, the width of the membrane, and the scale of the arch. Next, a greater degree of irregularity is considered to produce a wider range of design cases. From the regular anticlastic membranes that are symmetric about the two axes, one of the symmetries is removed. The two new parameters are employed: asymmetry about the transverse axis, creating trapezoid-shaped membranes, and asymmetry about the longitudinal axis, creating inclined membranes. Based on the findings of both case studies of regular and irregular anticlastic membrane tension structures, design aid charts and design equations for rectangular, trapezoid-shaped and inclined membranes are suggested. In the process of the development, computational accuracy has been verified by comparing the design and behavior prediction of the previously constructed membrane structures. The design aids will be useful for professional structural designers, particularly at their preliminary design phase.

1. INTRODUCTION

A lightweight structure utilizes materials optimally to resist external loads or prestress (e.g., the member is often subjected to tension rather than bending); therefore, the purpose of an optimization procedure to design the layout and form of a lightweight structure is to minimize bending or strain energy rather than weight (Bletzinger and Ramm, 2001). Additionally, each spatial structure is a prototype of itself, rather than a product generated from plants. Due to the lack of design guidance, the

[1)] Associate Professor
[2)] Formerly Master's Student at SNU (Currently Doctoral Student at the University of Stuttgart)

design of membrane structures is performed by experts' experience, engineering judgment and practicality. There are two obstacles to the more general future use of membrane structures in architecture; one is the lack of the special expertise required for the design, while the other is the lack of availability of a fabric which is easy to handle and has a long life span (Berger, 2012).

The motivation of this study originates from the need to understand the structural behavior of anticlastic membrane structures and the willingness to find any design limitations to help in the first stages of the project proposal. For this purpose, a detailed study is made for the different parameters of various anticlastic membrane panels. The geometry of regular anticlastic membranes supported by two arched edge beams and two straight transverse edge beams, with each pair of equal length beams being parallel, can be divided into three different parameters: the arch curvature, the width between arches, and the span length of the arches. Then, for a wider range of design possibilities, a greater degree of irregularity is further introduced. From the regular anticlastic membranes that are symmetric about the two axes, one of the symmetries is removed. In this way, two new parameters are introduced: asymmetry about the transverse axis and asymmetry about the longitudinal axis.

The primary objectives of this research are to show the computational development process and developed design aid charts that can be useful for designers during the preliminary design of regular and irregular anticlastic membranes supported by two nonparallel or asymmetric circular arches and two parallel straight non-equal length edge beams.

2. GEOMETRICALLY NONLINEAR FINITE ELEMENT PROGRAM

A non-commercialized geometrically nonlinear finite element program called NASS is used for both the steps of form-finding analysis and stress-deformation analysis (Gil Pérez et al., 2016). First, the element coordinates and stiffness equations are defined in the local axis and then shifted to the global axis. As no compressive stresses are resisted by the membrane structures, wrinkling or buckling may occur for an applied compressive stress value larger than the initial tension stress, and the procedure to treat element wrinkling is also included in the analysis.

In this implementation, constant-strain triangle elements proposed by Cook et al. (2007) and linear Lagrangian interpolation functions as shape functions are used. The geometrical nonlinearity of membrane elements is considered by including high-order terms. Assumption is a linear elastic material behavior for the stress-strain relation. The structural stiffness equation can be calculated by using the principle of virtual work and the shape functions.

Because nonlinear components that are dependent on displacement values are included in the stiffness matrix, a nonlinear system solution technique such as the Newton-Raphson method is employed (Heath, 2002). In addition, the nonlinear solution procedure and nonlinear finite element equations for the membrane structures with the assumption of linear elastic behavior for the stress-strain relation have been developed (Kim et al., 1997; Gil Pérez et al, 2016, 2017). The residual force vector converges to a zero vector as iterations are conducted in the nonlinear solution procedure.

In the form-finding analysis, the structure shape in equilibrium condition can only be determined by the geometric matrix, whereas geometric and elastic stiffness matrices are utilized for nonlinear stress-deformation analysis. The initial input data include the information of nodes, elements, loads, pressure, curvature, material, boundary conditions, load conditions, forced displacement, initial stress, force-displacement matrix, amount of imperfections, etc.

3. DESIGN AIDS FOR REGULAR ANTICLASTIC MEMBRANE STRUCTURES

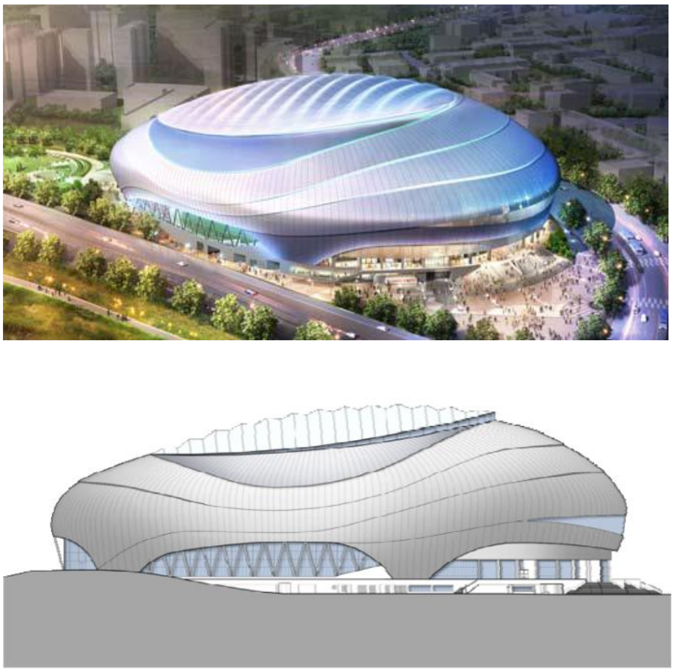

Fig. 1 Gocheok sky dome

First, the validation of the analytical tool NASS was provided by Gil Pérez et al. (2016) (Figs. 1 and 2). Then, a computational parametric study on regular anticlastic membranes supported between arches was performed by Gil Pérez et al. (2017). The primary three parameters that determine the geometry of this type of membranes, namely, the arch curvature, panel width, and arch scale, were considered. Based on the parametric study of different combinations of geometries, a design aid chart was obtained as shown in Fig. 3, and the following findings were obtained in the computational development process:

1) The effect of the scale of the arch is negligible as the maximum stresses are almost equivalent when the arch curvature and width of the panel are kept constant. This indicates that considering only the two parameters of arch curvature and panel width, any span can be covered.
2) The arch curvature influences the achievement of a balanced initial stress distribution when performing the form-finding analysis. If the curvature is increased, the uniformly distributed initial stress is more difficult to reach.

3) The width of the panel has a smaller influence on the achievement of the balanced initial stress distribution during the form-finding analysis. However, panels with narrower widths reach a uniform stress distribution more easily than the ones with wider spans between arches.
4) The stress-deformation analysis should be performed under downward loading (snow) and upward loading (wind). The analysis results indicate that the maximum stresses under downward loading are larger in the warp direction of the fabric but larger in the fill direction under upward loading.
5) As the fabric strips' ultimate tensile strength is smaller in the fill direction than in the warp direction, it is the fill direction that gives the limitations for the design. Additionally, as the upward loading produces larger stresses in the fill direction than the downward loading, the limitations are given by the upward loading case in the fill direction.
6) It is found that the smaller curvatures have larger stresses. However, when the curvature of the arch is increased, negative stresses appear in the panels. This means that the membrane can wrinkle and thus possible ponding problems can also appear.

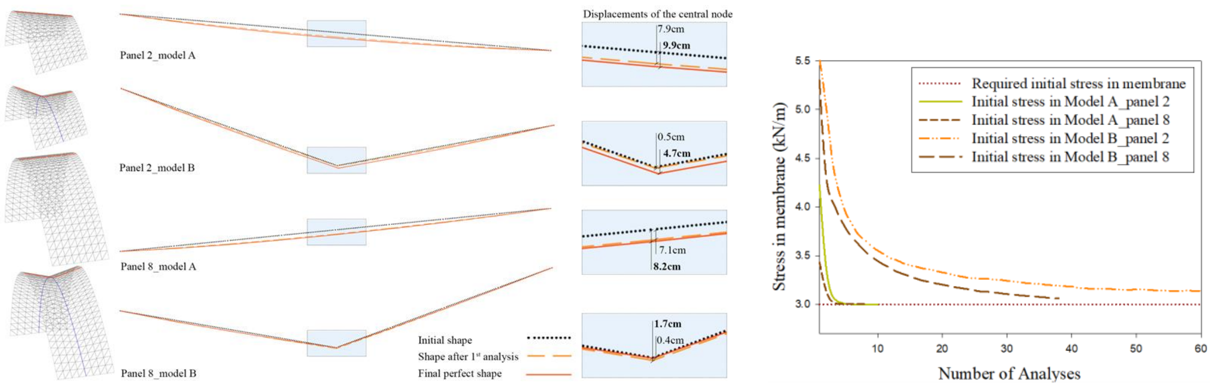

Fig. 2 Validation study using Gocheok sky dome

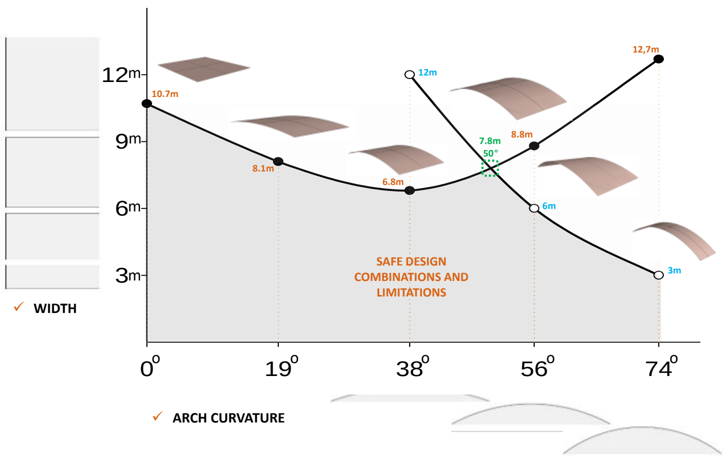

Fig. 3 Design aid for regular anticlastic membrane tension structures

3. DESIGN AIDS FOR IRREGULAR ANTICLASTIC MEMBRANE STRUCTURES

Table 1 Design aid for maximum midspan width (*M*) of trapezoid-shaped membranes

Curvature		0	19	38	56	74
m = slope		0.018	0.076	0.124	0.155	0.161
Max. β (degrees)						
n = max. stress with β = 0º (kN/m)	90	22.905	17.685	13.365	10.575	10.035
	80	23.085	18.445	14.605	12.125	11.645
	70	23.265	19.205	15.845	13.675	13.255
	60	23.445	19.965	17.085	15.225	14.865
	50	23.625	20.725	18.325	16.775	16.475
	40	23.805	21.485	19.565	18.325	18.085
	30	23.985	22.245	20.805	19.875	19.695
Max. width in the center M (m)	90	10.2	6.8	4.1	2.85	
	80	10.35	7.35	4.9	3.67	3.6
	70	10.5	7.88	5.71	4.74	4.84
	60	10.65	8.4	6.58	5.8	6.12
	50	10.8	8.93	7.47	6.96	7.65
	40	10.95	9.72	8.35	8.15	9.24
	30	11.1	10.55	9.28	9.3	11.27

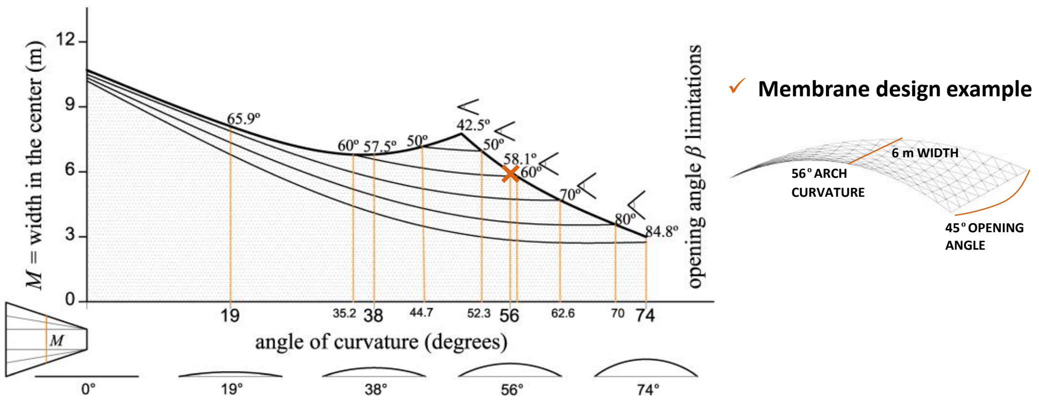

Fig. 4 Design aid for trapezoid-shaped anticlastic membrane tension structures and final design example

The assumptions are the same as those for the regular anticlastic membranes, which are rectangular in plan, as described by Gil Pérez et al. (2017). By introducing asymmetry about the transverse axis to the regular anticlastic membranes, trapezoid-shaped anticlastic membranes are created and a greater range of designs can be modeled. Various geometric parameters that define trapezoid-shaped membranes were analyzed (Hong et al., 2018). The values indicated in Table 1 and the curved lines inside the envelope in Fig. 4 are the result of the developed design aids. Of the two upper lines in Fig. 4, the right portion of the thicker line represents the criteria where negative stresses are found, that is, wrinkling of the membrane occurs. Here, it is possible to observe how the *M* (midspan width; see Fig. 4) varies for each opening angle and arch curvature. However, these limitations are only related to the results found for the downward loading condition in the fill direction. The limitations from both loading conditions should overlap. Figure 4 combines the graphs from Table 1 and the design aid graph in Fig. 3. It is noted that the limitations related to the upward loading conditions are independent of the opening angle. Any arch curvature can be combined with a mid-span width (*M*) inside the limited area (envelope), and the maximum opening angle can be checked following the curved lines.

A new parameter (see α in Fig. 5) is introduced that defines the asymmetry about the longitudinal axis for inclined anticlastic membranes. Having different scales in the arches and maintaining the width of the panel (i.e., arches are parallel), the inclination angle (α in Fig. 5) is formed between the horizontal line and the line that connects between the peaks of the 2 arches. A detailed explanation is provided by Hong et al. (2018) to find the relationships between the parameters that are needed for the design.

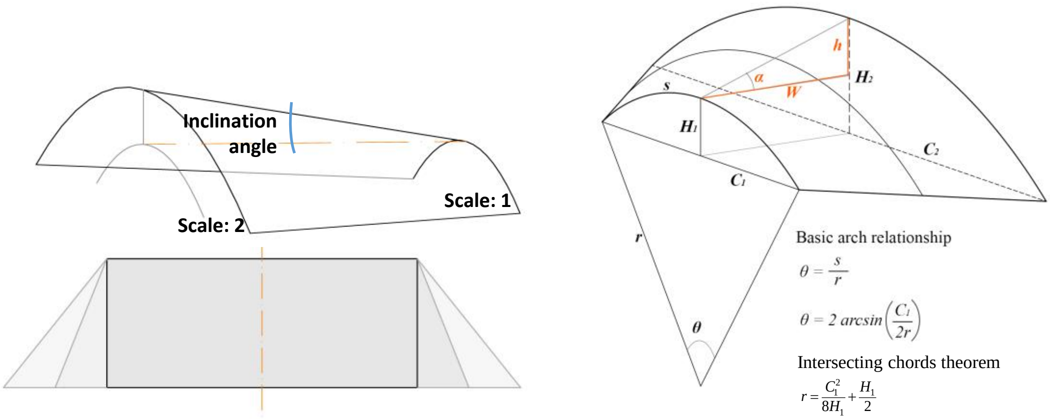

Fig. 5 Parameters defining scale of larger arch in consideration of inclination angle between two arches

Table 2 Design aid for maximum inclination angle (α) of inclined membranes

	Width (m)	2	3	4	5	6	7	8	9
	m = slope	0.216	0.227	0.256	0.306	0.319	0.319	0.319	0.319
	Arch curvature								
n = max. stress inclination on 0º (kN/m)	9.5	11.99	12.98	15.13	17.27	19.43	21.05	22.69	24.33
	19	12.49	13.74	16.12	18.51	20.89	22.56	24.23	25.9
	28.5	13.15	14.72	17.27	19.82	22.37	23.91	25.44	26.96
	38	13.75	15.62	18.17	20.72	23.27	24.58	25.88	27.18
	47	14.09	16.14	18.49	20.85	23.21	24.24	25.28	26.32
	56	14.25	16.37	18.38	20.39	22.41	23.17	23.93	24.7
	65	14.31	16.42	17.99	19.5	21.02	21.62	22.22	22.82
	74	14.34	16.37	17.44	18.36	19.27	19.8	20.33	20.85
Max. inclination angle α (degrees)	9.5	58.0	50.9	36.7	23.7	16.0	10.9	5.8	
	19	55.7	47.5	32.8	19.7	11.4	6.2	0.9	-4.3
	28.5	52.7	43.2	28.3	15.4	6.8	1.9	-2.9	-7.6
	38	49.9	39.2	24.8	12.4	3.9	-0.2	-4.2	-8.3
	47	48.3	36.9	23.6	12.0	4.1	0.9	-2.4	-5.6
	56	47.6	35.9	24.0	13.5	6.6	4.2	1.9	-0.5
	65	47.3	35.7	25.5	16.4	11.0	9.1	7.2	5.3
	74	47.2	35.9	27.7	20.1	16.5	14.8	13.2	11.5

All the parameters that influence the design of inclined anticlastic membranes were analyzed, including the arch curvature (θ), height (H_1), and width of the panel (W). Then, a computational parametric study on inclined anticlastic membranes was performed. Table 2 summarizes the values found from the analysis. As shown in the table, some combinations result in a negative inclination angle (α), which means that for that combination, inclined membranes may wrinkle with larger curvatures and wider membranes; thus, the models that wrinkle should also be removed from the safe combination cases. Finally, the developed design aid for inclined anticlastic tension membranes is graphically illustrated in Fig. 6.

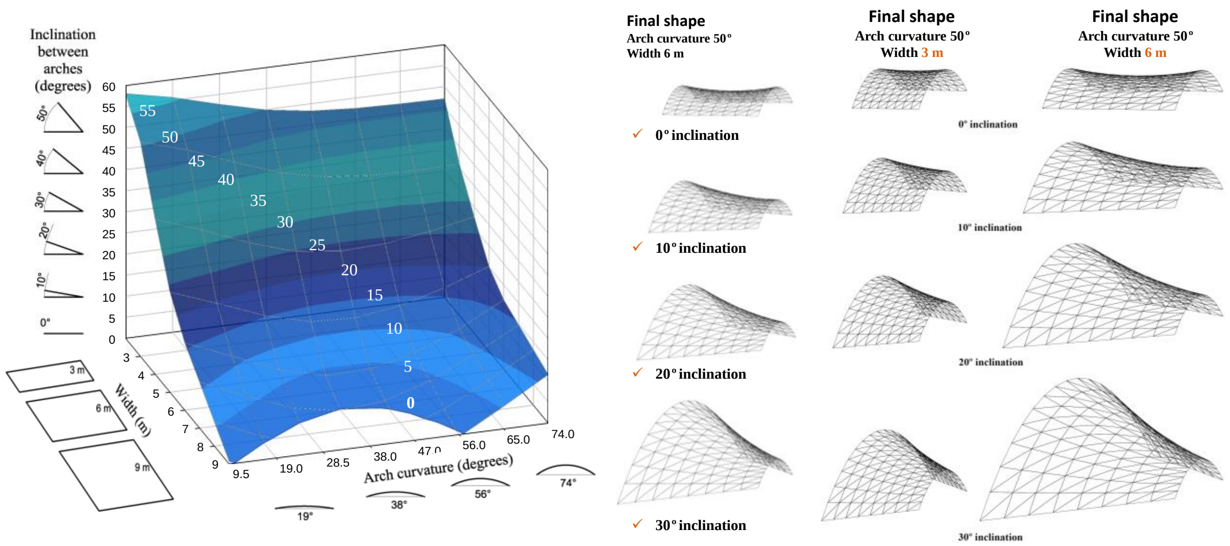

Fig. 6 Design aid for inclined anticlastic membrane tension structures and final design examples

As a result of the parametric study, the following findings were obtained:

1) The proposed trapezoid-shaped membranes have the same limitations as the regular membranes with the same width as that at the mid-span of the trapezoid-shaped membrane, regardless of the opening angle, because the maximum stress is always reached at the mid-span of the membrane.
2) However, the opening angle is limited by the behavior of the structure under downward loading in the fill direction. For the same arch curvature, the increase of the stress under downward loading in the fill direction is linear with the constant rate, and the same rate can be applied to any width.
3) The most critical case for inclined anticlastic membranes is observed under the upward loading in the fill direction. When the inclination angle is increased, the incremental rate of the stress in the fill direction under upward loading differs according to the membrane width.
4) However, the increasing rate is similar for different arch curvatures when the width is constant. This finding indicates that the maximum stress tends

to be reached across the transverse section at the mid-span region, which applies for all membranes regardless of the arch curvature, because the inclination angle is only dependent on the width of the membrane.

3. SUMMARY AND REMARKS

Descriptions for anticlastic tension membrane supported by two nonparallel or asymmetric circular arches and two parallel straight non-equal length edge beams are provided. This membrane is not synclastic but attached to arched and straight edge beams on all four sides. The primary outcomes are somewhat specific, because the initial prestressing force per width is fixed to be 5 kN/m and some other parameters are fixed as well.

The authors have exploited the developed computational program to investigate various cases of regular and irregular anticlastic membranes including trapezoid-shaped or inclined panels, and developed the design aid charts and tables. Such a computational development process should be emphasized, as similar approaches can be taken for further study by the authors and other researchers. The developed design aid itself is also meaningful and is a valuable contribution to the design practice, as it gives general ideas for practicing engineers and helps them get started designing anticlastic tension membrane structures.

REFERENCES

Berger, H. (2012), Creating Architecture with Tensile Membrane Structures, *Proceedings of the International Association for Shell and Spatial Structures 2012: From Spatial Structures to Space Structures*, Seoul, Korea.

Bletzinger, K.-U., Ramm, E. (2001), “Structural Optimization and Form Finding of Light Weight Structures,” *Computers & Structures*, 79, 2053–2062.

Cook, R.D., Malkus, D.S., Plesha, M.E., and Witt, R.J. (2007), Concepts and Applications of Finite Element Analysis, *John Wiley & Sons*.

Gil Pérez, M., Kang, T.H.-K., Sin, I.A., and Kim, S.D. (2016), “Nonlinear Analysis and Design of Membrane Fabric Structures: Modeling Procedure and Case Studies,” *ASCE Journal of Structural Engineering*, 142(11).

Gil Pérez, M., Kim, S.D., and Kang, T.H.-K. (2017), “Development of Design Aid for Barrel Vault Shaped Membrane Fabric Structures,” *Journal of Structural Integrity and Maintenance*, 2(1), 12–21.

Heath, M.T. (2002), “Scientific Computing: A Introductory Survey,” McGraw Hill, New York, NY.

Hong, S., Gil Pérez, M., and Kang, T.H.-K. (2018), “Case Studies of Irregular Anticlastic Membrane Structures with Asymmetry,” *ASCE Journal of Structural Engineering*, 114(8).

Kim, S.-D., Kang, M.-M., Kwun, T.-J., and Hangai, Y. (1997), “Dynamic Instability of Shell-Like Shallow Trusses Considering Damping,” Computers and Structures, 64(1-4), 481–489.