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Sensor Fault Diagnosis for Structural Health Monitoring using Weighted Principal Component Analysis

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Abstract

It is fundamental that the potential faulty sensor be detected, before implementing any structural health monitoring algorithm, to avoid unreliable health-evaluation result. A novel sensor fault detection method, based on the extension of traditional principal component analysis (PCA), is proposed in this paper. It is firstly demonstrated that the fault detection performance of each loading vector in the PCA transformation matrix is different from others for a specific sensor fault. A fault sensitive factor is then derived to quantify the fault detection performances, based on that a weighted fault detection statistic is built for each specific sensor. The Bayesian inference is combined with the weighted statistics to form a probabilistic fault detector. Case studies using a benchmark structure demonstrate that the proposed method is superior to the traditional approach.

Keywords: Weighted principal component analysis; Fault sensitivity; Bayesian inference; Sensor fault detection; Structural health monitoring.

1. INTRODUCTION

Structural health monitoring (SHM) can be applied to evaluate the health conditions of in-service civil structures and then help decision-makers confirm a proper maintainence plan for safe and sustainable operation. However, inaccurate evaluation results may be reflected by false alarms or missed detections if the monitoring data are distorted by various sensor faults. It is therefore necessary to detect potential sensor faults occurred in SHM systems before evaluating the health conditions of monitored structures (Huang *et al*. 2015; Huang *et al*. 2016). The principal component analysis (PCA)-based fault detection method is perhaps the most popular and broadly studied

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one due to its theoretical simplicity and computational efficiency. Nevertheless, the potential drawback is that the two traditional fault detection statistics, i.e., the T^2 and squared prediction error (SPE) statistics, of this technique are not sensitive to small or tiny faults that occur in any measurement variable. This paper presents an innovative method called weighted principal component analysis (WPCA) to establish a new fault detection technique that is more sensitive to sensor faults occurred in SHM systems. The remainder of this article is divided into four primary sections. First, the theoretical background of the PCA-based fault detection technique is reviewed. Second, a fault sensitive factor is deduced, through which the weighted fault detection statistic is established; Bayesian inference theory is employed to combine all the weighted statistics to decide whether there is a fault. Third, benchmark structure studies are used to demonstrate the efficacy. Finally, summaries and conclusions are given in detail.

2. Traditional PCA-based Fault Detection Method

The PCA-based fault detection method, which shows potential for SHM applications, has in recent years been widely studied in the field of industrial process monitoring (Qin 2012). A multidimensional dataset $\boldsymbol{X}=\left[\boldsymbol{x}_1,\boldsymbol{x}_2,...,\boldsymbol{x}_{\mathrm{N}}\right]\in\Re^{m\times n}$ is defined to represent a measurement section of the structural response. Assuming that the output of the monitored system is a zero-mean process, the PCA loading matrix can be obtained from the eigenvalue decomposition of the covariance matrix of the dataset:

$$\boldsymbol{C}=\mathrm{E}\left(\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right)=\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{T}} \tag{1}$$

where $\boldsymbol{C}\in\Re^{m\times m}$ is the covariance matrix, $\mathrm{E}(\cdot)$ represents the expectation operator, $\boldsymbol{\Lambda}=\mathrm{diag}\left(\lambda_1,\lambda_2,...,\lambda_m\right)$ is the diagonal matrix with m eigenvalues of $\boldsymbol{C}$, and $\boldsymbol{U}=\left[\boldsymbol{u}_1,\boldsymbol{u}_2,...,\boldsymbol{u}_m\right]$ denotes the PCA loading matrix with $\boldsymbol{u}_i$ representing the ith loading vector. The PCA transformation for a monitoring data $\boldsymbol{x}$ is represented as:

$$\boldsymbol{t}=\boldsymbol{U}^{\mathrm{T}}\boldsymbol{x} \tag{2}$$

According to the cumulative percent variance criterion (Chiang *et al*. 2001), defined as:

$$\sum_{i=1}^{r}\lambda_i \Bigg/ \sum_{i=1}^{m}\lambda_i\cdot 100\%\geq 85\% \tag{3}$$

the first r loading vectors are used to span principal subspace $\hat{\boldsymbol{U}}=\left[\boldsymbol{u}_1,\boldsymbol{u}_2,...,\boldsymbol{u}_r\right]\in\Re^{m\times r}$, and the last $(m-r)$ loading vectors are correspondingly used to span residual subspace $\tilde{\boldsymbol{U}}=\left[\boldsymbol{u}_{r+1},\boldsymbol{u}_{r+2},...,\boldsymbol{u}_m\right]\in\Re^{m\times(m-r)}$.

Two statistics, i.e., the T^2 statistic and the SPE statistic, can be separately defined on the principal subspace and the residual subspace for any certain monitoring data-point $\boldsymbol{x}$:

$$\mathrm{T}^2=\boldsymbol{x}^{\mathrm{T}}\left(\hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}^{-1}\hat{\boldsymbol{U}}^{\mathrm{T}}\right)\boldsymbol{x} \tag{4a}$$

$$\mathrm{SPE}=\boldsymbol{x}^{\mathrm{T}}\left(\tilde{\boldsymbol{U}}\tilde{\boldsymbol{U}}^{\mathrm{T}}\right)\boldsymbol{x} \tag{4b}$$

where $\hat{\boldsymbol{\Lambda}}=\mathrm{diag}\left(\lambda_1,\lambda_2,...,\lambda_r\right)$ is a diagonal matrix that contains the first r eigenvalues. A fault is judged to occur after one of these two statistics exceeds its corresponding control limit.

3. Establishment of WPCA-based Fault Detection Method

In the traditional PCA-based fault detection method, the fault is detected either by the T^2 statistic or by the SPE statistic. However, each loading vector has a different detection performance for different faults. A new statistic that considers the difference in the fault detection performance should be proposed. The T^2 statistic defined in Eq. (4a) is chosen in this paper and further represented as the following generalized form:

$$\mathrm{T}^2 = \boldsymbol{x}^\mathrm{T}\left(\hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}^{-1}\hat{\boldsymbol{U}}^\mathrm{T}\right)\boldsymbol{x} = \boldsymbol{x}^\mathrm{T}\left(\sum_{i=1}^{r}\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x} = \sum_{i=1}^{r}\boldsymbol{x}^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x} = \sum_{i=1}^{m}c_i\cdot\mathrm{T}_i^2 \tag{5}$$

where T_i^2 represents the T^2 statistic defined on the $i\mathrm{th}$ loading vector:

$$\mathrm{T}_i^2 = \boldsymbol{x}^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x} \tag{6}$$

and c_i represents a coefficient corresponding to T_i^2:

$$\begin{cases} c_i = 1\ , & i = 1,2,...,r \\ c_i = 0\ , & others \end{cases} \tag{7}$$

The weighting coefficient corresponding to each T_i^2 statistic should be determined according to its fault detection performance. Without a loss of generality, the ability of the $i\mathrm{th}$ loading vector to detect the $j\mathrm{th}$ sensor fault is considered. The $j\mathrm{th}$ sensor output after a fault occurs is assumed as the following form:

$$\boldsymbol{x} = \boldsymbol{x}^* + \delta_j\xi_j \tag{8}$$

where $\boldsymbol{x}^*$ represents the true sensor output value, δ_j represents the fault magnitude that occurred in the $j\mathrm{th}$ sensor, and ξ_j represents the fault-direction vector which is the $j\mathrm{th}$ column of an identity matrix:

According to Eq. (6), the T_i^2 statistic after the $j\mathrm{th}$ sensor gains a fault can be written as follows:

$$\mathrm{T}_i^2 = \boldsymbol{x}^{*\mathrm{T}}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x}^* + \left[\xi_j^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\xi_j\right]\delta_j^2 + 2\left[\xi_j^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x}^*\right]\delta_j \tag{9}$$

Therefore, the T_i^2 statistic can be represented as the summation type of two parts:

$$\mathrm{T}_i^2 = \mathrm{T}_i^{2*} + \Delta\mathrm{T}_i^2 \tag{10}$$

where $\mathrm{T}_i^{2*} = \boldsymbol{x}^{*\mathrm{T}}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x}^*$ represents the T_i^2 statistic of the monitoring data point without a sensor fault, and $\Delta\mathrm{T}_i^2 = \left[\xi_j^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\xi_j\right]\delta_j^2 + 2\left[\xi_j^\mathrm{T}\left(\boldsymbol{u}_i\lambda_i^{-1}\boldsymbol{u}_i^\mathrm{T}\right)\boldsymbol{x}^*\right]\delta_j$ represents an increment of the T_i^2 statistic caused by the sensor fault. Increment term $\Delta\mathrm{T}_i^2$ should be given more attention because its value actually decides whether a fault can be detected by the T_i^2 statistic.

The relationships $\boldsymbol{u}_i^\mathrm{T}\xi_j = \xi_j^\mathrm{T}\boldsymbol{u}_i = u_{ji}$ and $\boldsymbol{u}_i^\mathrm{T}\boldsymbol{x}^* = t_i^*$ is easy to obtain, where u_{ji} represents the $j\mathrm{th}$ row and $i\mathrm{th}$ column element of PCA loading matrix $\boldsymbol{U}$, and t_i^* is the projection of $\boldsymbol{x}^*$ onto the $i\mathrm{th}$ loading vector. The average increment $\mathrm{E}\left(\Delta\mathrm{T}_i^2\right)$ can further be represented as follows:

$$\mathrm{E}\left(\Delta \mathrm{T}_i^2\right)=\left(\frac{u_{ji}^2}{\lambda_i}\right)\delta_j^2+2\left(\frac{u_{ji}}{\lambda_i}\right)\mathrm{E}\left(t_i^*\right)\delta_j \tag{11}$$

It is easily obtained that $\mathrm{E}\left(t_i^*\right)=\boldsymbol{u}_i^{\mathrm{T}}\mathrm{E}\left(\boldsymbol{x}^*\right)=0$. Therefore, the expectation of $\Delta \mathrm{T}_i^2$ can be written as:

$$\mathrm{E}\left(\Delta \mathrm{T}_i^2\right)=\left(\frac{u_{ji}^2}{\lambda_i}\right)\delta_j^2 \tag{12}$$

A fault-sensitive factor can then be defined as:

$$f_{j,i}=\frac{u_{ji}^2}{\lambda_i} \tag{13}$$

When $f_{j,i}$ becomes larger, the fault that occurs in the jth sensor is more easily detected by the T_i^2 statistic.

A weighted summation type determined according to the fault sensitivity is preferable for detecting a certain sensor fault. The fault sensitive factors can be assembled into a column vector $\boldsymbol{f}_j=\left[f_{j,1},f_{j,2},\ldots,f_{j,m}\right]^{\mathrm{T}}\in\Re^m$, and all elements of this vector can be standardized into the interval $[-1,1]$ using the following equation:

$$f_{j,i}^{st}=\frac{f_{j,i}-f_{j,\mathrm{mid}}}{\frac{1}{2}\left(f_{j,\max}-f_{j,\min}\right)} \tag{14}$$

where $f_{j,\max}$ is the maximum element of $\boldsymbol{f}_j$, $f_{j,\min}$ is the minimum, and $f_{j,\mathrm{mid}}$ represents the middle value of $f_{j,\max}$ and $f_{j,\min}$. The Sigmoid function is used to determine the weighting coefficient:

$$S(\tau)=\frac{1}{1+e^{-a\cdot\tau}} \tag{15}$$

where τ represents a variable, and a is a parameter to be confirmed. For the jth sensor fault, the weighting coefficient corresponding to the ith loading vector is therefore determined as:

$$w_{j,i}=\frac{S\left(f_{j,i}^{st}\right)}{\sum_{i=1}^m S\left(f_{j,i}^{st}\right)} \tag{16}$$

The generalized T^2 statistic, i.e., the WPCA-based fault detection statistic, corresponding to the jth sensor fault, is then represented as the following equation:

$$\tilde{\mathrm{T}}_j^2=\sum_{i=1}^m w_{j,i}\cdot\mathrm{T}_i^2 \tag{17}$$

When a fault occurs in the jth sensor, weighted statistic $\tilde{\mathrm{T}}_j^2$ is more fault sensitive. If all T_i^2 statistics ($i=1,2,\ldots,m$) are assembled into a column vector as follows:

$$\boldsymbol{\psi}=\left[\mathrm{T}_1^2,\mathrm{T}_2^2,\ldots,\mathrm{T}_m^2\right]^{\mathrm{T}} \tag{18}$$

Eq. (17) is then written as:

$$\tilde{\mathrm{T}}_j^2=\boldsymbol{w}_j^{\mathrm{T}}\boldsymbol{\psi} \tag{19}$$

where $\boldsymbol{w}_j=\left[w_{j,1},w_{j,2},\ldots,w_{j,m}\right]^{\mathrm{T}}\in\Re^m$ is the weighting vector corresponding to the jth sensor. And if all weighted $\tilde{\mathrm{T}}_j^2$ statistics ($j=1,2,...,m$) are assembled into a column vector:

$$\boldsymbol{\Theta}=\left[\tilde{\mathrm{T}}_1^2,\tilde{\mathrm{T}}_2^2,...,\tilde{\mathrm{T}}_m^2\right]^{\mathrm{T}} \tag{20}$$

the computational formula of $\boldsymbol{\Theta}$ in the matrix production form is then represented as:

$$\boldsymbol{\Theta}=\left[\boldsymbol{w}_1^{\mathrm{T}}\boldsymbol{\psi},\boldsymbol{w}_2^{\mathrm{T}}\boldsymbol{\psi},...,\boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{\psi}\right]^{\mathrm{T}}=\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\psi} \tag{21}$$

where $\boldsymbol{W}=\left[\boldsymbol{w}_1,\boldsymbol{w}_2,...,\boldsymbol{w}_m\right]\in\Re^{m\times m}$ is the weighting matrix composed of all m weighting vectors.

To judge expediently whether there is a fault occurred in the sensor network, this section employs Bayesian inference theory (Bishop 2006) to combine all $\tilde{\mathrm{T}}_j^2$ statistics in vector $\boldsymbol{\Theta}$ into a single fault detection statistic from the probabilistic viewpoint. The posterior fault probability under $\tilde{\mathrm{T}}_j^2$ is given as:

$$P\left(\mathrm{F}\middle|\tilde{\mathrm{T}}_j^2\right)=\frac{P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{F}\right)P(\mathrm{F})}{P\left(\tilde{\mathrm{T}}_j^2\right)} \tag{22}$$

According to the law of total probability, the following equation is obtained:

$$P\left(\tilde{\mathrm{T}}_j^2\right)=P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{N}\right)P(\mathrm{N})+P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{F}\right)P(\mathrm{F}) \tag{23}$$

where symbols N and F, respectively, represent the normal and fault conditions of the sensor network. If the significance level is α, the prior probability of the normal and fault conditions of the sensor network is determined as: $P(\mathrm{N})=1-\alpha$ and $P(\mathrm{F})=\alpha$. The conditional probability of $\tilde{\mathrm{T}}_j^2$ under the normal or fault condition can be defined as follows:

$$P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{N}\right)=\exp\left(-\frac{\tilde{\mathrm{T}}_j^2}{v\tilde{\mathrm{T}}_{j,\lim}^2}\right) \tag{24a}$$

$$P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{F}\right)=\exp\left(-\frac{\tilde{\mathrm{T}}_{j,\lim}^2}{v\tilde{\mathrm{T}}_j^2}\right) \tag{24b}$$

where $\tilde{\mathrm{T}}_{j,\lim}^2$ is the control limit of the $\tilde{\mathrm{T}}_j^2$ statistic, and v is a turning parameter that is generally set to 1.

Therefore, the posterior fault probability under all elements of $\boldsymbol{\Theta}$ is given; i.e., the fault detection statistic based on Bayesian inference theory can be synthetically defined as the following weighted form:

$$P\left(\mathrm{F}\middle|\boldsymbol{\Theta}\right)=\sum_{j=1}^{m}\left\{\frac{P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{F}\right)}{\sum_{j=1}^{m}P\left(\tilde{\mathrm{T}}_j^2\middle|\mathrm{F}\right)}\cdot P\left(\mathrm{F}\middle|\tilde{\mathrm{T}}_j^2\right)\right\} \tag{25}$$

When the posterior fault probability $P\left(\mathrm{F}\middle|\boldsymbol{\Theta}\right)$ exceeds the original fault probability level of the sensor network, i.e., α, a fault can be judged to occur in the sensor network.

3. Case Study

To evaluate the efficacy of the proposed sensor fault detection method, a benchmark problem for SHM (Caicedo *et al*. 2004) is chosen in this section. The benchmark structure was established by the IASC-ASCE Structural Health Monitoring Task Group to enable researchers worldwide to compare the efficiencies of various SHM algorithms. The benchmark model has been created as a MATLAB software through which the structural response data could be obtained. There are 16 accelerometers installed to the benchmark structure to measure the acceleration responses. There are generally two types of sensor fault modes, i.e., the bias and gain, which are the typical modes of the additive and multiplicative sensor faults according to Abdelghani and Friswell (2004; 2007). Both of these sensor fault modes were simulated.

Table 1 AUC values of the three statistics for the bias fault

Sensor #	AUC values		
	T^2	SPE	$P(\mathrm{F}\|\Theta)$
3	0.5000	0.6328	**0.9843**
4	0.5001	0.6064	**0.9778**
7	0.5000	0.6438	**0.9893**
8	0.5000	0.6338	**0.9835**

There are 4 sensors, i.e., sensors 3, 4, 7 and 8, to be studied in this section. The bias fault type is simulated for demonstration. The ROC curve technique (Lu *et al*. 2009) is used to evaluate the fault detection performances of the three statistics. To quantify the fault detection performance of a statistic, the area under ROC curve (AUC) value is always employed. When the AUC value is equal to 0.5, the fault detection performance of the statistic gets worst; when the AUC value is equal to 1.0, the fault detection performance of the statistic gets best. Table 1 shows the AUC values of the three statistics for the bias fault, which indicate that the fault detection performance of the new proposed fault detection statistic is preferable to the traditional statistics.

The gain fault type is simulated to demonstrate the efficacy. Table 2 shows the AUC values of the three statistics for the gain fault. It is concluded that the fault detection performance of the new proposed statistic is preferable to that of the traditional ones.

Table 2 AUC values of the three statistics for the gain fault

Sensor #	AUC values		
	T^2	SPE	$P(\mathrm{F}\|\Theta)$
3	0.5058	0.6319	**0.8248**
4	0.5154	0.6053	**0.8170**
7	0.5082	0.6373	**0.8376**
8	0.5013	0.6379	**0.8231**

4. Conclusions

It is fundamental to detect various types of sensor faults before applying any SHM algorithm to assess the current health conditions of monitored structures. This paper proposed an innovative sensor fault diagnosis method to improve the traditional PCA-based approach with an application to SHM. The fault sensitivity of the statistic corresponding to each loading vector of PCA remained different for a specific sensor fault. A weighted summation type according to the fault sensitivity of each loading vector is preferable for detecting the specific sensor fault. To quantify the fault sensitivity, a fault sensitive factor was deduced from the statistical perspective, and a weighted fault detection statistic was then established. Bayesian inference theory was applied to integrate all weighted statistics corresponding to each sensor to form a probabilistic fault detector. Case study employing benchmark structure was considered and the bias or gain fault type were carried out in this research. The comparison results utilizing the ROC curve technique showed that the fault detection performance of the new proposed statistic was the best.

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