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Approximate equivalent SDOF system of MDOF structure with negative stiffness

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ABSTRACT

Negative stiffness (NS) can reduce the structural base shear, which is useful for structural vibration suppression. Therefore, it is necessary to consider NS effect and estimate the response of the structure with NS. This paper presents a new approximate method to convert a multi degree of freedom (MDOF) structure with NS in the 1st story (MNS1) to an approximate single degree of freedom (SDOF) bilinear elastic system (ASBS). The accuracy of approximation from the MNS1 to the ASBS is studied by the sensitivity method of mode shapes in which the relative changes in structural mode shapes are related to the variation of stiffness. An eight-story numerical example is employed to illustrate the accuracy of approximation by the proposed method.

1. INTRODUCTION

For the earthquake engineering, the most important function of structural stiffness is to resist ambient excitation and prevent structural inter-story deformation. Although the larger stiffness can resist the more external force, the inter-story shear force would be increased with the increasing of stiffness(Tsonos, 2014, Kalogeropoulos and Tsonos, 2014). The concept of stiffness weakening was proposed by Reinhorn et al. (2005) and Viti et al. (2006), in which the internal force or acceleration is attenuated by the reducing of structural stiffness. However, if the structure is actually weakened by reducing the strength of beams (or in some cases beams and columns), it can result in early yielding and permanent deformation, which would be very dangerous for structural safety. Nagarajaiah et al. (2010) proposed the "apparent weakening" that mimics the early "yielding" without inducing the structural inelastic behavior, and makes the structure remain elastic.

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In order to make sure the safety of a structure, the structural response should be estimated during structural design. Due to the advantage of NS to reduce the structural internal force, it is necessary to consider the NS property during estimating structural response. In civil engineering, many building codes adopted equivalent single-degree-of-freedom (SDOF) systems to evaluate the overall performance of the structure by mode superposition method (Chopra, 2001). However, a MDOF structure with NS that exhibits nonlinear property is hard to be transferred to an equivalent SDOF system.
This paper presents a new approximate method to convert the bilinear elastic MDOF to an equivalent SDOF. The MDOF structure with the NS only in the 1st story (MNS1) is considered, which is quite effective in reducing the base shear and displacement. Based on the mode superposition method, the conversion is derived to obtain the approximate SDOF bilinear elastic system (ASBS). An eight-story numerical example is employed to illustrate the accuracy of approximation by the proposed method.

2. MODEL DEFINITION

Different from the restoring force provided by the positive structural stiffness against the deformation, the negative stiffness (NS) “restoring force” assists deformation. However, it would be helpful to guarantee the stability of structures to maintain the original stiffness under some small excitation (Pasala et al., 2013). Thus, the practical negative stiffness will have the force-deformation relation as the middle figure in Fig. 1.

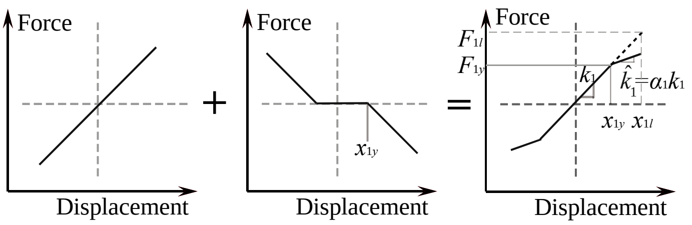

Fig. 1 Combination of the positive stiffness and the negative stiffness with gap results in bilinear stiffness.

Considering a MDOF shear-frame structure with NS in the 1st story (MNS1), the dynamics can be described as follows,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{F}(R_{1y}, \alpha_1, x_1) = -\mathbf{M}\mathbf{l}\ddot{x}_g \tag{1}$$

where $\mathbf{M}, \mathbf{C} \in \Re^{n \times n}$ are the mass, damping matrices of the MNS1, respectively; the parameters R_{1y}, α_1 and x_1, which are the “yielding” strength reduction factor, stiffness degradation, and the displacement for the 1st floor, respectively. These parameters as shown in Fig. 1 can be expressed by,

$$R_{1y} = \frac{F_{1l}}{F_{1y}} = \frac{x_{1l}}{x_{1y}} \tag{2}$$

$$\alpha_1 = \frac{\hat{k}_1}{k_1} \tag{3}$$

where x_{1l} (or F_{1l}) is the peak value of the 1st floor deformation (or restoring force); x_{1y} (or F_{1y}) is the first floor "yielding" deformation ("yielding" restoring force) and $\hat{k}_1$ is the stiffness of structure after "yielding". **F** in Eq.(1). is the restoring force vector, which can be expressed in a piecewise formulation,

$$\mathbf{F}(R_{1y},\alpha_1,x_1)=\begin{cases}\mathbf{Kx}, & if\ x_1\le x_{1y}\\ \mathbf{Kx}_y+\hat{\mathbf{K}}\left(\mathbf{x}-\mathbf{x}_y\right), & if\ x_1>x_{1y}\end{cases} \tag{4}$$

where $\mathbf{x}_y$, **K** and $\hat{\mathbf{K}}$ represents the structural relative displacement vector, stiffness matrix for linear elastic structure and the stiffness matrix when x_1 reaches the "yielding" displacement x_{1y}.

3. CONVERSION FROM MNS1 TO ASBS

For the linear elastic structure with the matrix parameters **M**, **C** and **K**, the natural frequencies and mode shapes can be expressed as ω_1, ω_2, …, ω_n, and $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$,…, $\mathbf{\Phi}_n$. The co-ordinate q_1, q_2,…, q_n are governed by,

$$\ddot{q}_i+2\zeta_i\omega_i\dot{q}_i+\omega_i^2q_i=-\Gamma_i\ddot{x}_g\left(t\right) \tag{5}$$

where ζ_i is the damping ratio for the i-th mode and Γ_i is expressed by,

$$\Gamma_i=\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{Ml}}{M_i} \tag{6}$$

$$M_i=\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M\Phi}_i \tag{7}$$

For the structure with the stiffness $\hat{\mathbf{K}}$ in Eq.(5), the natural frequencies and mode shapes will be denoted as $\hat{\omega}_1$, $\hat{\omega}_2$,…, $\hat{\omega}_n$, and $\mathbf{\Psi}_1$,$\mathbf{\Psi}_2$,…, $\mathbf{\Psi}_n$. The co-ordinates are denoted as p_1, p_2,…, p_n. Thus, in the bilinear elastic structure in Eq.(1), the mode expansion of displacement **x** has the form as the following,

$$\begin{aligned}\mathbf{x}=\sum_{i=1}^{n}\left(\mathbf{\Phi}_iq_i+\mathbf{\Psi}_ip_i\right)=\mathbf{\Phi}_1q_1+\mathbf{\Phi}_2q_2+\\ \ldots+\mathbf{\Phi}_nq_n+\mathbf{\Psi}_1p_1+\mathbf{\Psi}_2p_2+\ldots+\mathbf{\Psi}_np_n\end{aligned} \tag{8}$$

As the derivation of Eq.(5) based on the superposition (Chopra, 2001), substituting Eq.(8) into Eq.(1), pre-multiplying by $\mathbf{\Phi}_i$, and then is dividing both sides by M_i gives,

$$\ddot{q}_i+\Xi_m+2\zeta_i\omega_i\dot{q}_i+\Xi_c+\omega_i^2q_i+\Xi_k=-\Gamma_i\ddot{x}_g \tag{9}$$

where

$$\Xi_m=\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M\Psi}_1}{M_i}\ddot{p}_1+\ldots+\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M\Psi}_j}{M_i}\ddot{p}_j+\ldots+\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{M\Psi}_n}{M_i}\ddot{p}_n \tag{10a}$$

$$\Xi_c = \frac{\mathbf{\Phi}_i^{\mathrm{T}} \mathbf{C} \mathbf{\Psi}_1}{M_i} \dot{p}_1 + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}} \mathbf{C} \mathbf{\Psi}_j}{M_i} \dot{p}_j + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}} \mathbf{C} \mathbf{\Psi}_n}{M_i} \dot{p}_n \tag{10b}$$

$$\Xi_k = \frac{\mathbf{\Phi}_i^{\mathrm{T}} \hat{\mathbf{K}} \mathbf{\Psi}_1}{M_i} p_1 + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}} \hat{\mathbf{K}} \mathbf{\Psi}_j}{M_i} p_j + \ldots + \frac{\mathbf{\Phi}_i^{\mathrm{T}} \hat{\mathbf{K}} \mathbf{\Psi}_n}{M_i} p_n \tag{10c}$$

The coupling phenomenon of co-ordinates q_i and p_i is hidden in Eqs.(10). Neglecting the coupling co-ordinates in Eq.(10) results in the following,

$$\Xi_m \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}} \mathbf{M} \mathbf{\Psi}_i}{M_i} \ddot{p}_i = \ddot{p}_i \tag{11a}$$

$$\Xi_c \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}} \mathbf{C} \mathbf{\Psi}_i}{M_i} \dot{p}_i = 2\zeta_i \omega_i \dot{p}_i \tag{11b}$$

$$\Xi_k \approx \frac{\mathbf{\Phi}_i^{\mathrm{T}} \hat{\mathbf{K}} \mathbf{\Psi}_i}{M_i} p_i = \hat{\omega}_i^2 p_i \tag{11c}$$

According to the expression Eq.(1), neglecting the coupling co-ordinates gives the controlling equation of ASBS as,

$$\ddot{\tilde{q}}_i + 2\zeta_i \omega_i \dot{\tilde{q}}_i + f_i \left(r_{iy}, \lambda_{id}, \tilde{q}_i \right) = -\Gamma_i \ddot{x}_g \tag{12}$$

where the new co-ordinate $\tilde{q}_i$ for the bilinear elastic system is equal to p_i+q_i; the factors of r_{1y}, λ_{1d} like the parameters R_{1y}, α_1 defined in Eq.(2) and Eq.(3) which are the "yielding" strength reduction factor, frequency degradation factor for the ASBS, respectively, and can be expressed by,

$$r_{iy} = \frac{q_{il}}{q_{iy}} = \frac{f_{il}}{f_{iy}} \tag{13}$$

$$\lambda_{id} = \frac{\hat{\omega}_i^2}{\omega_i^2} \tag{14}$$

where q_{il} (or f_{il}) is the maximal co-ordinate value (or restoring force) of the linear elastic system; q_{iy} (or f_{iy}) is the "yielding" co-ordinate value (or restoring force), which has the following relation with x_{1y} in Eq.(2),

$$\phi_{1i} q_{iy} = x_{1y} \tag{15}$$

where ϕ_{1i} is the 1st element of $\mathbf{\Phi}_i$. So the restoring force f_i in Eq. (2) can be expressed by,

$$f_i\left(r_{iy},\lambda_{id},\tilde{q}_i\right)=\begin{cases}\omega_i^2\tilde{q}_i,\ \ if\ \tilde{q}_i\le q_{iy}\\ \omega_i^2 q_{iy}+\hat{\omega}_i^2\left(\tilde{q}_i-q_{iy}\right),\ \ if\ \tilde{q}_i>q_{iy}\end{cases} \tag{16}$$

4. NUMERICAL VALIDATION

In order to illustrate the accuracy for the approximations, Eqs.(11), an eight-story structure example is employed (Li et al., 2015). The damping ratio for this structure is assumed to be 5%. This paper considers that the additional negative stiffness property only exists in the 1st story, and other stories maintain the linear elastic property.
The MNS1 response can be estimated by the ASBS in Eq.(12). Due to the approximate equations Eqs.(11), the ASBS would lead to some errors in the response of the MNS1. This section will analyze the accuracy. The 1st floor displacement and base shear are considered, which can be estimated by,

$$\tilde{x}_{ji}=\begin{cases}\tilde{q}_i\Phi_{ji},\ \ if\ \tilde{q}_i\le q_{iy}\\ \left(\tilde{q}_i-q_{iy}\right)\Psi_{ji}+q_{iy}\Phi_{ji},\ \ if\ \tilde{q}_i>q_{iy}\end{cases} \tag{17}$$

$$\tilde{V}_i=\begin{cases}\mathbf{E}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_i\tilde{q}_i,\ \ if\ \tilde{q}_i\le q_{iy}\\ \mathbf{E}^{\mathrm{T}}\hat{\mathbf{K}}\mathbf{\Psi}_i\left(\tilde{q}_i-q_{iy}\right)+\mathbf{E}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}_i q_{iy},\ \ if\ \tilde{q}_i>q_{iy}\end{cases} \tag{18}$$

where *i* represents the *i*-th mode; *j* is the *j*-th floor; $\tilde{x}_{ji}$ and $\tilde{V}_i$ are the estimated *j*-th floor displacement and base shear under *i*-th mode, respectively.

The combined displacement and base shear for the first three modes are compared with the real response in Fig. 2.

(a) (b)

Fig. 2 Comparisons of real and estimated displacement on 1st floor and base shear

In Fig. 2, the errors of peak values for the 1st floor displacement and the base shear are 5.43% and 0.92%, respectively. Errors from the evaluation by Eq.(12) are due to the approximation Eqs.(11), which means that the approximation in Eq. (11) is reasonable.

5. CONCLUSIONS

This paper develops a new method using the single degree of freedom bilinear elastic system (ASBS) to approximately estimate the response of the multi degree of freedom (MDOF) structure with the negative stiffness (NS) in the 1st story (MNS1) based on mode superposition method, which neglects the coupling phenomenon in the co-ordinates.

An eight-story numerical example is employed to illustrate the accuracy of approximation of the proposed method. The results shows that the approximate procedure provides good estimates of floor displacements and base shears.

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