*The 2016 World Congress on*
***The 2016 Structures Congress (Structures16)***
*Jeju Island, Korea, August 28-September 1, 2016*

# Parametric Kalman Filter with Unknown Inputs for Probabilistic Damage Detection of Uncertain Structures under Unknown Input

*Lijun Liu[1)], Ying Lei[4)], Longfei Wang[2)] and Han Su[3)]

[1), 2), 3), 4)] *School of Architecture and Civil Engineering, Xiamen University, China*
[4)] *ylei@xmu.edu.cn*

## ABSTRACT

System identification and damage detection for structural health monitoring have received considerable attention. Time domain analysis methodologies based on measured vibration data, such as the recursive least-squares estimation or parametric Kalman filter, have been studied and shown to be useful. However, the traditional parametric Kalman filter approach requires that all the external excitation (input) data be available. On the other hand, structural uncertainties are inevitable for civil infrastructures, it is necessary to develop approaches for probabilistic damage detection of structures with uncertainties. In this paper, a parametric Kalman filter with unknown inputs is proposed for the simultaneous identification of structural parameters and the unmeasured external inputs. Analytical recursive solutions for the proposed parametric Kalman filter with unknown inputs are derived based on the traditional parametric Kalman filter approach. Then, it is used for probabilistic damage detection of structures by considering the uncertainties of structural parameters. The damage index and the damage probability are derived from the statistical values of the identified structural parameters of intact and damaged structure. Some numerical examples are used for validating the proposed approach.

## 1. INTRODUCTION

One of the important tasks of structural health monitoring (SHM) is to identify the state of the structures and to detect structural damage for the reliability and safety of structures. An early detection of local damages in structures will be really important for evaluating the reliability and safety of structures. Therefore, based on measured vibration data, the detection of structural damage has been received considerable attention recently. When a structure is damaged, such as cracking in a certain structural element, the stiffness of the damaged component will be reduced. So, the variations of structural parameters could indicate the structural damage. Hence, identifications of structural parameters and the tracking of their variations due to

---

[1)] Assistant Professor
[2), 3)] Master Student
[4)] Professor, Corresponding author

damages are important objectives of structural health monitoring (Yang *et al*.2007).

Literature reviews on system identification and damage detection have been available (Zhong *et al*.2003, Jiang *et al*.2011 ). Various approaches in time domain analyses have been developed, such as the methods of least-squares estimation (Yang *et al*.2004,2005), sequential nonlinear least-squares estimation (Yang *et al*.2006a), the extended Kalman filter (Yang *et al*.2006b), the finite element model updating and structural damage identification based on OMAX data (operational modal analysis) with eXogenous forces (Reynders *et al*.2010), response surface metamodels for structural damage identification (Rutherforda *et al*. 2005 and Fang *et al*.2011), a two-stage Kalman estimation approach for the identification of nonlinear structural parameters (Lei *et al*. 2011). However, these approaches above are only applicable when the information of external excitations to structures is available. In practice, it is difficult or even impossible to directly measure all external excitations on the structures, especially dynamic load. The information of external excitation is important in SHM. So, it is necessary to develop the algorithms for the structural damage detection with unknown external excitations.

There have been some approaches proposed for simultaneous identification of structural damage and unknown external excitations in the last two decades, e.g, numerical iterative procedures based on the classical least squares estimation or extended Kalman filter for identification of the constant structural parameters (Haldar *et al*. 1994,1997,2004 ), the recursive least squares estimation with unknown inputs (RLSE-UI) approach for damage identification of structures (Yang *et al*.2007) However, the derivations of these approaches are quite involved, e.g., the mathematical derivations of the refereed RLSE-UI by Yang *et al* (2007) were presented in both the paper txt and the Appendix with many page spaces. But the final recursive estimation expressions are analogous to those of the parameter Kalman filter (PKF), which implies the direct extension of PKF for simultaneous identification of structural damage and unknown external excitations.

Detection of structural damage in civil engineering involves a large number of uncertainties which result from environment measurement noise, modeling error, and uncertainties in structures. These errors and uncertainties can result in mistake or low accuracy in damage detection. So, the uncertainties in structures limit the successful use of those deterministic damage detection methods. Some approaches with consideration of the uncertainties have been developed for structural damage detection. However, it is necessary to develop the algorithms with consideration of the uncertainties in structures for the structural damage detection with unknown external excitations.

In this paper, a parametric Kalman Filter with unknown input (PKF-UI) for probabilistic damage detection of structures with uncertainties under unknown input is investigated. The approach is a direct extension of the conventional parametric Kalman Filter. Finally, some numerical examples are used to demonstrate and validate the performances of proposed PKF-UI for simultaneous identification of uncertain structural damage and unknown external excitations, respectively.

## 2. THE Parametric Kalman Filter with unknown Input (PKF-UI)

When external inputs to a structure are unknown, the equation of motion can be written by

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(\mathrm{t}) + \boldsymbol{F}\left[\boldsymbol{x}(\mathrm{t}), \dot{\boldsymbol{x}}(\mathrm{t}), \boldsymbol{\theta}\right] = \boldsymbol{\eta}^u \boldsymbol{f}^u(\mathrm{t}) \tag{1}$$

where $\ddot{\boldsymbol{x}}$ $\dot{\boldsymbol{x}}$ and $\boldsymbol{x}$ are vectors of structural acceleration, velocity and displacement respectively, $\boldsymbol{\theta}$ is a *m*-dimensional time-invariant parametric vector involving unknown parameters to be estimated, including stiffness, damping, and nonlinear parameters, $\boldsymbol{F}\left[\boldsymbol{x}(\mathrm{t}), \dot{\boldsymbol{x}}(\mathrm{t}), \boldsymbol{\theta}\right]$ is a force vector which can be linear or nonlinear function of the displacements, velocities and the structural parameters, $\boldsymbol{f}^u(\mathrm{t})$ is an unmeasured *p*-dimensional external excitations vector, and $\boldsymbol{\eta}^u$ is the corresponding influence matrix associated with the unknown external excitations $\boldsymbol{f}^u(\mathrm{t})$.

When all structural responses are observed, the observation equation associated with the equation of motion in Eq.(1) can be expressed as

$$\boldsymbol{y}(\mathrm{t}) = \boldsymbol{\varphi}\left[\boldsymbol{x}(\mathrm{t}), \dot{\boldsymbol{x}}(\mathrm{t})\right]\boldsymbol{\theta} + \boldsymbol{\eta}^u \boldsymbol{f}^u(\mathrm{t}) + \boldsymbol{\upsilon}(\mathrm{t}) \tag{2}$$

in which $\boldsymbol{y}(\mathrm{t}) = \boldsymbol{\eta}^u \boldsymbol{f}^u(\mathrm{t}) - \boldsymbol{M}\ddot{\boldsymbol{x}}(\mathrm{t})$, $\boldsymbol{\varphi}$[ ] is the observation matrix composed of the system response vectors, and $\boldsymbol{\upsilon}(\mathrm{t})$ is a measurement noise vector, which is assumed a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{R}$(t).

The discrete equation for the observation equation in Eq.(2) is expressed as

$$\boldsymbol{y}_{\mathrm{k}+1} = \boldsymbol{\varphi}_{\mathrm{k}+1}\boldsymbol{\theta} + \boldsymbol{\eta}^u \boldsymbol{f}^u_{k+1} + \boldsymbol{\upsilon}_{k+1} \tag{3}$$

For a structure with time-invariant parameters, it is know that

$$\dot{\theta} = 0 \tag{4}$$

Based on Eqs.(3) - (4), the structural unknown parametric vector $\boldsymbol{\theta}$ can be recursively estimated by the conventional KF as follows,

$$\hat{\boldsymbol{\theta}}_{k+1|k+1} = \hat{\boldsymbol{\theta}}_{k|k} + \boldsymbol{K}_{k+1}(\boldsymbol{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_{k|k} - \boldsymbol{\eta}^u \hat{\boldsymbol{f}}^u_{k+1|k+1}) \tag{5}$$

where $\hat{\boldsymbol{\theta}}_{k|k}$ denotes the estimated parametric vector at time $t = k\Delta t$, $\hat{\boldsymbol{\theta}}_{k+1|k+1}$ and $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ denotes the estimated values of estimated parametric vector $\boldsymbol{\theta}$ and unknown external excitations vector $\boldsymbol{f}^u(\mathrm{t})$ □at time $t = (k+1)\Delta t$. $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix at time $t = (k+1)\Delta t$.

Under the condition that the number of response measurements (sensors) is not less than the number of unknown external excitations, $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ can be estimated by minimizing the following error vector as,

$$\Delta_{k+1} = \boldsymbol{y}_{k+1} - \varphi_{k+1}\hat{\boldsymbol{\theta}}_{k+1|k+1} - \eta^{u}\hat{\boldsymbol{f}}^{u}_{k+1|k+1} \tag{6}$$

By inserting the expression of $\hat{\theta}_{k+1|k+1}$ in Eq.(5) into the above error vector in Eq.(6), $\Delta_{k+1}$ can be rewritten by

$$\Delta_{k+1} = \left(\mathbf{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\left(\boldsymbol{y}_{k+1} - \varphi_{k+1}\hat{\theta}_{k|k}\right) - \left(\mathbf{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\eta^{u}\hat{\boldsymbol{f}}^{u}_{k+1|k+1} \tag{7}$$

Then, $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ can be estimated from above Eq.(7) by least-squares estimation as

$$\hat{\boldsymbol{f}}^{u}_{k+1|k+1} = \boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\left(\boldsymbol{Y}_{k+1} - \varphi_{k+1}\hat{\boldsymbol{\theta}}_{k|k}\right) \tag{8}$$

where $\boldsymbol{S}_{k+1} = \left[\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\eta^{u}\right]^{-1}$

By inserting Eq.(3) into Eq.(8), the error of the estimated $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ is given by

$$\hat{\boldsymbol{e}}^{f}_{k+1|k+1} = \boldsymbol{f}^{u}_{k+1} - \hat{\boldsymbol{f}}^{u}_{k+1|k+1} = \boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\left(\varphi_{k+1}\hat{\boldsymbol{e}}^{\theta}_{k|k} + \boldsymbol{v}_{k+1}\right) \tag{9}$$

in which $\hat{\boldsymbol{e}}^{\theta}_{k|k} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k|k}$

Based on Eq.(3) and Eq.(5), it is known that the error $\hat{\boldsymbol{e}}^{\theta}_{k+1|k+1}$ can be estimated by

$$\hat{\boldsymbol{e}}^{\theta}_{k+1|k+1} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{k+1|k+1} = \hat{\boldsymbol{e}}^{\theta}_{k|k} - \boldsymbol{K}_{k+1}\left(\varphi_{k+1}\hat{\boldsymbol{e}}^{\theta}_{k|k} + \eta^{u}\hat{\boldsymbol{e}}^{f}_{k+1|k+1} + \boldsymbol{v}_{k+1}\right) \tag{10}$$

By inserting $\hat{\boldsymbol{e}}^{f}_{k+1|k+1}$ in Eq.(9) into $\hat{\boldsymbol{e}}^{\theta}_{k+1|k+1}$ in Eq.(10), $\hat{\boldsymbol{e}}^{\theta}_{k+1|k+1}$ can be expressed by:

$$\hat{\boldsymbol{e}}^{\theta}_{k+1|k+1} = \left(\boldsymbol{I}_l + \boldsymbol{K}_{k+1}\eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\varphi_{k+1}\right)\left(\boldsymbol{I}_l - \boldsymbol{K}_{k+1}\varphi_{k+1}\right)\hat{\boldsymbol{e}}^{\theta}_{k|k} - \boldsymbol{K}_{k+1}\left[\boldsymbol{I}_l - \eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\right]\boldsymbol{v}_{k+1} \tag{11}$$

Then, the covariance matrix $\hat{\boldsymbol{P}}^{\theta}_{k+1|k+1}$ is derived by

$$\begin{aligned}\hat{\boldsymbol{P}}^{\theta}_{k+1|k+1} = &\left(\boldsymbol{I}_l + \boldsymbol{K}_{k+1}\eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\varphi_{k+1}\right)\left(\boldsymbol{I} - \boldsymbol{K}_{k+1}\varphi_{k+1}\right)\hat{\boldsymbol{P}}^{\theta}_{k|k}\left(\boldsymbol{I} - \boldsymbol{K}_{k+1}\varphi_{k+1}\right)^{T}\left(\boldsymbol{I}_l + \boldsymbol{K}_{k+1}\eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\varphi_{k+1}\right)^{T}\\ &+ \boldsymbol{K}_{k+1}\left[\boldsymbol{I}_l - \eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\right]\boldsymbol{R}_{k+1}\left[\boldsymbol{I}_l - \eta^{u}\boldsymbol{S}_{k+1}\eta^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{k+1}\boldsymbol{K}_{k+1}\right)\right]^{\mathrm{T}}\end{aligned} \tag{12}$$

To minimize the error covariance matrix $\hat{\boldsymbol{P}}^{\theta}_{k+1|k+1}$, Kalman gain matrix $\boldsymbol{K}_{k+1}$ should be selected as

$$\boldsymbol{K}_{k+1} = \hat{\mathbf{P}}^{\theta}_{k|k}\varphi^{\mathrm{T}}_{k+1}(\varphi_{\mathrm{k}+1}\hat{\mathbf{P}}^{\theta}_{k+1|k}\varphi^{\mathrm{T}}_{k+1} + \boldsymbol{R}_{k+1})^{-1} \tag{13}$$

Therefore, the estimation of $\hat{\boldsymbol{P}}^{\theta}_{k+1|k+1}$ in Eq.(12) can be simplified as：

$$\hat{\boldsymbol{P}}^{\theta}_{k+1|k+1} = \left(\boldsymbol{I}_l + \boldsymbol{K}_{k+1}\boldsymbol{\eta}^u\boldsymbol{S}_{k+1}\boldsymbol{\eta}^{uT}\boldsymbol{R}^{-1}_{k+1}\varphi_{\mathrm{k}+1}\right)\left(\boldsymbol{I} - \boldsymbol{K}_{k+1}\varphi_{\mathrm{k}+1}\right)\hat{\boldsymbol{P}}^{\theta}_{k|k} \tag{14}$$

In summary, the derivation of the proposed parametric KF-UI is completely based on the classical KF and the recursive procedures of the proposed parametric KF-UI are analogous to those of the classical KF described in the above section. Therefore, the proposed PKF-UI is a direct extension of the classical KF, which simplifies the complex derivations in previous Least-Squares Estimation with Unknown Excitations (Yang *et al*. 2007).

## 3. DAMAGE IDENTIFICATION OF UNCERTAIN STRUCTURES BASED ON PKF-UI

Structural damage detection in civil engineering involves a large number of uncertainties which result in the uncertainties of the estimation. Hence, the identified structural parametric vector $\Omega$ and the unknown external excitations $\boldsymbol{f}^{\mathrm{u}}$ are uncertain based on the proposed PKF-UI.

$$\hat{\boldsymbol{f}}^u_{k+1|k+1}(\theta^{\mathrm{u}}) = \boldsymbol{S}_{k+1}(\theta^{\mathrm{u}})\boldsymbol{\eta}^{uT}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_l - \varphi_{\mathrm{k}+1}(\theta^{\mathrm{u}})\boldsymbol{K}_{k+1}(\theta^{\mathrm{u}})\right)\left(\boldsymbol{Y}_{k+1} - \varphi_{\mathrm{k}+1}(\theta^{\mathrm{u}})\hat{\boldsymbol{\theta}}_{k|k}(\theta^{\mathrm{u}})\right) \tag{15}$$

$$\hat{\boldsymbol{\Omega}}_{k+1|k+1}(\theta^{\mathrm{u}}) = \hat{\boldsymbol{\Omega}}_{k|k}(\theta^{\mathrm{u}}) + \boldsymbol{K}_{k+1}(\theta^{\mathrm{u}})(\boldsymbol{y}_{k+1} - \boldsymbol{\varphi}_{k+1}(\theta^{\mathrm{u}})\hat{\boldsymbol{\Omega}}_{k|k}(\theta^{\mathrm{u}}) - \boldsymbol{\eta}^u\hat{\boldsymbol{f}}^u_{k+1|k+1}(\theta^{\mathrm{u}})) \tag{16}$$

The probability distribution of uncertain structural parameters could be obtained when the information of these parameters are sufficient. So, at time $t = \boldsymbol{k}\Delta t$, $\Omega$ and $\boldsymbol{f}^{\mathrm{u}}$ can be expanded at corresponding mean value $\bar{\theta}^u$ of uncertain parameters $\theta^{\mathrm{u}}$ by Taylor series expansion to the first order as,

$$\boldsymbol{f}^u_k(\boldsymbol{\theta}^u) \approx \boldsymbol{f}^u_k(\bar{\boldsymbol{\theta}}^u) + \frac{\partial \boldsymbol{f}^u_k}{\partial \boldsymbol{\theta}^u}\Big|_{\boldsymbol{\theta}^u = \bar{\boldsymbol{\theta}}^u}(\boldsymbol{\theta}^u - \bar{\boldsymbol{\theta}}^u) \tag{17}$$

$$\Omega(\boldsymbol{\theta}^u) \approx \Omega(\bar{\boldsymbol{\theta}}^u) + \frac{\partial \Omega_k}{\partial \boldsymbol{\theta}^u}\Big|_{\boldsymbol{\theta}^u = \bar{\boldsymbol{\theta}}^u}(\boldsymbol{\theta}^u - \bar{\boldsymbol{\theta}}^u) \tag{18}$$

where $\boldsymbol{f}_k^u(\bar{\boldsymbol{\theta}}^u)$ and $\Omega_k(\bar{\boldsymbol{\theta}}^u)$ are the structural identified parametric vector and the unknown external excitations at $\theta^u = \bar{\theta}^u$ , which can be obtained by PKF-UI. $\frac{\partial \boldsymbol{f}_k^u}{\partial \boldsymbol{\theta}^u}|_{\theta^u=\bar{\theta}^u}$ and $\frac{\partial \Omega_k}{\partial \boldsymbol{\theta}^u}|_{\theta^u=\bar{\theta}^u}$ are the sensitivity matrix of uncertain structural parameters of $\boldsymbol{f}^u$ and $\Omega$ at time $t = \boldsymbol{k}\Delta t$ .

Hence, Probability density functions of $\Omega$ can be achieved when probability distribution of uncertain parameters $\theta^u$ is known.

When confidence level of the structural identified parametric vector $\boldsymbol{\Omega}_i^*(\theta^u)$ in undamaged model is $1-\alpha_i$

$$prob(L \le \boldsymbol{\Omega}^* < \infty) = 1-\alpha_i \tag{19}$$

where $L$ is Lower bounds of confidence intervals.

Probability of damage is defined by

$$\mathrm{PDE} = prob(-\infty < \boldsymbol{\Omega}^d \le L\ ) \tag{20}$$

where $\boldsymbol{\Omega}^d$ is the structural identified parametric vector in damaged model.

The degree of damage is defined by

$$\frac{\boldsymbol{\Omega}^* - \boldsymbol{\Omega}^d}{\boldsymbol{\Omega}^*} \times 100\% \tag{21}$$

## 4 NUMERICAL EXAMPLES

A 10-story shear building is used as an numerical example. Parameters of the building are assumed as: floor mass $m_i$=[67.955 , 65.485 , 63.079 , 62.591 , 61.918 , 60.485 , 59.922 , 58.418 , 57.484 , 56.837] kg ; floor original stiffness $k_i$ =[2.713 , 2.685 , 2.657 , 2.648 , 2.639 , 2.629 , 2.604 , 2.589 , 2.576 , 2.558]$\times 10^5$ N/m ($i$ =1,2,…,10);

An input of ground excitation in K-T spectrum is unknown. All the velocity and displacement and 5% noisy acceleration measurements at every story are used in the proposed PKF-UI. Density $\rho \sim N(\rho_0, \sigma^2)$ , $\rho_0 = 7850 kg/m^3$ , $\sigma = 0.01\rho_0$ .Rayleigh damping $\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}$, where $\alpha$ and $\beta$ are unknown.

In the damage pattern A, the stiffness of the third floor reduces 10% decrease. In the damage pattern B, the stiffness of the 4th floor reduces 10% decrease and the 7th floor reduces 15% decrease. Unknown parametric vector to be identified is

$\boldsymbol{\theta} = \{k_1, k_2, \ldots, k_n, \beta k_1, \beta k_2, \ldots, \beta k_n, \alpha\}^T$ ,($n$=10).The unknown input of ground excitation in K-T spectrum will also be estimated.

The undamaged and damaged parameters, identified damage degree and damage probability of each story for pattern A and pattern B are listed in Table 1 and Table 2.

Table 1. Undamaged and damaged structural parameters and damage probability of each story for pattern A

| Story No. | Undamaged Stiffness (N/m) | Identified undamaged stiffness (N/m) | Error ( % ) | Damaged Stiffness (N/m) | Identified damaged stiffness (N/m) | Error(%) | Identified damage degree (%) | PDE (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 271300 | 271307 | 0.003 | 271300 | 271191 | -0.040 | -0.04 | 5.14 |
| 2 | 268500 | 268502 | 0.001 | 268500 | 268396 | -0.039 | -0.04 | 5.13 |
| 3 | 265700 | 265722 | 0.008 | 239130 | 239043 | -0.036 | **-10.04** | **97.09** |
| 4 | 264800 | 264806 | 0.002 | 264800 | 264695 | -0.040 | -0.04 | 5.14 |
| 5 | 263900 | 263897 | -0.001 | 263900 | 263825 | -0.028 | -0.03 | 5.09 |
| 6 | 262900 | 262930 | 0.011 | 262900 | 262812 | -0.033 | -0.04 | 5.15 |
| 7 | 260400 | 260434 | 0.013 | 260400 | 260307 | -0.036 | -0.05 | 5.16 |
| 8 | 258900 | 258810 | -0.035 | 258900 | 258803 | -0.038 | 0.00 | 5.01 |
| 9 | 257600 | 257508 | -0.036 | 257600 | 257411 | -0.073 | -0.04 | 5.12 |
| 10 | 255800 | 255665 | -0.053 | 255800 | 255525 | -0.107 | -0.05 | 5.18 |

Table 2. Undamaged and damaged structural parameters and damage probability of each story for pattern B

| Story No. | Undamaged Stiffness (N/m) | Identified undamaged stiffness (N/m) | Error ( % ) | Damaged Stiffness (N/m) | Identified damaged stiffness (N/m) | Error(%) | Identified damage degree (%) | PDE (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 271300 | 271307 | 0.003 | 271300 | 271214 | -0.032 | -0.03 | 5.12 |
| 2 | 268500 | 268502 | 0.001 | 268500 | 268417 | -0.031 | -0.03 | 5.11 |
| 3 | 265700 | 265722 | 0.008 | 265700 | 265623 | -0.029 | -0.04 | 5.13 |
| 4 | 264800 | 264806 | 0.002 | 238320 | 238240 | -0.034 | **-10.03** | **97.07** |
| 5 | 263900 | 263897 | -0.001 | 263900 | 263843 | -0.021 | -0.02 | 5.07 |
| 6 | 262900 | 262930 | 0.011 | 262900 | 262828 | -0.027 | -0.04 | 5.13 |
| 7 | 260400 | 260434 | 0.013 | 221340 | 221275 | -0.029 | **-15.04** | **100.00** |
| 8 | 258900 | 258810 | -0.035 | 258900 | 258829 | -0.027 | 0.01 | 4.98 |
| 9 | 257600 | 257508 | -0.036 | 257600 | 257450 | -0.058 | -0.02 | 5.08 |
| 10 | 255800 | 255665 | -0.053 | 255800 | 255587 | -0.083 | -0.03 | 5.10 |

Figs. 1-2 show damage probability of each story for pattern A and pattern B. Figs.3-4 show probability density functions of the 1st and 3rd story for pattern A and pattern B. Fig. 5 shows the comparison of identified mean value of ground acceleration with the actual value.

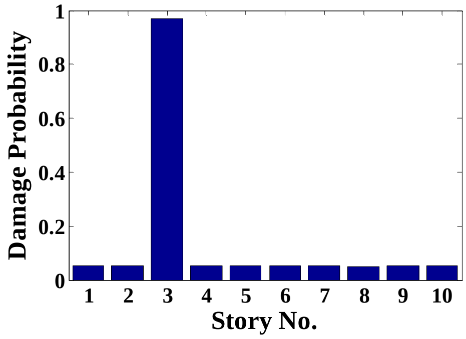



Fig.1: Damage probability of each story for pattern A

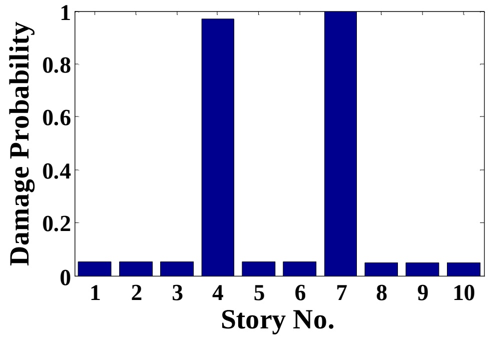
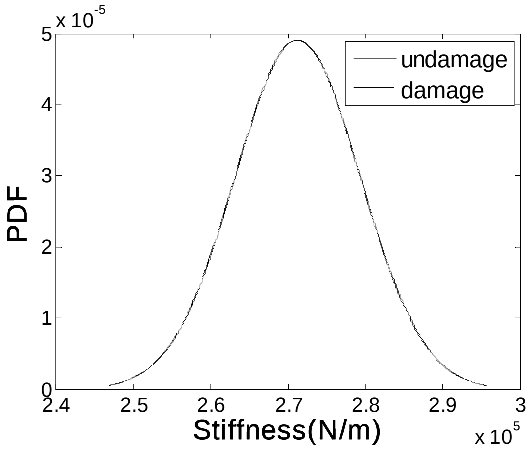
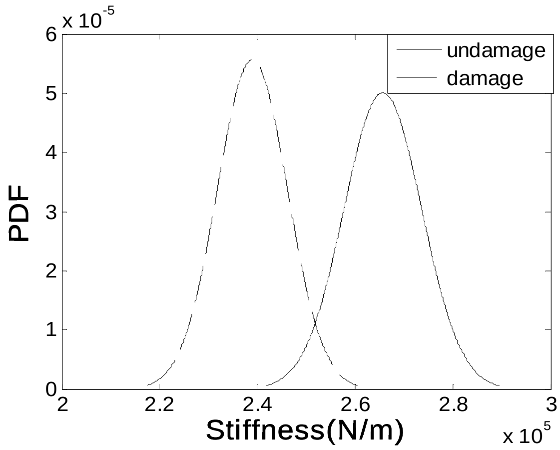



Fig.2: Damage probability of each story for pattern B

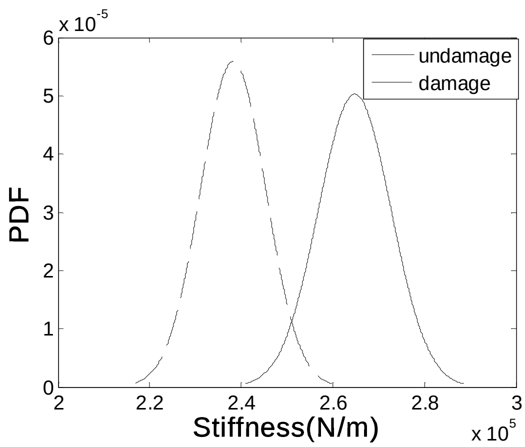



（a） PDF of $1^{st}$ story

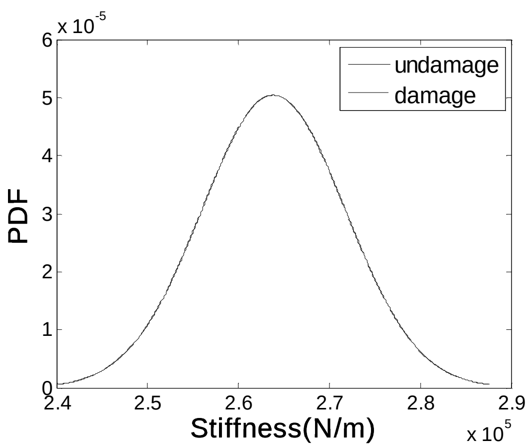



（b）PDF of 3th story

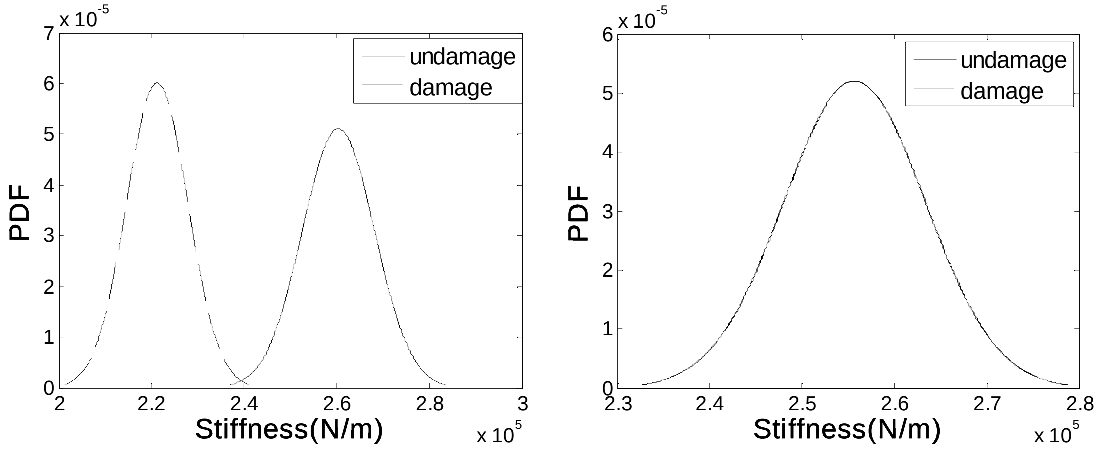
Fig.3: Probability density functions of each story for pattern A




（a） PDF of 4th story




（b）PDF of 5th story




（c）PDF of 7th story （d）PDF of 10th story

Fig.4: Probability density functions of each story for pattern B

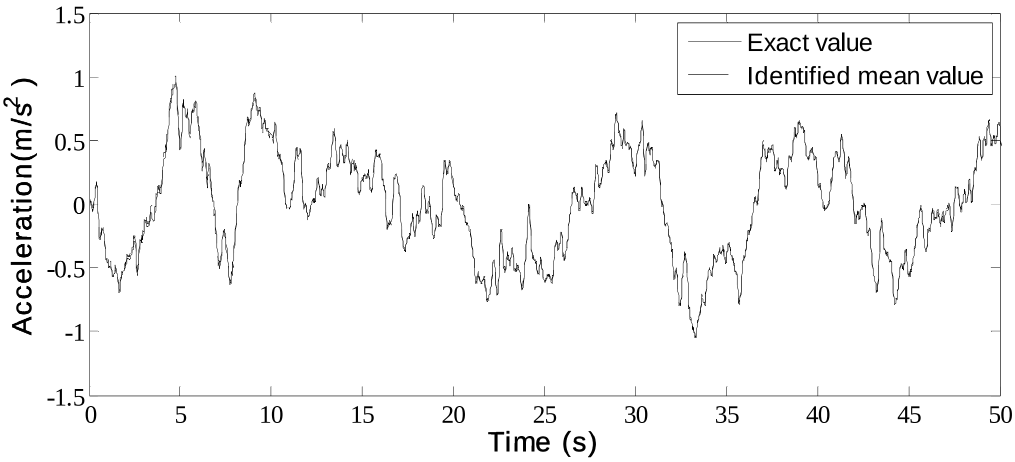



Fig.5 : Comparison of identified mean value of ground acceleration and exact value

## 5. CONCLUSIONS

The conventional parametric Kalman filter (PKF) is only applicable when the information of external excitations to structures is available. Some improved methodologies have been proposed. However, the derivations of these approaches are quite involved. On the other hand, structural uncertainties are inevitable for civil infrastructures, it is necessary to develop approaches for probabilistic damage detection of structures. In this paper, a parametric Kalman filter with unknown inputs is proposed for the simultaneous identification of structural parameters and the unknown external inputs based on the conventional PKF. Then, it is used for probabilistic damage detection of structures by considering the uncertainties of structural parameters. The damage index and the damage probability are derived from the statistical values of the identified structural parameters of intact and damaged structure. Finally, some numerical examples are used for validating the proposed method.

**— Proceeding Paper**

## REFERENCES

Beck J L, Katafygiotis L S.(1998) “Updating Models and Their Uncertainties. I: Bayesian Statistical Framework”. *Journal of Engineering Mechanics*, **124**(4),455-461.

Fang S E, Perera R. (2011) “Damage identification by response surface based model updating using D-optimal design”, *Mechanical Systems and Signal Processing*, **25** (2) : 717-733.

Hao S, Betti R. A(2015) “Hybrid Optimization Algorithm with Bayesian Inference for Probabilistic Model Updating”, *Computer-Aided Civil and Infrastructure Engineering*, **30**(8),602-619.

Jiang S.F., Wu Z. Q.,(2011) “Structural health monitoring and intelligent information processing technology and its application”, *China Building Industry Press*.

Lei Y, Jiang Y Q.(2011), “A Two-Stage Kalman Estimation Approach for the Identification of Nonlinear Structural Parameters”, *Procedia Engineering*, **14**(2259):3088-3094.

Li X Y, Law S S.(2008) “Damage Identification of Structures Including System Uncertainties and Measurement Noise”. *Aiaa Journal*, **46**(46),263-276.

Ling, X., and Haldar, A. (2004). “Element level system identification with unknown input with Rayleigh damping.” *J. Eng. Mech*., **130**(8),877−885.

Mustafa S, Debnath N, Dutta A.(2015) “Bayesian probabilistic approach for model updating and damage detection for a large truss bridge”, *International Journal of Steel Structures*, **15**(2),473-485.

Reynders E, Teughels A, De Roeck G.(2010) “Finite element model updating and structural damage identification using OMAX data”, *Mechanical Systems and Signal Processing*, **24** (5) ,1306-1323.

Rutherford A.C., Inman D.J, Park G., Hemez F.M., (2005) “Use of response surface metamodels for identification of stiffness and damping coefficients in a simple dynamic system”, *Shock & Vibration*, **12**(5), 317-331.

Wang, D., and Haldar, A. (1994). “Element-level system identification with unknown input”, *J. Eng. Mech.*, **120**(1), 159-176.

Wang, D., and Haldar, A. (1997). “System identification with limited observations and without input”. *J. Eng. Mech*., **123**(5), 504-511.

Yang, J. N., and Lin, S.(2005) “Identification of parametric variations of structures based on least squares estimation and adaptive tracking technique”, *J. Eng. Mech*., **131**(3), 290 – 298.

Yang J. N., Pan S, and Lin S, (2007) “Least-Squares Estimation with Unknown Excitations for Damage Identification of Structures,” *Journal Of Engineering Mechanics ASCE* , **133**(1),12-21.

Zong Z. H,. Ren W.X., Ruan Y., (2003) “Recent Advances in Research on Damage Diagnosis for Civil Engineering Structures”, *China Civil Engineering Journal*, **36**(5),105-110.