The 2016 World Congress on
The 2016 Structures Congress (Structures16)
Jeju Island, Korea, August 28-September 1, 2016

On-line Integrated Identification and Reliability Evaluation of uncertain Structures under Unknown Excitation

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ABSTRACT

To ensure the serviceability and safety of structures, it is important to conduct structural identification/damage detection and reliability evaluation. In most of previous investigations, structural system identification/damage detection and reliability evaluation are treated separately. Research efforts devoted to the methodologies that accept the monitoring data as input and produce as output the reliability of the concerned building structure are very few in comparison with the myriad of literature addressing on structural system identification. It is necessary to combine structural system identification/damage detection and reliability evaluation. Moreover, in practical application, it is requested to conduct on-line integrated identification and reliability evaluation of uncertain structures under unknown excitation inputs. In this paper, on-line integrated identification and reliability evaluation of uncertain structures under unknown excitation is investigated. The proposed method is based on the embedment of structural identification/damage detection by the recursive extended Kalman filter with unknown input, which was recently proposed by the authors for simultaneous structural identification and unknown excitation inputs with partial observations of acceleration and displacement responses, into in the procedure of structural reliability evaluation. Both structural component reliability with only one limit state function and system reliability with multi-limit state functions are studied for stochastic building structures. Some numerical examples are used for the validation of the proposed method. The evaluated results of structural component reliability and structural system reliability are compared with those by the Monte Carlo simulation (MCS). It is shown that the proposed method is effective for on-line integrated identification and reliability evaluation of uncertain structures under unknown excitation.

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1. INTRODUCTION

It is well known that structural system identification/damage detection and structural reliability evaluation are important issues to ensure the serviceability and safety of structures (Mustafa *et al.* 2015, Lei *et al*. 2012, 2014, Chen and Li 2005, Li and Chen 2006, Su *et al*. 2016). However, in most of previous investigations, structural system identification/damage detection and reliability evaluation are investigated separately. When uncertainties are taken into account, the stiffness parameters of all the elements in a building structure identified are random parameters (Li and Law 2008, Pothisiri *et al.* 2003, Xia *et al.* 2002, Law and Li 2010). Under this circumstance, it is prohibitive to evaluate structural reliability by the current reliability analysis methods for civil structures. It is necessary to embed structural system identification/damage detection in the procedure of structural reliability evaluation.

Recently, Zhang et al. (2010) have investigated integrated system identification and reliability evaluation of stochastic building structures by combining a statistical moment-based system identification method (Zhang *et al.* 2009, Xu *et al.* 2009,) and a probability density evolution equation-based reliability evaluation method (Chen and Li 2005, Li and Chen 2006). The integrated methods accept the measurement responses as input and produce as output the reliability of the concerned instrumented building structure. Structural system identification is embedded in the procedure of the reliability evaluation in the proposed methods.

However, it is still essential to conduct on-line integrated identification and reliability evaluation of uncertain structures. Moreover, since it is difficult or even impossible to measure all structural external excitations under actual operating conditions, it is requested to study on-line integrated identification and reliability evaluation of structures under unknown excitation. In this paper, a method is proposed for this purpose. Based on the extended Kalman Filter with unknown input (EKF-UI), which was recently proposed by the authors (Liu *et al.* 2016) for simultaneous structural identification and unknown excitation inputs with partial observations of acceleration and displacement responses, identification of structural system and external excitation inputs are embedded into in the procedure of structural reliability evaluation. Both structural component reliability with only one limit state function and system reliability with multi-limit state functions are studied for stochastic building structures (Zhang *et al.* 2010). Some numerical examples are used for the validation of the proposed method. The evaluated results of structural component reliability and structural system reliability are compared with those by the Monte Carlo simulation (MCS).

2. Brief introduction of the EKF-UI

The equation of motion of a n-DOF building structure can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{\eta}\,\boldsymbol{f}+\boldsymbol{\eta}^{u}\,\boldsymbol{f}^{u} \quad (1)$$

where $\ddot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$ are the acceleration, the velocity and the displacement vector, $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the matrixes of mass, damp and stiffness. $\boldsymbol{f}$ and $\boldsymbol{f}^{u}$ are the known and unknown external excitation matrixes, $\boldsymbol{\eta}$ and $\boldsymbol{\eta}^{u}$ are the location matrixes of excitation.

By introducing an extended state vector $\boldsymbol{Z}$ defined as

$$\boldsymbol{Z} = \left[\boldsymbol{x}^T(t), \dot{\boldsymbol{x}}^T(t), \boldsymbol{\theta}^T \right]^T \tag{2}$$

where $\boldsymbol{\theta}$ is the unknown structural parameters. Eq.(1) can be converted into state equation as

$$\dot{\boldsymbol{Z}}_e = \begin{Bmatrix} \dot{\boldsymbol{x}}(t) \\ -\boldsymbol{M}^{-1}[(\boldsymbol{C})_\theta \dot{\boldsymbol{x}}(t) + (\boldsymbol{K})_\theta \boldsymbol{x}(t) - \boldsymbol{\eta} \boldsymbol{f} - \boldsymbol{\eta}^u \boldsymbol{f}^u] \\ \boldsymbol{0} \end{Bmatrix} = \boldsymbol{g}[\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u] \tag{3}$$

in which, $(\boldsymbol{C})_\theta$ and $(\boldsymbol{K})_\theta$ are the damping and stiffness matrixes, and $\boldsymbol{g}$ is a nonlinear function.

At the same time, $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}^u_{k|k}$ are estimated value of $\boldsymbol{Z}_k$ and $\boldsymbol{f}^u_k$. The first-order Tylor expansion is done on the $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}^u_{k|k}$ points and get that

$$\boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u) \approx \boldsymbol{g}(\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}^u_{k|k}) + \boldsymbol{G}_{k|k}(\boldsymbol{Z} - \hat{\boldsymbol{Z}}_{k|k}) + \boldsymbol{B}^u_{k|k}(\boldsymbol{f}^u - \hat{\boldsymbol{f}}^u_{k|k}) \tag{4}$$

in which

$$\boldsymbol{G}_{k|k} = \left. \frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}} \right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k};\, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}} \quad ; \quad \boldsymbol{B}^u_{k|k} = \left. \frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u} \right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k};\, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}} \tag{5}$$

Similar to EKF (Lei 2011), the prediction equation is

$$\tilde{\boldsymbol{Z}}_{k+1|k} = \hat{\boldsymbol{Z}}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} \boldsymbol{g}(\hat{\boldsymbol{Z}}_{t|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}^u_{k|k}) dt \tag{6}$$

The non-linear discrete equation of observation equation is shown as

$$\boldsymbol{y}_{k+1} = \boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}^u_{k+1}) + \boldsymbol{v}_{k+1} \tag{7}$$

The first-order Tylor expansion is done to this equation in $\tilde{\boldsymbol{Z}}_{k+1|k}$ and $\hat{\boldsymbol{f}}^u_{k|k}$ to get that

$$\boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}^u_{k+1}) = \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^u_{k|k}) + \boldsymbol{H}_{k+1|k}(\boldsymbol{Z}_{k+1} - \tilde{\boldsymbol{Z}}_{k+1|k}) + \boldsymbol{D}^u_{k+1|k}(\boldsymbol{f}^u_{k+1} - \hat{\boldsymbol{f}}^u_{k|k}) \tag{8}$$

in which

$$\boldsymbol{H}_{k+1|k} = \left. \frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}} \right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k};\, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}} \quad ; \quad \boldsymbol{D}^u_{k+1|k} = \left. \frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u} \right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k};\, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}} \tag{9}$$

From the filter equation, the state estimation is known that

$$\hat{\boldsymbol{Z}}_{k+1|k+1} = \tilde{\boldsymbol{Z}}_{k+1|k} + \boldsymbol{K}_{k+1}\left[\boldsymbol{y}_{k+1} - \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^u_{k|k}) - \boldsymbol{D}^u_{k+1|k}(\hat{\boldsymbol{f}}^u_{k+1|k+1} - \hat{\boldsymbol{f}}^u_{k|k}) \right] \tag{10}$$

in which $\hat{\boldsymbol{Z}}_{k+1|k+1}$ and $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ are estimation of $\boldsymbol{Z}_{k+1}$ and $\boldsymbol{f}^u_{k+1}$ when the observation of $(y_1, y_2, \ldots, y_k)$ is known, $\boldsymbol{K}_{k+1}$ is Kalman gain matrix (Liu *et al.* 2016).

And the unknown input can be estimated as,

$$\hat{\boldsymbol{f}}^{u}_{k+1|k+1}=\boldsymbol{S}_{k+1}\boldsymbol{D}^{u\mathrm{T}}_{k+1|k}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_m-\boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1}\right)\left[\boldsymbol{y}_{k+1}-\boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k},\boldsymbol{f}_{k+1},\hat{\boldsymbol{f}}^{u}_{k|k})+\boldsymbol{D}^{u}_{k+1|k}\hat{\boldsymbol{f}}^{u}_{k|k}\right] \tag{11}$$

in which, $\boldsymbol{S}_{k+1}=\left[\boldsymbol{D}^{uT}_{k+1|k}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_m-\boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1}\right)\boldsymbol{D}^{u}_{k+1|k}\right]^{-1}$

Thus, in the EKF-UI, the structural state vector and unknown excitation inputs can be identified simultaneously in real time.

3. On-line integration of identification and reliability evaluation

Due to the inevitable uncertainties, the dynamic displacement response of structures is a random process. Eq.(1) can be rewritten as

$$\boldsymbol{M}(\boldsymbol{\Theta})\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}(\boldsymbol{\Theta})\dot{\boldsymbol{x}}(t)+\boldsymbol{K}(\boldsymbol{\Theta})\boldsymbol{x}(t)=\boldsymbol{\eta}\,\boldsymbol{f}+\boldsymbol{\eta}^{u}\,\boldsymbol{f}^{u} \tag{12}$$

where $\boldsymbol{\Theta}$ is the random parameter vector of n_θ order which reflects the uncertainty in the structural identification procedure, with the known probability density function $p_\Theta(\theta)$.

The structural displacement response can be expanded by Taylor series to the first order as

$$\boldsymbol{x}(\boldsymbol{\Theta},t)=\boldsymbol{x}(\boldsymbol{\Theta}_0,t)+\sum_{i=1}^{Nq}\left.\frac{\partial\boldsymbol{x}(\boldsymbol{\Theta},t)}{\partial\Theta_i}\right|_{\Theta_i=\Theta_{i0}}(\Theta_i-\Theta_{i0}) \tag{13}$$

where $\boldsymbol{\Theta}_0$ is the mean value of $\boldsymbol{\Theta}$, Nq is the number of the vector $\boldsymbol{\Theta}$.

When Rayleigh damping is adopted with structural first modal damping ratio being uncertain parameter, it is known by taking the derivative with respect to the first-modal damping ratio damping ratio in Eq.(13) that

$$\boldsymbol{M}\frac{\partial\ddot{\boldsymbol{x}}}{\partial\xi_1}(t)+(a\boldsymbol{M}+b\boldsymbol{K})\frac{\partial\dot{\boldsymbol{x}}}{\partial\xi_1}(t)+\boldsymbol{K}\frac{\partial\boldsymbol{x}}{\partial\xi_1}(t)=-\boldsymbol{M}\dot{\boldsymbol{x}}\frac{\partial a}{\partial\xi_1}-\boldsymbol{K}\dot{\boldsymbol{x}}\frac{\partial b}{\partial\xi_1} \tag{14}$$

where the two Rayleigh damping coefficients a, b and $\partial a/\partial\xi_1$, $\partial b/\partial\xi_1$ are obtained by

$$\begin{Bmatrix}a\\b\end{Bmatrix}=\frac{2\omega_1\omega_2}{\omega_2^2-\omega_1^2}\begin{bmatrix}\omega_2 & -\omega_1\\ -\dfrac{1}{\omega_2} & \dfrac{1}{\omega_1}\end{bmatrix}\begin{Bmatrix}\xi_1\\\xi_2\end{Bmatrix} \tag{15}$$

$$\frac{\partial a}{\partial\xi_1}=\frac{2\omega_1\omega_2^2}{\omega_2^2-\omega_1^2},\quad \frac{\partial b}{\partial\xi_1}=\frac{2\omega_1}{\omega_1^2-\omega_2^2} \tag{16}$$

in which ω_1 and ω_2 are the first two natural frequency.

The relationship between the displacement response and the uncertainties is set up according to the Eq.(13). Therefore, the probability that structural displacement response does not exceed a certain threshold can be calculated. Then, the reliability can be evaluated.

When the reliability index is chosen as the relative displacement at a certain DOF or an inter-story drift, structural component reliability can be evaluated by

$$R(t) = P\left\{\left|x_{top}(\Theta,t)\right| \le x_T, t \in [0,t]\right\} \tag{16a}$$

or

$$R(t) = P\left\{\left|x_{\text{inter}-story}(\Theta,t)\right| \le x_T, t \in [0,t]\right\} \tag{16b}$$

In addition, when all the inter-story drifts of the multi-story shear building are required not to exceed the corresponding threshold, a family of limit state functions should be considered. Denote the inter-story drifts from the floor to the top by $Z_1(\Theta, t)$, $Z_2(\Theta, t)$, . . . , $Z_n(\Theta, t)$, Structural system reliability can be evaluated by

$$R(t) = P\left\{\bigcap_{i=1}^{N}\left\{Z_i(\Theta,t) < x_T, t \in [0,t]\right\}\right\} \tag{17}$$

where $P\{\cdot\}$ is the probability of the random event, x_T is the threshold value.

3. Numerical Study

In this paper, a six-story shear frame structure, as shown in Fig.1, is chosen to verify the feasibility of the integration method based on time moment. The mass of the frame is m= {600 550 500 450 400 350}*kg*, and the story stiffness is k_i= {1.4 1.3 1.1 0.9 0.7 0.5} ×10^6 *N/m*. The Rayleigh damping is assumed, and the first modal damping ratio ξ_1 is considered as a random parameter with a lognormal distribution due to the uncertainty. The mean value of ξ_1 is 3%, and the standard deviation is σ=10%ξ_1.

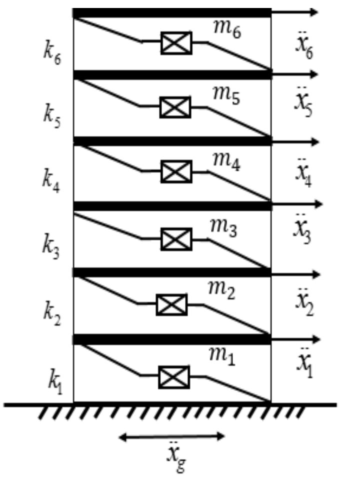

Fig. 1 A 6-floor shear frame building structure

The external excitation to the shear building is the ground acceleration generated by the Kanai–Tajimi spectrum

$$\ddot{x}_g(t) = \ddot{x}_{gG}(t) \cdot g(t) \tag{18}$$

where $\ddot{x}_{gG}(t)$ is the colored Gaussian noises, the KT spectral density function has the form of

$$S_g(\omega) = \frac{1 + 4\xi_g^2 (\frac{\omega}{\omega_g})^2}{\left[1 - (\frac{\omega}{\omega_g})^2\right]^2 + 4\xi_g^2 (\frac{\omega}{\omega})^2} S_0 \tag{19}$$

in which ω_g , ξ_g and S_0 are the characteristics and the intensity of the Kanai–Tajimi spectrum. These parameters are selected as ω_g =15.0*rad/s*, ξ_g =0.6, S_0=4.64×10^{-4}$m^2/rads^3$. The time duration of the simulated acceleration is 15s and the sampling frequency is 1000Hz.

Structural damage scenario is assumed as a story stiffness deduction of 10% at 1st story, and the measured accelerations of the 1st, 3rd and 4th floors are polluted by 5% noise.

Figs.2-3 show structural component reliabilities in terms of the threshold of top displacement of the undamaged structure and the damaged structure. As seen from these figures, the reliabilities increase when the threshold enlarges and the reliabilities decrease for the damaged structure. In these figures, the evaluated structural component reliability results are also compared with those by Monte Carlo simulation (MCS) and it is noted that evaluated structural component reliabilities are in good agreement with those by MCS.

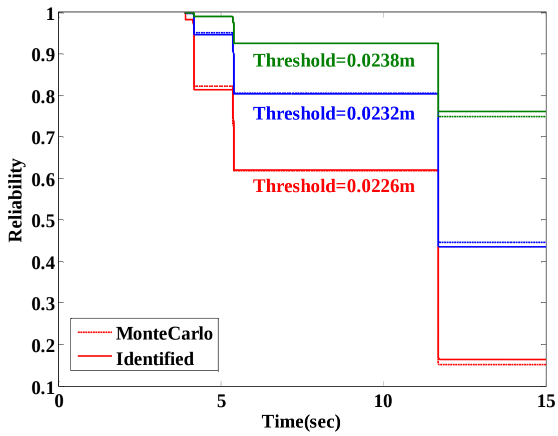

Fig.2 Component reliability of the undamaged structure

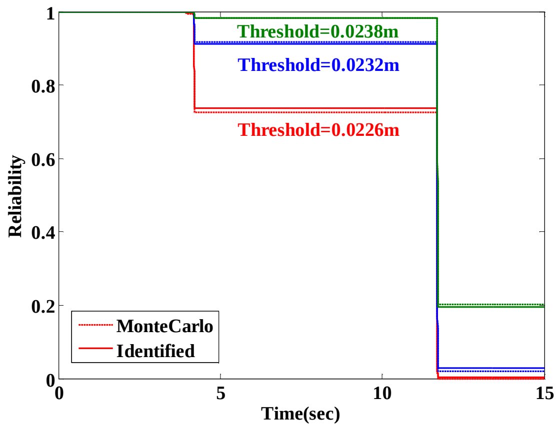

Fig.3 Component reliability of the damaged structure

Figs.4-5 show the comparisons of the evaluated structural system reliability by the proposed method with those by Monte Carlo simulation (MCS). The failure probability of the whole structure equals to the maximum value of those of story drifts. It is shown that that evaluated structural system reliabilities by the proposed method are in good agreement with those by MCS.

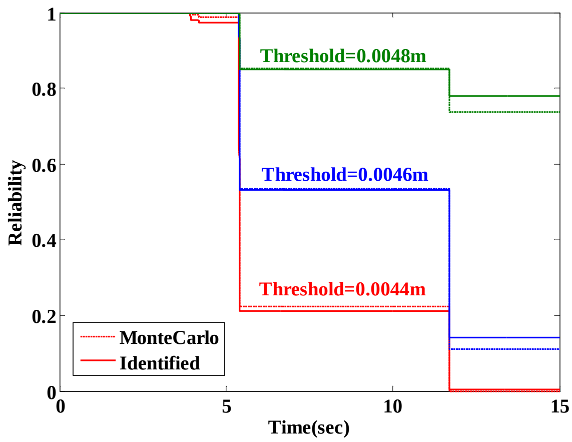

Fig.4 System reliability of undamaged structure

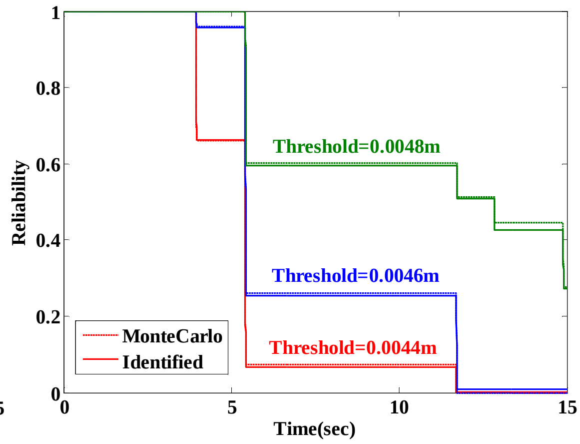

Fig.5 System reliability of damaged structure

Figs.6-7 show the identification results of the unknown ground excitation which indicates that the error between the exact value and identified value is acceptable.

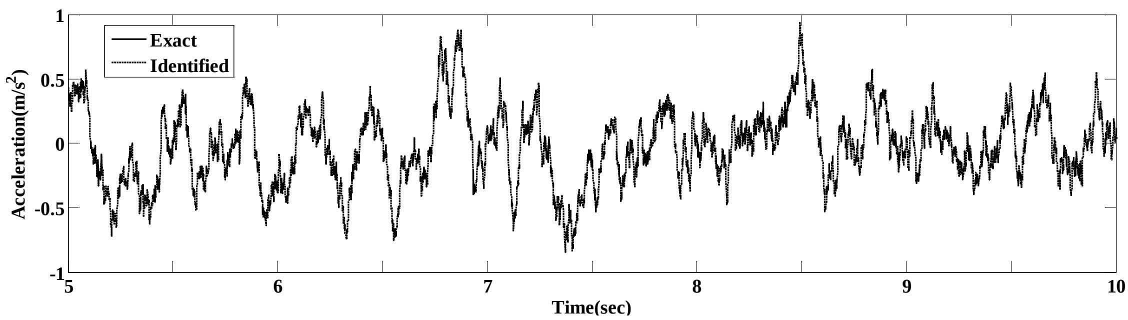

Fig. 6 Comparison of ground excitation in 5~10 seconds

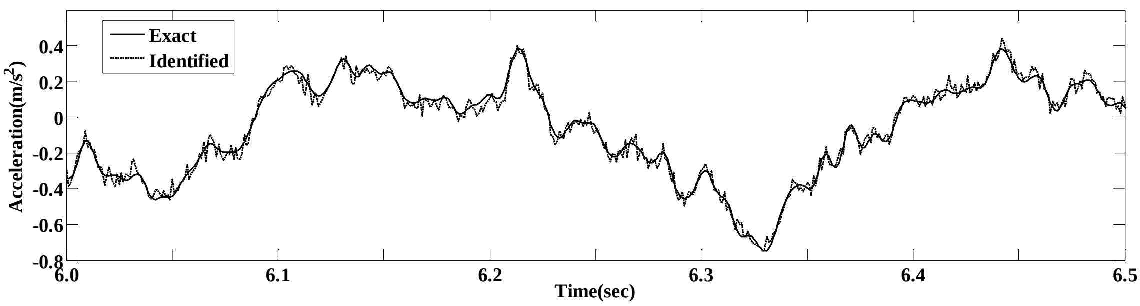

Fig.7 Details of the identified ground excitation

4. CONCLUSIONS

In this paper, a method is proposed for on-line integrated identification and reliability evaluation of uncertain structures under unknown excitation. Based on the extended Kalman Filter with unknown input (EKF-UI), which was recently proposed by the authors, identification of structural system and external excitation inputs are embedded into in the procedure of structural reliability evaluation. Both component reliability with only one limit state function and system reliability with multi-limit state functions of stochastic building structures are studied. To demonstrate the feasibility and effectiveness of the proposed methods, numerical investigation has been conducted to evaluate structural component reliability and structural system reliability of a six-story shear building structure under unknown ground excitation. Structural damage is also included. Compared with the results of MCS, the proposed method is viable and comparatively accurate.

ACKNOWLEDGEMENTS

This research is supported by the National Natural Science Foundation of China through Grant No. 51378445.

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