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The fluctuating wind field simulation analysis for building structure based on the random Fourier spectrum

Li Lin[1,2,3], A.H.S. ANG[3], Hai-tao Hu[1], Dan-dan Xia[1] and Fu-Qiang He[1, 4]

[1]*School of Civil & Architecture Engineering, Xiamen University of Technology, Xiamen, Fujian, 361024, China*

[2]*College of Civil Engineering, Hunan University, Changsha, Hunan,410082,China*

[3]*Department of Civil & Environmental Engineering, University of California, Irvine, CA, 92697, USA*

[4]*Fujian Provincial Key Laboratory of Fire Retardant Materials, 361000, China*

Abstract

The accurate calculation to stochastic wind field is the foundation to analyze wind-induced structure response and reliability. In this research, the spatial correlation of structure wind field was considered based on the time domain method. A calculation method for the stochastic wind field based on mutual stochastic Fourier spectrum is proposed. Furthermore, the programs for generation the wind field sample were programmed also in this paper. Combing with the previous analysis of regional wind speed distribution, the wind speed time history sample was calculated, and the efficiency can therefore be verified. Results show that the proposed method and programs could provide an efficient simulation for the further structure wind caused respond analysis and help to determine the related parameters easily.
Key words: Stochastic wind field, mutual stochastic Fourier spectrum, wind speed time history, time domain method

1. Introduction

Both structural response analysis and the further structural dynamic reliability analysis under the wind load are the main purposes of wind-resistant research. For a building structure, the wind velocity of a spatial point could be described by a stochastic process. However, for a structure with large volume, the correlation of wind speed between different positions should also be presented. In this situation, the spatial wind speed should be regarded as a stochastic speed field. Therefore, it is how to systemically model the reasonable time-domain speed field that is the foundation to analyze the stochastic response and reliability of the structure induced by the wind.

To the definite structure system, the typical stochastic vibration was always used to the wind resistant analysis, which includes frequency domain method and time domain method. Frequency domain method is more convenient for direct structure pulse respond analysis based on the fluctuation wind speed spectrum. However, the linear structure is demand to using this method, which limits the exactitude of the structure characters analysis. Time domain method could be used to directly calculate the biggest value time history of structure stress and displacement. The structure model needn’t to be simplified for the time domain method, but it takes lot of time to do the calculation. Obviously, the time domain method is more directly. In this paper, basing on the random Fourier spectrum, a fluctuating wind field simulation analysis method was put forward to calculate the respond spectrum of a high-rise building.

2. Mutual stochastic Fourier spectral fluctuating wind field modeling method and program

2.1 Stochastic characters

Stochastic characters include annual 10 minutes average maximum wind speed U_{10}, roughness length Z_0, and spectral coherence functions $\gamma_{ij}\left(C_1, C_x, C_y, C_z\right)$.

The annual 10 minutes average maximum wind speed U_{10} should be analyzed based on the original wind data history. The distribution and its parameters could be determined by the GUPP method very easy. (Lin Li,2015)

According to the *Chinese Load Code for the Design of Building Structures*, the roughness length can be quantitatively decided regarding to the geometrically condition. Certainly, it is a rough method and has large errors. Taking the uncertainty of roughness of the ground into consideration, the gradient wind velocity can be tested in the specific locations. Then sample data of the real ground roughness z_0 can be calculated based on the widely applied gradient distributions such as exponential and logarithmic distributions. Based on the distribution analysis, the parameters and distribution type can be determined, the mean and standard error can therefore be obtained as well. Applying the stochastic modeling method, the probability distribution density can be obtained. The value of z_0 can be subsequently determined through the comparison with traditional statistic methods and hypothesis testing

Spectral coherence function $\gamma_{ij}\left(C_1, C_x, C_y, C_z\right)$ relates to the coefficients of C_1,C_2,C_3 and C_4. Based on the statistical analysis of tested gradient wind speed, the distribution of related parameters C_1, C_2, C_3 and C_4 can be determined by the correlation of experiential equation S(*z*, *n*) of fluctuating wind spectrum, Fourier spectrum and power spectrum.

2.2 Stochastic wind field modeling

Stochastic wind speed is assumed to be a stationary random field, which can be separated into a mean value component $\bar{V}(z)$ and a zero mean value fluctuation component $V(x,y,z,t)$. (Xiangting Zhang; 1990)

$$V(x,y,z,t) = \bar{V}(z) + V(x,y,z,t) \tag{1}$$

2.2.1 Mean wind speed

Mean value component $\bar{V}(z)$ is described by the wind speed profile function, which is the function of average wind speed varied on the altitude. Generally logarithmic law function and exponential law function are two common wind speed profile functions. The logarithmic law is used in this research as:

$$\bar{V}(z) = \frac{1}{k} u_* \ln(z/z_0) \tag{2}$$

k is Von Karman constant, which can be taken as 0.4, z_0 is ground roughness length(*m*), u_* is shear velocity, which is defined by:

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \tag{3}$$

In this function, τ_0 is ground shear force, ρ is air density.

Actually, τ_0 is hard to obtain, so the u_* is calculated by the known wind speed $\bar{V}(z')$ on the specific height z' which can be 10 meters:

$$u_* = \frac{k\bar{v}(z')}{\ln(z'/z_0)} \tag{4}$$

Therefore, the function of wind speed profile showed as function 5.

$$\bar{V}(z) = V(z_{10}) . \frac{\ln(z/z_0)}{\ln(10/z_0)} \tag{5}$$

2.2.2 Power spectrum of Fluctuation wind field

There are some classic power spectrum such as Von Carman spectrum, Davenport spectrum and Simiu spectrum (Simiu E., 1961). Davenport spectrum (Davenport A. G., 1992) is used in *Chinese Building*

Code (Ministry of Housing and Urban-Rural Development of PRC, 2012), which is independent of height:

$$\frac{nS(z,n)}{u_*^2} = \frac{4f^2}{(1+f^2)^{4/3}} \tag{6}$$

In function, $f = \frac{1200n}{U_{10}}$, U_{10} (m/s) is the average wind speed of 10 meters height.

Basing on this, the unilateral spectrum function is:

$$S(z,n) = \frac{4f^2 u_*^2}{\left(1+f^2\right)^{4/3}} \tag{7}$$

$$S(w) = \frac{460800\times\omega}{\pi\times\left[\ln(\frac{10}{Z_0})\right]^2\times\left[1+\left(\frac{600\times\omega}{\pi\times U_{10}}\right)^2\right]^{4/3}} \tag{8}$$

Or

$$S(n) = \frac{921600\times n}{\left[\ln(\frac{10}{Z_0})\right]^2\times\left[1+\left(\frac{1200\times n}{U_{10}}\right)^2\right]^{4/3}} \tag{9}$$

As to two-sides spectrum the function 7 should be revised to：

$$S(z,n) = \frac{4f^2 u_*^2}{2\times\left(1+f^2\right)^{4/3}} \tag{10}$$

2.2.3 Mutual stochastic Fourier spectrum of Fluctuation wind field

Spatial stochastic wind speed field is a continued spatial field. However, the discrete wind load is used for the actual analysis. To the specific spatial point$\left(x_j, y_j, z_j\right)$, *j*=1, 2, 3, ……, *n*, stochastic process $V_j(t)$ is the wind speed time history of the j point. (Shinozuka, 1996 1998)

$$V_j(t) = V_j\left(x_j, y_j, z_j, t\right) \tag{11}$$

Therefore, there are *n* stochastic processes of the spatial points to form *n* directions wind speed stochastic vectors as$\left\{V_1(t), V_2(t), \cdots, V_j(t)\right\}$.

The mutual stochastic Fourier spectrum of a *n*-direction zero mean value stochastic process is:

$$F\left(z_0,U_{10},n\right)=\begin{bmatrix} F_{11}\left(z_0,U_{10},n\right) & F_{12}\left(z_0,U_{10},n\right) & \dots & F_{1n}\left(z_0,U_{10},n\right) \\ F_{21}\left(z_0,U_{10},n\right) & F_{22}\left(z_0,U_{10},n\right) & \dots & F_{2n}\left(z_0,U_{10},n\right) \\ \vdots & \dots & \ddots & \vdots \\ F_{n1}\left(z_0,U_{10},n\right) & F_{n2}\left(z_0,U_{10},n\right) & \dots & F_{nn}\left(z_0,U_{10},n\right) \end{bmatrix} \quad (12)$$

Diagonal elements $F_{ii}\left(z_0,U_{10},n\right)$ are composed by self-power Fourier spectrum:

$$F_{ii}\left(z_0,U_{10},n\right)=F_{V_i}\left(z_0,U_{10},n\right)^2,\ \text{i=1,2,…,n;} \quad (12)$$

The off-diagonal elements $F_{ij}\left(z_0,U_{10},n\right)$ are composed by mutual stochastic Fourier spectrum:

$$F_{ij}\left(z_0,U_{10},n\right)=F_{V_i}\left(z_0,U_{10},n\right)F_{V_j}\left(z_0,U_{10},n\right)\gamma_{ij}\left(z_0,U_{10},C_1,C_x,C_y,C_z,n\right),i\ ,j=1,2,..n \quad (13)$$

U_{10} (m/s) is the average wind speed of 10 meters height, $z_0(m)$ is ground roughness length, and $n(Hz)$ is the frequency, C is the attenuation coefficients of correlation function. To the specific construction site and the 10 meters height wind speed conditions, the elements of the mutual stochastic Fourier matrix are the variables with respect to the variable n.

$$F_{ii}\left(n\right)=F_{V_i}\left(n\right)^2,\ i=1,2,\dots,n; \quad (14)$$

$$F_{ij}\left(n\right)=F_{V_i}\left(n\right)F_{V_j}\left(n\right)\gamma_{ij}\left(n\right),\ i,\ j=1,\ 2,..,\ n; \quad (15)$$

Self-power spectrum density function is a real value even function of n, as:

$$\begin{cases} F_{ij}^0\left(z_0,U_{10},n\right)=F_{ij}^0\left(z_0,U_{10},-n\right) \\ F_{ij}^0\left(z_0,U_{10},n\right)=F_{ji}^0\left(z_0,U_{10},n\right) \end{cases} \quad (16)$$

Then the stochastic Fourier spectrum is the symmetric matrix.

2.2.4 FFT technique for wind speed spectral

FFT technique can be used to reduce the cost of wind speed spectral. Mutual stochastic Fourier spectrum $F\left(z_0,U_{10},n\right)$ could be resolved by Cholsky method as follow.

$$F\left(z_0,U_{10},n\right)=I\left(z_0,U_{10},n\right)I^T\left(z_0,U_{10},n\right) \quad (17)$$

I(n) is the lower triangular matrix:

$$I\left(z_0,U_{10},n\right)=\begin{bmatrix} I_{11}\left(z_0,U_{10},n\right) & 0 & 0 & 0 \\ I_{21}\left(z_0,U_{10},n\right) & I_{12}\left(z_0,U_{10},n\right) & 0 & 0 \\ \vdots & \dots & \ddots & 0 \\ I_{n1}\left(z_0,U_{10},n\right) & I_{n2}\left(z_0,U_{10},n\right) & \dots & I_{nn}\left(z_0,U_{10},n\right) \end{bmatrix} \quad (18)$$

To distinguish to the power *n*, the *nn* is used to describe the special dimensions of the discrete wind history. The wind speed history of each dimension is as follows:

$$V_j(t) = \sqrt{2}\sqrt{2\pi}\sum_{m=1}^{j}\sum_{l=1}^{N} I_{jm}(\omega_{ml})\sqrt{\Delta n}\cos\left[2\pi n_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}\right],\ j = 1, 2, \ldots, nn \tag{19}$$

Which is unilateral power spectrum：

$$V_j(t) = \sqrt{2\Delta\omega}\sum_{m=1}^{j}\sum_{l=1}^{N} I_{jm}(\omega_{ml})\cos\left[\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}\right],\ j = 1, 2, \ldots, nn \tag{20}$$

If it is two-sided power spectrum, the coefficient should be $2\sqrt{2\pi}$, and then the wind speed history as follow:

$$V_j(t) = 2\sqrt{2\pi}\sum_{m=1}^{j}\sum_{l=1}^{N} I_{jm}(\omega_{ml})\sqrt{\Delta n}\cos\left[2\pi n_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}\right],\ j = 1, 2, \ldots, nn \tag{21}$$

Which is ：

$$V_j(t) = 2\sqrt{\Delta\omega}\sum_{m=1}^{j}\sum_{l=1}^{N} I_{jm}(\omega_{ml})\cos\left[\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}\right],\ j = 1, 2, \ldots, nn \tag{22}$$

In this paper, single-side spectrum is taken into considered.

The fluctuation wind speed time history now is rewritten as Equation 23.

$$V_j(p\Delta t) = \mathrm{Re}\left\{\sum_{m=1}^{j} G_{jm}(q\Delta t)\exp\left[i\left(\frac{2\pi m\Delta n}{nn}(p-1)\Delta t\right)\right]\right\},\ P = 1, \ldots, M \times nn,\ j = 1, \ldots, nn$$

$$\Delta\omega = \omega_u / N$$

$$\Delta t = 2\pi / (M\Delta\omega) \tag{23}$$

In order to avoid aliasing, the time step Δt has to obey the condition:

$$\Delta t \le 2\pi / 2\omega_u$$

The following condition is established between *N* and *M* (R. Popescu; 1998):

$$M \ge 2N$$

Take *q* as the remainder of p/M, q=1,…,*M*.

$$G_{jm}(q\Delta t) = \sum_{l=1}^{M} B_{jm}(2\pi l\Delta n)(i(2\pi(l-1)\Delta n)((q-1)\Delta t)) \tag{24}$$

In which,

$$B_{jm}(2\pi l\Delta n) = B_{jm}(l\Delta\omega) = \begin{cases} \sqrt{2\Delta\omega} I_{jm}\left(2\pi\left(l-1\right)\Delta n - \frac{m\Delta n}{nn}\right)\exp\left(i\phi_{ml}\right), 1 \le l \le N \\ 0, N \le l \le M \end{cases} \tag{25}$$

Defined $ml = \omega_{ml} = 2\pi\left(l-1\right)\Delta n + \frac{m\Delta n}{nn}$ (26)

In which (Bracewell, R. N., 1986): $I_{jm}\left(\omega_{ml}\right) = \left|I_{jm}\left(\omega_{ml}\right)\right| \exp\left(i\theta_{jm}\left(\omega_{ml}\right)\right)$,

$$\theta_{jm}\left(\omega_{ml}\right) = \arctan\left\{\frac{\mathrm{Im}\left[I_{jm}\left(\omega_{ml}\right)\right]}{\mathrm{Re}\left[I_{jm}\left(\omega_{ml}\right)\right]}\right\}, \quad j = 1, \ldots, nn, \quad m = 1, \ldots, j \tag{27}$$

However, I is the real matrix, then I_m is zero.

$$\theta_{jm}\left(\omega_{ml}\right) = 0, \text{ and } \exp\left(i\theta_{jm}\left(\omega_{ml}\right)\right) = 1, \tag{28}$$

So $I_{jm}\left(\omega_{ml}\right) = \left|I_{jm}\left(\omega_{ml}\right)\right|$

$B_{jm}(2\pi l\Delta n)$ could be rewritten as:

$$B_{jm}(2\pi l\Delta n) = B_{jm}(l\Delta\omega) = \begin{cases} \sqrt{2\Delta\omega}\left|I_{jm}\left(2\pi\left(l-1\right)\Delta n - \frac{m\Delta n}{nn}\right)\right|\exp\left(i\phi_{ml}\right), 1 \le l \le N \\ 0, N \le l \le M \end{cases} \tag{29}$$

Based on the two equation ahead,

$G_{jm}^{(q)} = \sum_{l=1}^{m} B_{jm}^{(l)} \exp\left(\frac{2\pi i\left(l-1\right)\left(q-1\right)}{M}\right)$, $G_{jm}^{(q)}$ is $B_{jm}^{(l)}$ the discrete Fourier transform.

In which,

$$G_{jm}^{(q)} = G_{jm}\left(q\Delta t\right), \quad B_{jm}^{(l)} = B_{jm}(2\pi l\Delta n) = B_{jm}(l\Delta\omega) \tag{30}$$

In those functions, nn is the wind speed spectrum dimension, n_u is the upper cut-off frequency beyond which the power spectral density function may be assumed to be zero for either mathematical or physical reasons. The measure unit of n_u is HZ. According to the energy scaling to describe the number of n_u, which function is $p = \frac{\int_0^{n_u} S(n)dn}{\int_0^{\infty} S(n)dn} \times 100\%$. P is required to be 1, according to the research $n_u \ge 3$, the energy could cover 95%. ml is the sample value of angular frequency, shown as Equation 31.

$$ml = \omega_{jl} = 2\pi\left(l-1\right)\Delta n + \frac{m2\pi\Delta n}{nn} = \left(l-1\right)\Delta\omega + \frac{m\Delta\omega}{nn} \tag{31}$$

To satisfy the ergodic, ω_{jl} , j=1~nn， l=1~M, so the period of the $V_j(t)=V_j(p\Delta t)$ is $T_0 = p\Delta t = nn\times M\times\Delta t$， $p=1,2,\dots,nn\cdot M$.

2.2.5 Programming of fluctuation wind speed simulation

The program of the fluctuation wind speed field sample includes two parts, one is mutual stochastic Fourier spectrum program, and the other one is wind speed field sample program. Mutual stochastic Fourier spectrum program was made to help generate the mutual stochastic Fourier spectrum and resolving it. Wind speed field sample program help to generate the wind speed field sample, which includes the *nn* direction fluctuation wind speed time history.

In this paper, the two programs were programmed by Matlab software, and the programming flow chart can be seen as Figure 1 to Figure2.

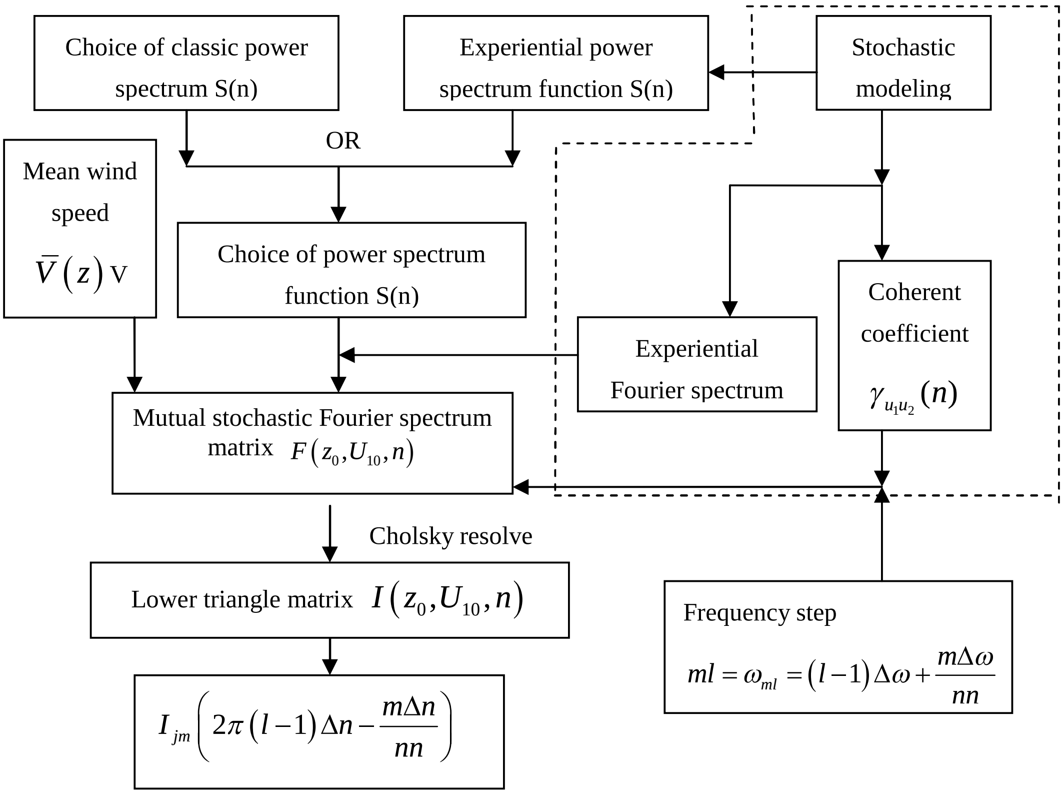


Figure 1 Mutual stochastic Fourier spectrum and its resolve

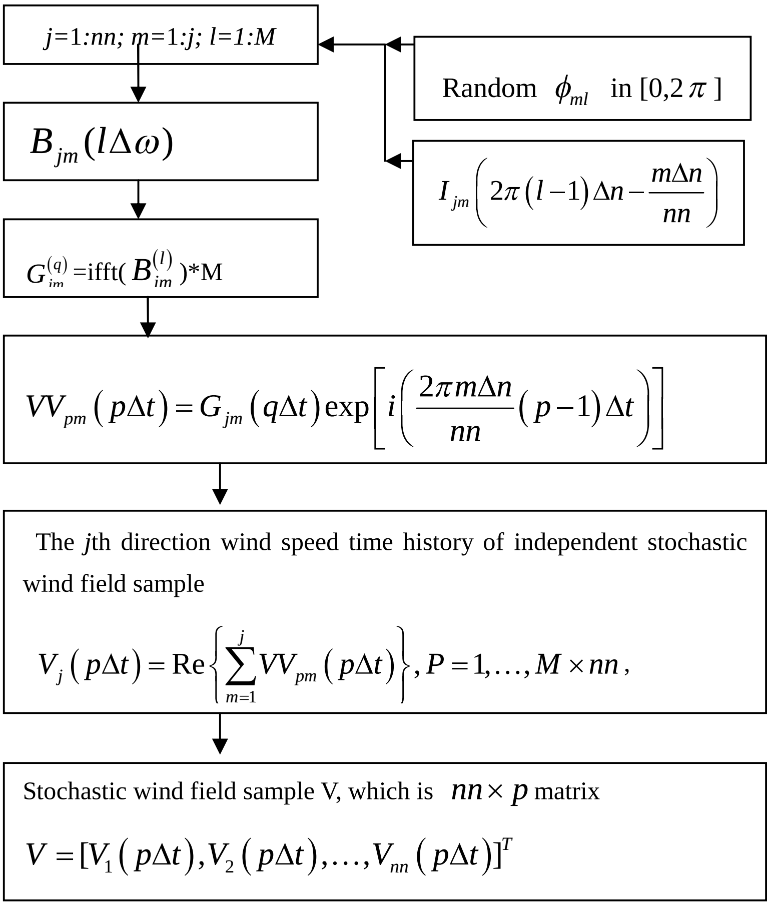
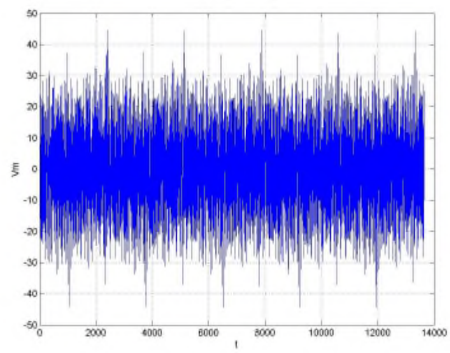


Figure 2 Wind speed field sample program

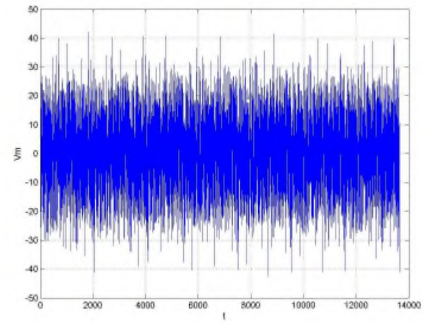
Analysis result of fluctuation wind speed field simulation

Based on the program of wind speed field, fluctuation wind speed power spectrum and time history data are calculated. The main parameters of the analysis includes dimension *nn*, ground roughness length z_0, upper cut-off frequency n_u, modeling time *M*, and annual 10 minutes average maximum wind speed on 10 meter height U_{10}. In this example, the *nn*=20, z_0=0.01(m), n_u=3(Hz), M=4096. Based on the research of the annual 10 minutes average maximum wind speed in south east of China, the distribution should be Pearson-III. Therefore, U_{10} was considered as the maximum wind speed depends on 50 years return period, which is 31(m/s). (Lin Li, 2015)

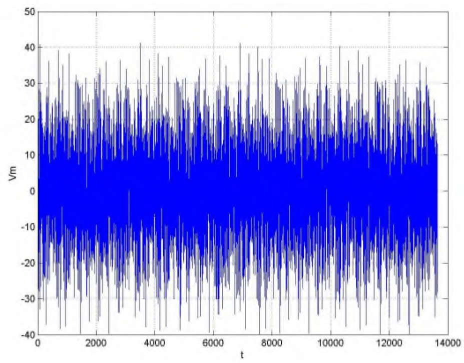
The results of the wind speed time history and power spectrum can be seen in the Figure 3-Figure4. For the sake of analysis, the 5^{th}, 7^{th}, $16^{th}$and 20^{th}dimensions are selected to be simulated. Figure 3 indicates the fluctuation wind speed time history. To verify the accuracy of results, the corresponding wind speed power spectrums in logarithm coordinate as can be seen in the Figure 4. In the Figure 4, the blue line indicates the simulated spectrum while the red dashed line represents the target spectrum which is Davenport spectrum in the paper. It can be seen that the two spectrums meet well with each other, indicating the accuracy of the proposed method.



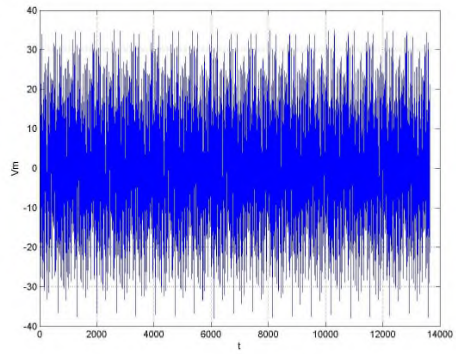
(a): 5th dimension



(b): 7th dimension

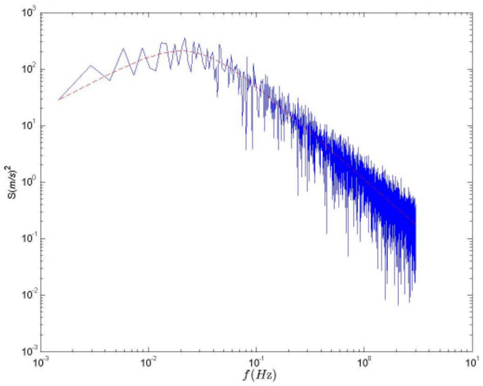


(c): 16th dimension

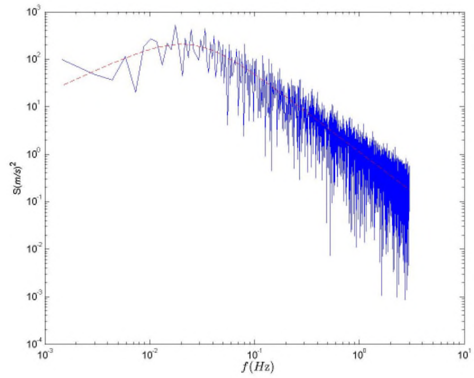


(d): 20th dimension

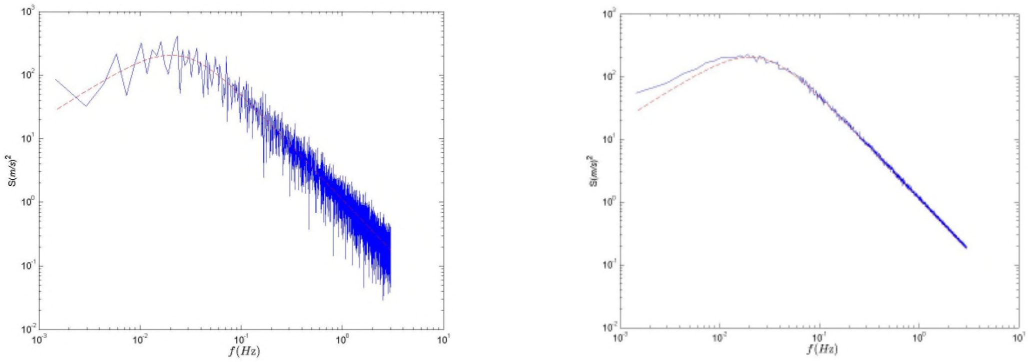
Figure 3 Fluctuating wind speed time history



(a): 5th dimension



(b): 7th dimension



(c): 16th dimension (d): 20th dimension

Figure 4 Fluctuating wind power spectrum in logarithm coordinate

3. Conclusions and discussions

A reasonable wind field time history is of great significance to the analysis of stochastic wind field-induced structural response and reliability. Based on the mutual stochastic Fourier spectrum, a calculation method for stochastic wind field is proposed in the paper. The detailed derivation of equations and programming are introduced in the paper as well. Using the numerical software MATLAB, the simulation results are presented. The simulation spectrums fit target spectrums such as Davenport spectrum very well shown as Figure 4. The plots of simulation spectrums of every dimension match the target spectrums well, especial for the dimension points which near the place corresponding to reference wind speed. The accuracy of the fluctuating wind power spectrum indicates the efficiency of the proposed calculation method.

However, some of the parameters of the analysis are still not deterministic, including the ground roughness length, shear velocity, coefficient of ground roughness, three dimensions correlation coefficients, and so on. Since the validation of proposed method can be guaranteed from the results in the paper. Some further work can be conducted to improve the accuracy of the analysis progress and programming for fluctuating wind field simulation. Among which, the wind field test is an efficient way to determined those parameters for specific wind field analysis.

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