

## **Damage detection for shear buildings using interpolated curvatures of frequency response functions**

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### **ABSTRACT**

Building structures may be experienced natural hazards such as earthquakes and windstorms over their life span and induce severe damage. Thus, structural health monitoring is becoming increasingly important. While techniques of damage detection have been widely developed for structural health monitoring in recent years, the influence of the foundation soil is rarely considered. This paper presents a new method to predict damage locations for a shear building embedded in soil medium by evaluating the curvature of interpolated frequency response functions (FRFs). A parametric analysis is also performed to investigate the effectiveness of the new method on damage localizations while the effects of stiffness loss, noise levels, and damage scenarios are also evaluated. It is observed that the proposed method provides higher probability of detection and lower probability of false alarm to predict damage locations for the shear building in comparisons to the existing method. Moreover, the proposed method is found to have better prediction performance as the FRFs of the structure include a moderate level of random noise. The proposed method may be further applied to structural health monitoring for building structures.

### **1. INTRODUCTION**

Vibration-based damage detection (VBDD) methods have been extensively developed in recent years. Most of them investigated the changes of the dynamic responses to detect, locate and quantify structural damage. Numerous methods (Pandey et al. 1991, Hearn and Testa 1991, Salawu 1997, Kim et al. 2003, Ge and Eric 2005) presented damage indicators based upon the changes of natural frequencies

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and mode shapes between the intact and damaged structure for detecting the structural damage. Those methods need using modal identification techniques to obtain modal parameters of the structure. However, Ewins (2000) found that that experimentally extracted mode shapes may be uncertain and inaccurate. Thus, only a few mode shapes could be accurately obtained from modal identification process. Recently, several methods have been developed using frequency response functions (FRFs) without the need of any modal identification techniques. The FRF-based VBDD methods investigate the variations related to FRF to all frequencies in the measurement range. Sampaio et al. (1999) firstly proposed the FRF curvature method to evaluate the curvature of FRF shapes as a damage indicator. Maia et al. (2003) further presented the FRF-based damage index method based upon the curvature of FRF shapes. Ratcliffe (2000) applied gapped-smoothing method to capture the damage-caused abnormality on the curvature of FRF shapes for damaged structure. Liu et al. (2009) presented a scheme of using FRF curvatures for structural damage localization. The existing methods commonly suggested that the curvature of FRF shapes is directly evaluated by using the central difference approximation. However, Crue and Salgado (2009) indicated that small regularities in the dynamic response not related to damage are also magnified by using the central difference approximation. Kim et al. (2006) had also indicated that the central difference approximation needs a sufficient spatial resolution to obtain accurate the second derivatives.

The previous work of the authors (Shi and Chen 2015) developed an analytical method to calculate the curvature of the vibration responses for shear structures. Using the curvatures evaluated from the proposed method, this study further develops an updated FRF-curvature method to find damage locations for a 12-story shear structure embedded in a soil medium. The effectiveness of the proposed method is to be compared with that of the existing method through the receiver operating characteristics curves. This study also investigates the effects of damage scenarios, noise levels, and shear-wave velocities of soil on the accuracy of damage localization.

## 2. CURVATURE-BASED DAMAGE INDICATOR

In the FRF curvature method (Sampaio et al. 1999), a curvature-based indicator  $S$  is evaluated by adding up a number of absolute differences between the FRF curvatures of the damaged and undamaged structures along the chosen frequency range, as shown below.

$$S = \sum_{j=1}^M |v_d''(\omega_j) - v_o''(\omega_j)|, \quad (1)$$

where  $M$  is the total number of chosen frequencies,  $v$  is the FRF measured,  $v_d''$  is the FRF curvature of the damaged structure and  $v_o''$  is that of the undamaged structure,  $\omega_j$  denotes the  $j$ -th chosen frequency. The existing method suggested that the FRF curvature is directly evaluated using the central difference approximation, i.e.,

$$v_i'' = \frac{(v_{i+1} - 2v_i + v_{i-1}))}{h^2}, \quad (2)$$

where  $v$  is the displacement FRF and  $h$  is the distance between two successive measured locations. This study proposes an updated FRF curvature method while the FRF curvatures are calculated by finding the second derivative of a displacement function  $\Phi(z)$ , as shown below

$$v_i'' = \Phi''(z) = 6\lambda_0 z + 2\lambda_1, \quad \Phi(z) = \lambda_0 z^3 + \lambda_1 z^2 + \lambda_2 z + \lambda_3, \quad (3)$$

Note that the displacement function is a cubic polynomial to interpolate the FRFs measured at the floor level for building structures. An illustrative example is shown in Fig. 1 for a 3-story building. The function coefficients  $\lambda_j$  are determined by the FRFs and the story heights considering the general compatibility criterion for the FRF shape. The proposed interpolation technique was shown in the previous work (Shi and Chen 2015). This study uses both of the finite difference method and the proposed analytical method to evaluate the FRF curvatures for the damage indicator  $S$ . The effectiveness of the updated FRF curvature method on damage localization is also to be investigated and compared with that of the existing method.

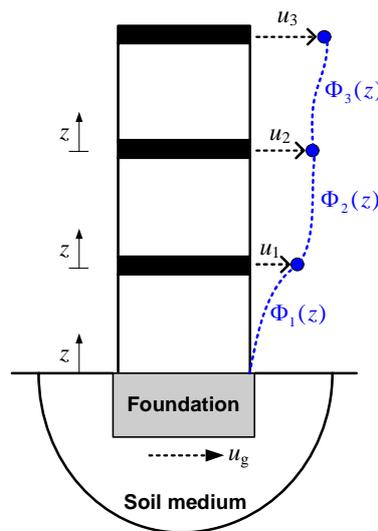


Fig.1 Displacement functions for a 3-story building

### 3. NUMERICAL STUDIES

This paper performs numerical studies to investigate the effectiveness of the updated FRF curvature method on damage localization. Consider a 12-story shear structure embedded in a uniform elastic soil medium. The system parameters for the upper structure, foundation and the soil medium are listed in Table 1. This study analyzes three damage scenarios shown in Table 2. The damage scenarios could simulate successive damages located at the lower stories. In addition, each damage scenario is designed to model the change in the lateral stiffness of the damaged story

as a percentage reduction of the undamaged one. A computer program SASSI (Lysmer et al. 1981) is used in this study to analyze the displacement FRF for the considered building-soil structure undergoing translational excitations. In the dynamic analysis, a frequency range of 0.2 – 20 Hz is chosen. The finite difference method and the proposed method are used to evaluate the curvatures of the FRFs analyzed from SASSI. The numerical results are shown below to investigate the accuracy of the FRF curvature method for finding the damaged story in the building-soil structure.

Table 1 Modeling parameters of the building-soil structure

Zone	Parameter	Value
Upper structure	Story mass (Mg)	100
	Story stiffness (kN/m)	251000
	Story height (m)	1F and 2F: 4, Others: 3
	Model damping ratio	2%
Square Foundation	Side length (m)	20
	Embedment depth (m)	10
	Foundation mass (Mg)	1000
Soil Medium	Poisson's ratio	0.33
	Shear-wave velocity (m/s)	400
	Material density (Mg/m <sup>3</sup> )	2

Table 2 Simulated damage scenarios

Scenario 1		Scenario 2		Scenario 3	
Story	Stiffness loss (%)	Story	Stiffness loss (%)	Story	Stiffness loss (%)
1	20	1	20	1	20
		2	20	2	20
				3	20

### 3.1 Deterministic Identification Results

In consideration of the numerical example, Figs. 1 and 2 display the floor level against the damage indicator  $S$  using the deterministic FRFs analyzed from the computer program. From the results of the finite difference method, a significant peak can be found in Fig. 1 for scenario 1 whose damage locates at the first story. However, the damage indicators of the undamaged floors are abnormally found to be greater than that of the damaged base floor for scenario 2 and 3 with successive damages. Sampaio et al. (1999) indicated that the location of the damage has the largest damage indicator  $S$ . Consequently, Fig. 1(b) and Fig. 1(c) show an unclear identification for successive damage locations, which may be made by the truncate error on the FRF curvatures evaluated from the finite difference method. However, as the proposed method is applied to evaluate the FRF curvatures, Fig. 2 shows that the damage locations may have noticeably greater damage indicators than that of undamaged ones for all of damage scenarios. Hence, the analysis results indicates that the damage indicator  $S$  using the proposed method may give a more clear identification of successive damage locations than that using the finite difference method.

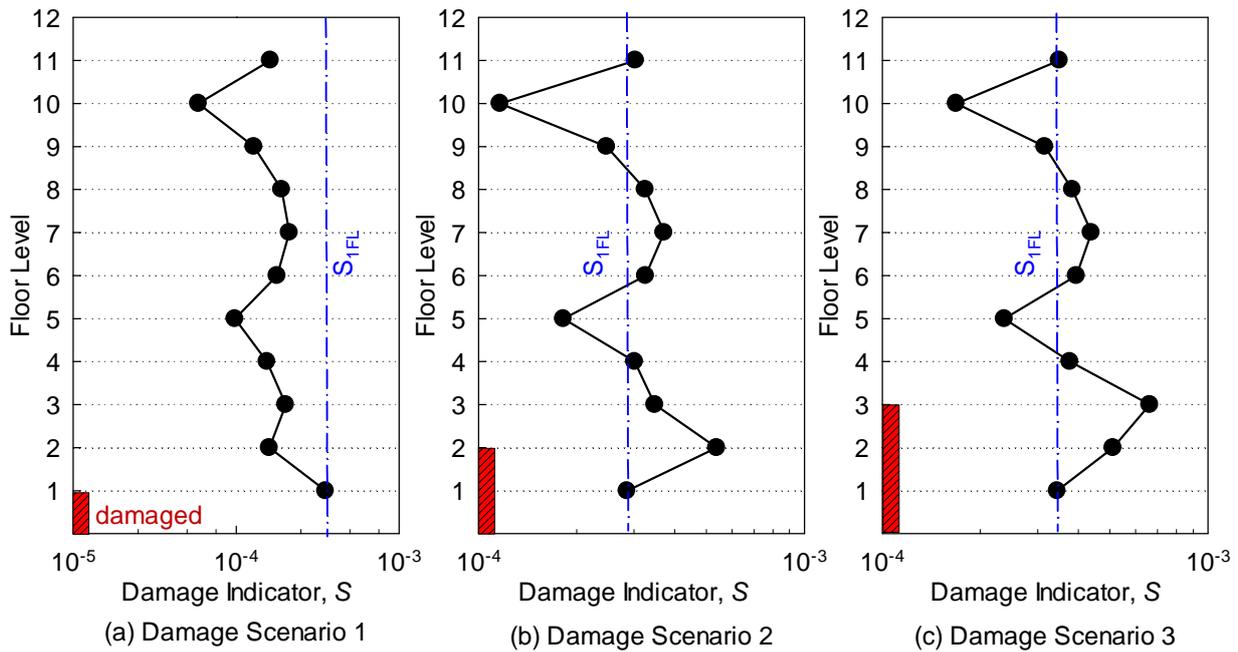


Fig.2 Damage indicator  $S$  using the FRF curvatures from the finite difference method

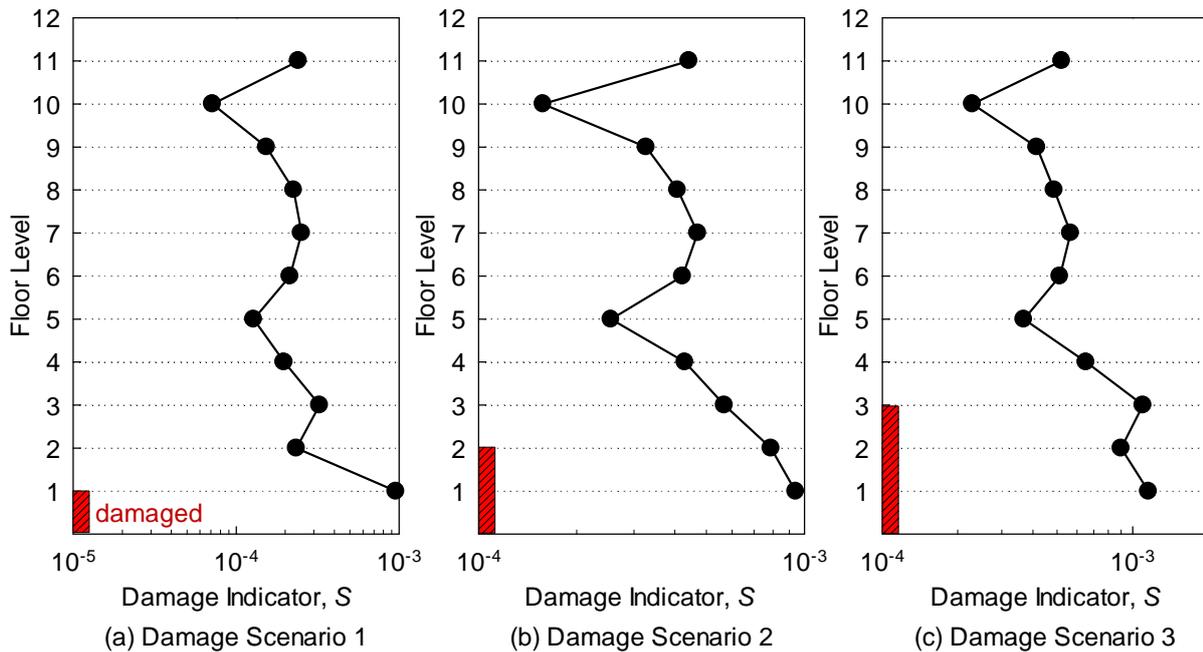


Fig.3 Damage indicator  $S$  using the FRF curvatures from the proposed method

### 3.2 Probabilistic Identification Results

In practical vibration monitoring, it may be unavoidable to explore the structural damage with measurement data contaminated by noise. This study also investigates the damage detection performance using the noise-polluted FRF. In general, any single type of random noise may be experienced in the field due to the complexity of the

vibration measurement process. Thus, a uniformly distributed noise with zero mean is to be considered in this study. In consideration of the shear structure, assume that each FRF has a signal-to-noise ratio (SNR) as a result of adding a series of uniformly distributed random numbers to the FRF obtained from the computer program. The following study considers two SNRs with 80 and 40 dB to simulate a slight and a moderate level of random noise, respectively. Besides, this study further use the receiver operating characteristics (ROC) curves to quantify and evaluate the probabilistic detection performance for the FRF curvature method. ROC curve is a useful technique to visualize and evaluate the performance of classifiers and commonly used in medical decision making. An overview to ROC curves has been presented by Fawcett (2006). An ROC curve is typically created by plotting the true positive rate against the false positive rate for each potential threshold. The true positive rate can be deemed as a probability of detection to indicate the probability of positives correctly classified. In addition, FPR is also known as a probability of false alarm to represent the probability of negatives incorrectly classified. Each point on the ROC curve represents a specific threshold, moving from the most strict at the bottom left corner to the most lenient at the top right corner. In this study, a threshold  $T$  is defined below to calculate the ROC curve.

$$T = \mu_s + \alpha \times \sigma_s, \quad (4)$$

where  $\mu_s$  and  $\sigma_s$  are the mean and standard deviation for the damage indicator  $S$  and  $\alpha$  is a scalar used to control the threshold. The following study chooses  $\alpha$  ranging from 3 to -3 with a step size 0.01 to analyze the ROC curve.

Figs. 4–6 show the ROC curves formed by averaging the results of  $10^5$  test sets for the three damage scenarios while each figure displays the results for two noise levels. Observe that the ROC curve using the proposed method reaches at the perfect point where the probability of detection is 100% and the probability of false alarm is 0. It is obvious that, as the proposed method is applied to evaluate the FRF curvatures, the three damage scenarios may be correctly found using the damage indicator  $S$  even though the displacement FRFs are contaminated by a moderate level of noise. In contrast, as a moderate level of noise with SNR = 40 dB is considered, observe from Figs. 4–6 that the ROC curve using the finite difference method may noticeably lie down the curve using the proposed method. The ROC curve also shows that the finite difference method may give a considerable probability of false alarm than that by the proposed method to achieve a given probability of detection. Moreover, it is also found that the ROC curve using the finite difference method could be close to the diagonal line using random guessing. The results clearly show that the existing FRF curvature method could not perform much well than random guessing for damage localization as the FRFs are contaminated by a moderate level of noise. Therefore, the proposed method may give better results than the finite difference method to estimate the FRF curvatures for correctly finding damage locations.

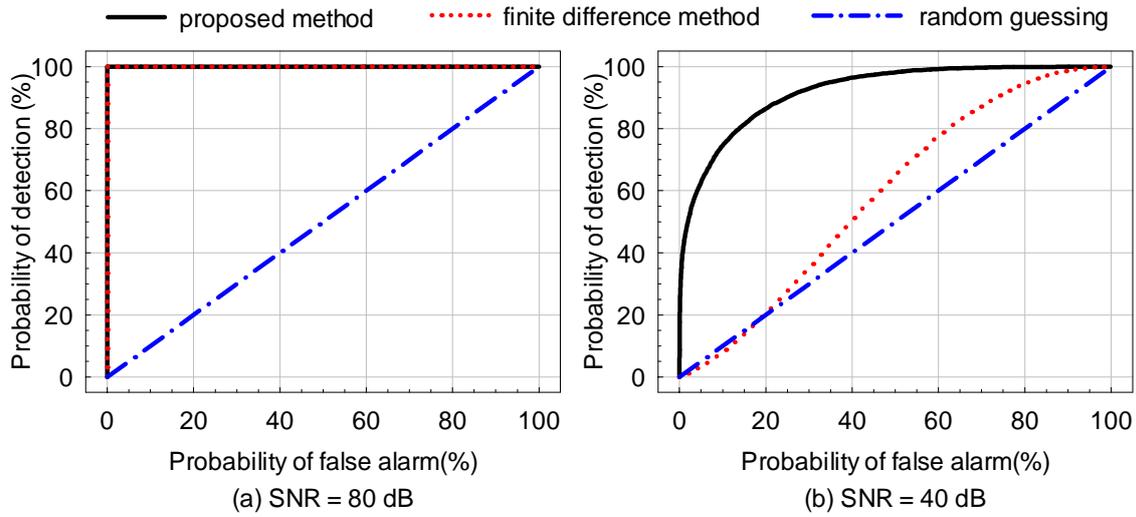


Fig.4 ROC curves for damage scenario 1

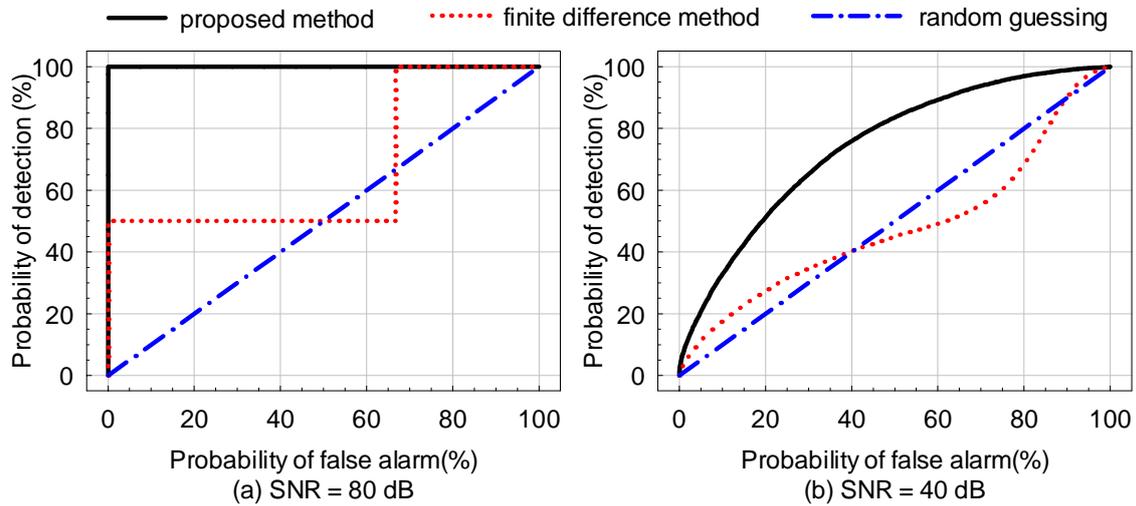


Fig.5 ROC curves for damage scenario 2

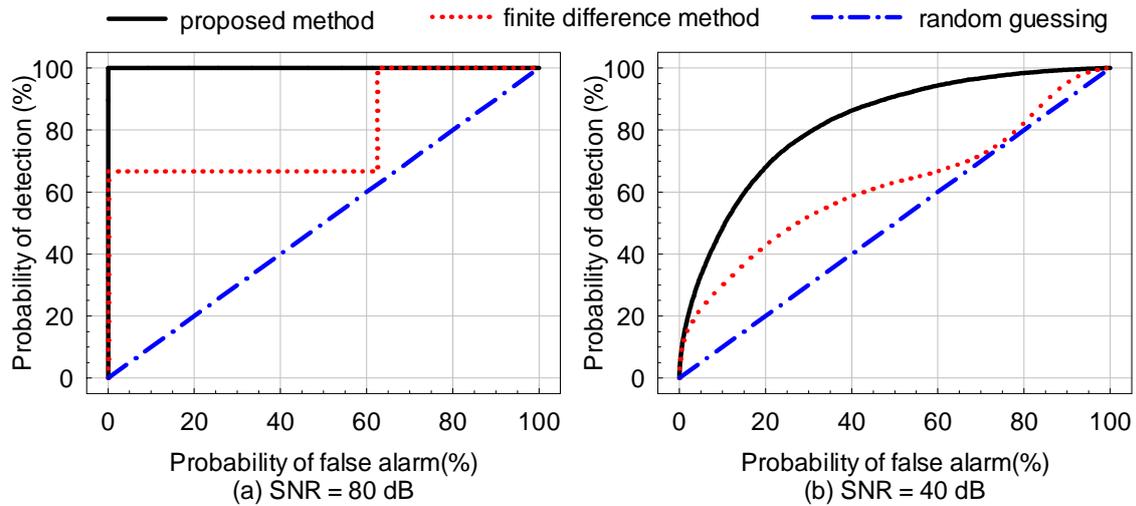


Fig.6 ROC curves for damage scenario 3

For the building-soil structure, the FRFs of the upper structure are influenced by the shear-wave velocity of soil  $V_s$  which has been widely used to measure the stiffness of the soil medium in practice. This study further investigates the effect of the shear-wave velocity on the accuracy of the FRF-curvature method. For the three damage scenarios, Figs. 7-9 display the cost of the probability of false alarm as the probability of detection reaches 95%. In this study, the shear-wave velocity of soil is considered as 1000 m/sec for stiff soil and 100 m/sec for soft soil. Observe that the FRF-curvature method may have a low probability of false alarm as the proposed method is used to evaluate the curvatures. In addition, for damage scenario 1, a significant difference between the results of the two methods is found from SNR = 45 to 55 dB for the case of stiff soil, as shown in Fig. 7(a). However, a similar noticeable difference is observed from SNR = 35 to 45 dB for the case of soft soil, as shown in Fig. 7(b). Moreover, from Figs. 8 and 9, it is obvious that the probability of false alarm changes noticeably as the shear-wave velocity of soil varies. The analysis results clearly indicate that the shear-wave velocity of soil may significantly affect the accuracy of the FRF-curvature method through the soil-structure interaction of the building system. Hence, the effect of the soil medium shall be properly considered on damage detection using the FRF curvature method.

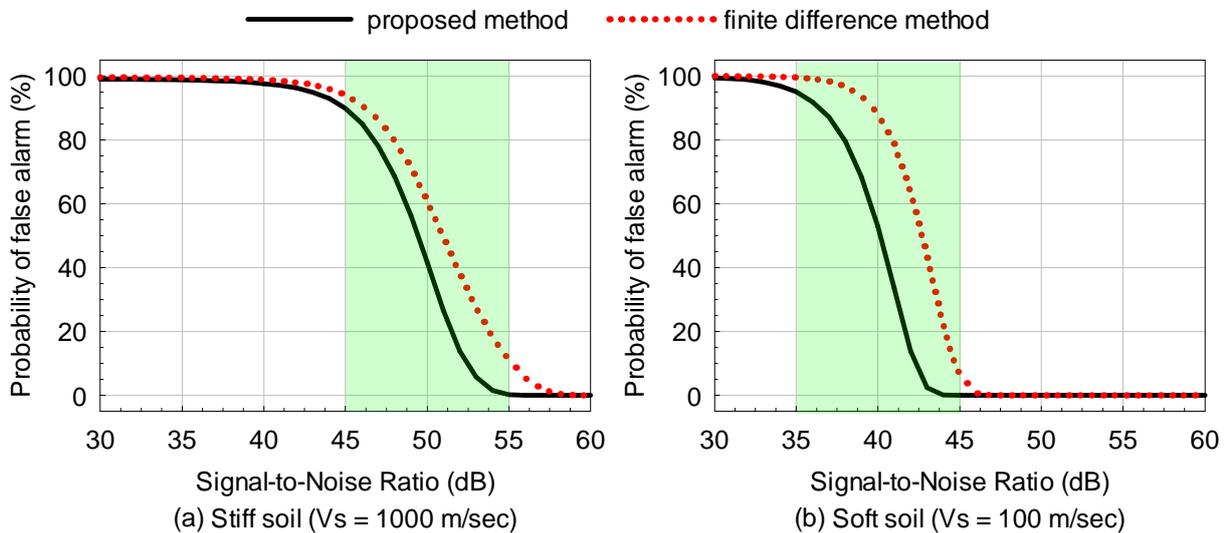


Fig.7 The probability of false alarm against SNR for damage scenario 1

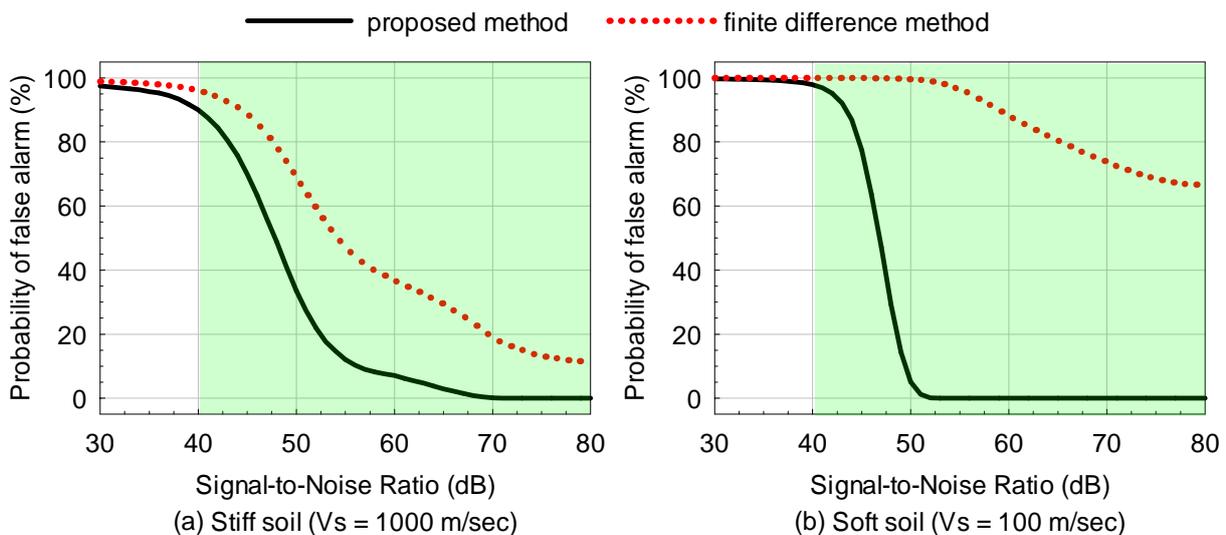


Fig.8 The probability of false alarm against SNR for damage scenario 2

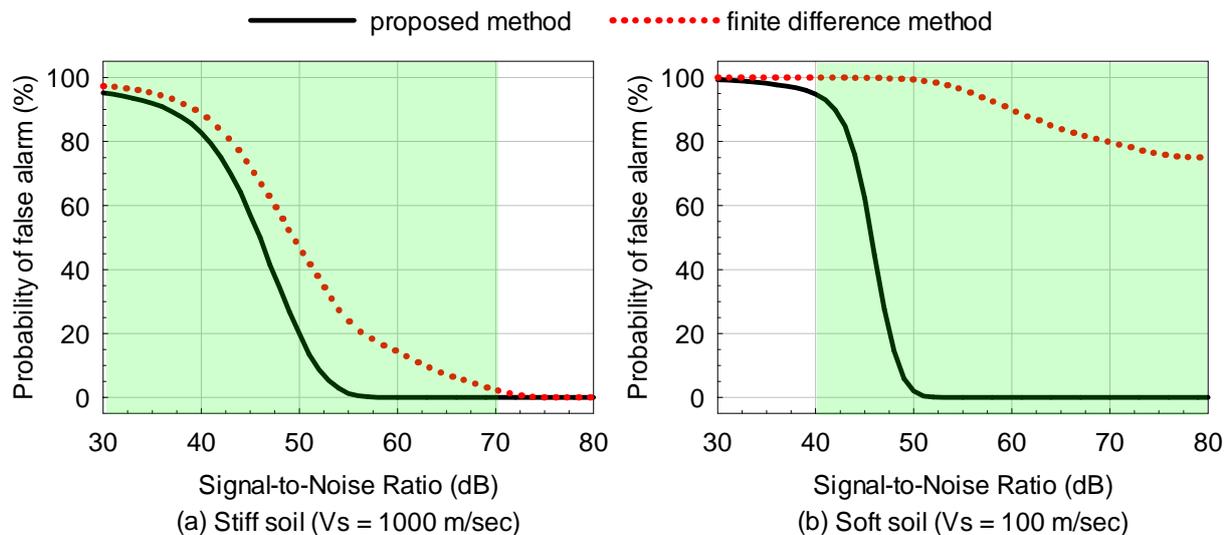


Fig.9 The probability of false alarm against SNR for damage scenario 3

#### 4. CONCLUSIONS

This study presents an updated FRF curvature method for damage detection while the curvatures are evaluated by finding the second derivate of the interpolated frequency response functions. This study also investigates the effectiveness of the proposed method on damage localizations for a 12-story building-soil structure. As the simulated noise is under a moderate level, it is found that the proposed method may pay a lower probability of false alarm to make correct detections on damaged trials in comparison with the existing method. Also, observe that the soil property may significantly affect the accuracy of the FRF-curvature method through the soil-structure interactions of the building system. Therefore, the proposed method could be effectively applied to find damage detections for building structures.

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