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Group Sparse Optimization Based-Compressive Sensing of Vibration Data Using Wireless Sensors for Structural Health Monitoring

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ABSTRACT

For most of vibration signals of civil infrastructures have sparse characteristic, namely, only a few modes contribute to the vibration of the structures. Additionally, the measured vibration data by the sensors placed on different locations of structure almost has same sparse structure in the frequency domain. Based on this group sparsity of the vibration data of structure, the group sparse optimization based compressive sensing (CS) for wireless sensors are proposed. Different from the Nyquist sampling theorem, the data is first acquired by non-uniform low rate random sampling method according to the CS theory. Then, the group sparse optimization algorithm is developed to reconstruction the original data from incomplete measurements. The field tests on Xiamen Haicang Bridge with wireless sensors are carried out to illustrate the ability of the proposed approach. The results show that even using 10% random sampling data, the original data can be reconstructed by the proposed group sparse optimization method with small reconstruction error.

1. INTRODUCTION

The structural health monitoring (SHM) technology has developed about one decades and a lot of civil infrastructures have been installed the SHM systems in the world (Ou and Li, 2010; Li and Ou, 2016). In the wired sensors based SHM system, the wired connection between sensors and data acquisition system reduces the reliability of SHM system, increasing the system cost, and cause great difficulties for the maintenance and replacement. Wireless sensors and wireless network have intelligent data processing capabilities with embedding algorithm and do not have cables that will great reduce the sensor installation cost.

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In SHM, great efforts have been made to explore wireless sensing systems and some academic and commercial smart sensor prototypes have been developed and used in the field of SHM (Lynch and Loh, 2006; Spencer Jr, et al. 2015). Comparing with wired sensor, the wireless sensor and sensor networks need additional energy acquisition technology to ensure the power supply of sensor. In addition, in the long-term monitoring of the structure, the large amounts of data acquisition and wireless transmission are likely to cause the instability of wireless sensor network. The wireless data transmission process will consume most of the energy of sensor battery. Therefore, it is necessary to embed data compression algorithm to reduce the amount of data transmission, as far as possible to minimize energy consumption and prolong the service life of wireless sensor. However, traditional data compression method based on the sampling theorem has its limitations: first complete collection of data, and then the data is compressed. For wireless sensor, data compression process will consume part of the energy. Therefore, new methods data compression methods are needed to effectively improve the wireless sensors and wireless sensor network for long-term SHM.

In this paper, Compressive sensing (CS) is employed to reduce the data sampling of wireless sensors, which data provides a new sampling theory to reduce data acquisition with non-uniform low rate random sampling method, which said that the sparse or compressible signals can be exactly reconstructed from highly underdetermined sets of measurements under the assumption of signal sparsity and under certain conditions on the measurement matrix (Donoho, 2006; Candes, 2006). The potential of CS for the SHM has been investigated widely and many applications of CS have been presented (Bao et al., 2009； Mascarenas et al. 2013).

2. COMPRESSIVE SENSING BASED ON GROUP SPARSE OPTIMIZATION ALGORITHM

Different form the traditional Nyquist uniform sampling, the CS enables non-uniform low rate random sampling. The Nyquist sampling needs to sense *N* samples of a signal to avoid information loss; however, CS randomly sensing much fewer $M \ll N$. But according to the CS theory, if the original is sparse, CS is able to exactly recover it from far fewer incoherent random measurements than what is required by sampling theorem.

Suppose there are *K* sensors which are placed in the structure. The acceleration is measured at discrete time t_j, $j=1,\ldots,M$ by each sensor. Here, we assume the total time span of samples is $[0,T]$ and the sample time is uniformly distributed, i.e. $t_j = jT/M$, $j=1,\ldots,M$. By collecting all the data together, we get a $M\times K$ matrix, $\mathbf{Y}$,

$$\mathbf{U}=\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1K} \\ u_{21} & u_{22} & \cdots & u_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N1} & u_{N2} & \cdots & u_{NK} \end{bmatrix} \quad (1)$$

where u_{mk} is the data measured by *k*th sensor at time t_m.

Then, the signal matrix, $\mathbf{U}$ is usually an incomplete matrix. Let $\mathbf{\Omega}=\left\{(m,k):U_{m,k} \text{ is available}\right\}$ and $P_\Omega : R^{M\times K} \to R^{M\times K}$ is the zero padding operator, that is $\mathbf{Y}=P_\Omega \mathbf{U}$,

$$y_{m,k}=\begin{cases} u_{m,k}, & (m,k)\in\Omega \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Then the problem we are facing is from the given signal $\mathbf{Y}=P_\Omega \mathbf{U}\in R^{M\times K}$, to calculate the original signal matrix $\mathbf{U}$.

The signal matrix $\mathbf{U}$ can be represented as

$$\mathbf{U}=\mathbf{\Psi X} \tag{3}$$

where $\mathbf{\Psi}$ is a Fourier matrix

$$\mathbf{\Psi}=\begin{bmatrix} e^{i2\pi t_1/T} & e^{i4\pi t_1/T} & \cdots & e^{i2M\pi t_1/T} \\ e^{i2\pi t_2/T} & e^{i4\pi t_2/T} & \cdots & e^{i2M\pi t_2/T} \\ \vdots & \vdots & \ddots & \vdots \\ e^{i2\pi t_M/T} & e^{i4\pi t_M/T} & \cdots & e^{i2M\pi t_M/T} \end{bmatrix} \tag{4}$$

where $\mathbf{X}\in R^{M\times K}$ are the Fourier coefficients of original signal only has a few nonzero rows. This representation gives us alternative way to recover the signal matrix $\mathbf{U}$, which is so called joint sparsity.

Considering the measurement noise, the Eq. (3) is changed to

$$\mathbf{U}=\mathbf{\Psi X}+\boldsymbol{\varepsilon} \tag{5}$$

where $\boldsymbol{\varepsilon}\in R^{M\times K}$ is Gaussian noise matrix.

To take advantage of the joint sparsity, usually, we try to minimize the $\|\cdot\|_{p,q}$ norm of $\mathbf{X}\in R^{M\times K}$, where

$$\|\mathbf{X}\|_{p,q}=\left(\sum_{m=1}^{M}\|\mathbf{x}_m\|_p^q\right)^{1/q} \tag{6}$$

One of the most often choice of p, q is $p=2$, $p=1$, which is also the value of p, q we used in this paper. Then the Fourier coefficients matrix $\mathbf{X}$ is recovered by solving the following optimization problem:

$$\min_{\mathbf{X}\in C^{M\times K}} \|\mathbf{X}\|_{2,1}+\frac{\mu}{2}\left\|P_\Omega(\mathbf{\Psi X})-P_\Omega(\mathbf{U})\right\|_2^2 \tag{7}$$

Once the optimal solution $\mathbf{X}_{rec}$ is obtained, the recovered signal matrix is given by

$$\mathbf{U}_{rec}=\mathbf{\Psi X}_{rec} \tag{8}$$

3 FIELD TEST OF BRIDGE

3.1 Description of the test

The field test on Xiamen Haicang Bridge is carried out. The bridge is a steel box girder suspension bridge with a span distribution of 230m+648m+230m as shown in Figure 1. The test schemes are shown in Figure 2, which shows the tests are repeated with 9 times and the totally test points are 70, where the test point No. 30 are selected as the reference point. Nine commercial wireless velocity sensors as shown in Figure 2 are used in this test and the sampling frequency for data acquisition is 100Hz.

Figure 1. Xiamen Haicang bridge

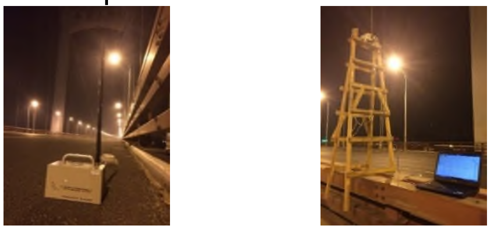
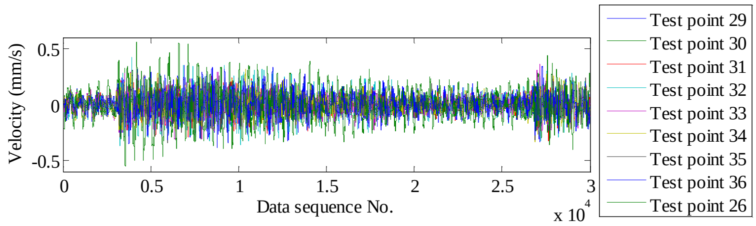


Figure 2. The wireless sensor node and wireless base station

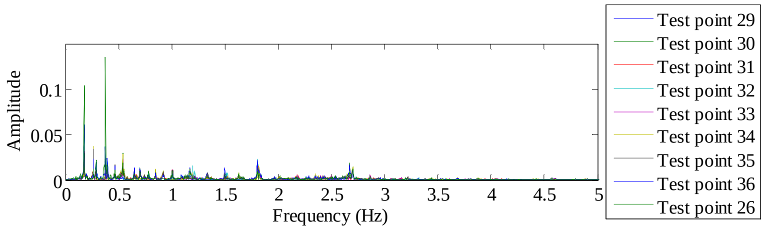
The representative dataset measured from the bridge are shown in Figure 3(a) and the corresponding Fourier spectrum are shown in Figure 3(b), which shows the multiple sensors data almost has similar sparsity in the frequency domain. To further illustrate the similar sparsity of multiple sensors data, the cross correlation of the Fourier amplitude spectrum are calculated by:

$$\gamma_{XY} = \frac{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2}\sqrt{\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2}} \tag{9}$$

The results show that the minimal and maximal cross correlation coefficient is 0.5117 and 0.9589, respectively. Most of the cross correlation coefficient within a range of [0.7, 0.95]. These cross correlation coefficients further indicate the group sparsity of the signal.



(a) The measured velocity data of test points 29-36

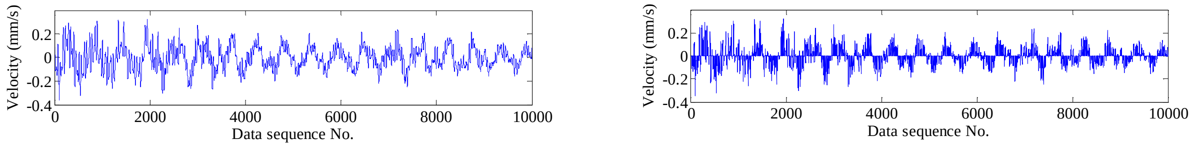


(b) The Fourier specturm of the measured velocity data of test points 29-36

Figure 3. The typitical measurements and Fourier specturm

3.2 Data sampling by CS

The typical measured velocity data by wireless sensor is shown in Figure 4(a). We simulate the non-uniform low-rate random sampling of CS, the data with 10% and 30% samples are shown in Figure 4 (a, b).

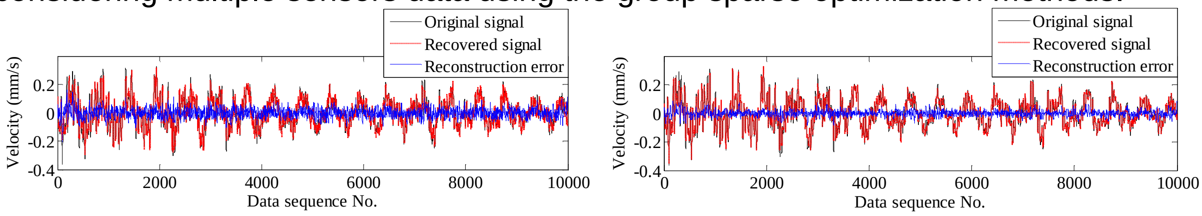


(a) (b)

Figure 4. The sampling data by CS: (a) original data; (b) 30% samples

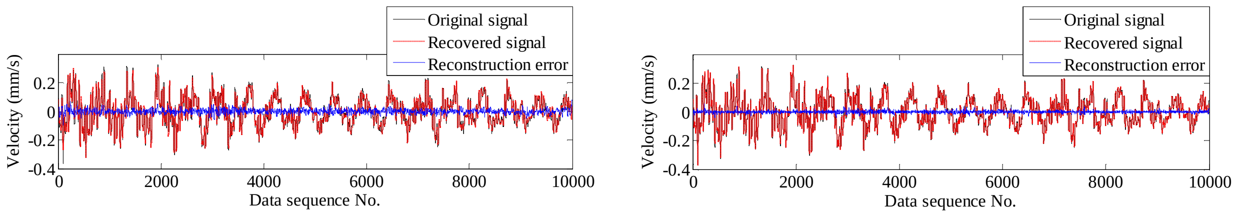
3.3 Data reconstruction results

The data reconstruction results for 10% and 30% samples are shown in Figures 5 and 6, in which the Figures 5(a) and 6(a) are the reconstruction results using only one sensor data and Figures 5(b) and 6(b) are the reconstruction results with group sensors data. These figures show that the smaller reconstruction errors can be achieved by considering multiple sensors data using the group sparse optimization methods.



(a) (b)

Figure 5. Data reconstruction results from 10% samples: (a) reconstruction from single sensor data; (b) reconstruction from group sensors data



(a) (b)

Figure 6. Data reconstruction results from 30% samples: (a) reconstruction from single sensor data; (b) reconstruction from group sensors data sets

4. CONCLUSIONS

This paper proposed a group sparse optimization method for compressive sensing data reconstruction of wireless sensors for structural health monitoring, which used the group sparsity of structural vibration data of multiple sensors to improve the data reconstruction accuracy.

Field tests results on Xiamen Haicang Bridge show that even using 10% random sampling data, the original data can be reconstructed using the group sparse optimization method with small reconstruction error. Comparing with data reconstruction, the smaller reconstruction errors can be achieved by considering multiple sensors data using the group sparse optimization methods than the method only using single sensor data.

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REFERENCES

[1] Ou, J. and Li, H. (2010), “Structural health monitoring in mainland China: review and future trends”, *Struct. Health Monit.*, **9**(3), 219-232.
[2] Li H. and Ou J. (2016), “The state of the art in structural health monitoring of cable-stayed bridges”, *J. Struct. Health Monit.*, 2016, **6**(1), 43-67
[3] Lynch, J. P. and Loh, K. J. (2006), “A summary review of wireless sensors and sensor networks for structural health monitoring”, *Shock Vib. Digest*, **38**(2), 91-130.
[4] Spencer Jr, B. F., Jo, H., Mechitov, K. A., Li, J., Sim, S. H., Kim, R. E. and Agha, G. (2015), “Recent advances in wireless smart sensors for multi-scale monitoring and control of civil infrastructure”, *J. Struct. Health Monit.*, 1-25.
[5] Donoho, D. (2006), “Compressed Sensing”, *IEEE T. Signal Proces.*, **52**(4),1289-1306.
[6] Candès, E.J. (2006), “Compressive Sampling”, Proceedings of the International Congress of Mathematicians, Madrid, Spain,1433-1452.
[7] Bao, Y. Beck, J.L. and Li, H. (2011), “Compressive sampling for accelerometer signals in structural health monitoring”, *Struct. Health Monit.*, **10**(3), 235-246
[8] Mascarenas, D., Cattaneo, A., Theiler, J., and Farrar, C. (2013). “Compressed sensing techniques for detecting damage in structures”, *Struct. Health Monit.*, **12**(4), 325-338.