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Modelling Error of Using a Shear Model to Model a Complex Building Structure

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ABSTRACT

Because of the simplicity of shear model, it is one of most widely adopted structural dynamic models for building structures. However, due to the difference of structural dynamics, shear model may not be able to fully represent the dynamic responses of real building very well. To the best of the authors' knowledge, little research have been carried out to study the modelling error of using a shear model to model a complex building structure. In this paper, this issue is investigated. First, the principal of equivalent interstory shear force is introduced, which provides a new way of studying the modelling error of shear model. Second, it is proved that if the interstory shear forces of a simple model (e.g. shear model) are equal to those of building structure, then the simple model will have no modelling error, in terms of the horizontal responses of building floors. Third, a novel reduce model is proposed for building structures, which have the same interstory shear forces as the building structure. In the new model, the effect of beam-column joint rotation on the interstory shear forces is reflected by some average rotational responses that can be computed by a weighted linear combination of building floors' horizontal responses. Fourth, an optimization strategy is proposed to obtain the weight factors used to calculate the average rotation response. Finally, a numerical example is utilized to demonstrate that the new proposed reduced model is able to represent the dynamic responses of a frame structure much better than the shear model.

1. INTRODUCTION

As important infrastructures, high-rise buildings play crucial roles in the normal

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operation of our modern society. However, the aging of construction materials gradually deteriorates structural carrying capacity, which poses a threat to the safety of structures, especially for those located in seismic active or hurricane prone area. Therefore, research community have devoted a lot of endeavor to developing accurate and efficient structural health monitoring techniques. Among many choice of SHM methods, vibration-based methods have attracted a lot of attention, due to the easy accessibility of structural vibration responses and the potential capacity of these method to detect multiple damage in a large structure (Doebling, 1996; Beck, 2004; Johnson, 2004).

According to whether or not a physical structural model (i.e., finite element model) is used, vibration-based SHM methods can be further classified into non-model based and model based methods. Non-model based methods usually make use of the change of structural dynamic features (e.g., natural frequency, mode shape, dynamic flexibility, etc.) to detect structural damage, while the model based methods generally utilize structural identification technique to direct estimate the parameters of the structural model. Compared with non-model based method, model based methods can determine the location and severity of structural damage by directly comparing the estimated structural parameters before and after the occurrence of structural damage. Therefore, model based methods are more preferred than non-model based methods.

From mathematical point of view, structural identification is an ill-conditioned problem and is difficult to provide accurate estimating results when applied to complexed structures. Therefore, while performing structural identification, a simplest possible model that can represent the major characteristics of the structure (Zhang, 2014) is usually preferred. For frame structure studied herein, a shear model is frequently adopted due to its simplicity. The estimated story stiffness parameters of the shear model are then used to diagnose the damage status of the frame structure. However, due to the difference of structural dynamics, shear model may not be able to fully represent the dynamic responses of real building very well. To the best of the authors' knowledge, little research have been carried out to study the modelling error of using a shear model to model a complex building structure, which will be investigated in this paper.

This paper is organized as follows: First, the principal of equivalent interstory shear force is introduced, which provides a new way of studying the model erroring of shear model. Second, it is proved that if the interstory shear forces of a simple model (e.g. shear model) are equal to those of building structure, then the simple model will have no modelling error, in terms of the horizontal responses of building floors. Third, a novel reduce model is proposed for building structures, which have the same interstory shear forces as the building structure. In the new model, the effect of beam-column joint rotation on the interstory shear forces is reflected by some average rotational responses that can be computed by a weighted linear combination of building floors' horizontal responses. Fourth, an optimization strategy is proposed to obtain the weight factors used to calculate the average rotation response. Finally, a numerical example is utilized to demonstrate that the new proposed reduced model is able to represent the dynamic responses of a frame structure much better than the shear model.

2. Principal of Equivalent Interstory Shear Force

Modelling error is defined as the discrepancy between the responses of real physical system and those predicted by the model. From system identification point of view, people always prefer to find a simplest possible model that can represent the major characteristics of structural responses observed. For frame building structures, usually only floor horizontal responses are measured; thus, people always want to find a simple model for frame buildings, which can reflect the floor horizontal responses and, simultaneous, the parameterization of which is simple. In this regard, shear structure is probably the most adopted model. However, it is well known that the shear structure ignores beam-column joint rotations of the frame; therefore, there are modelling errors exist for the shear structure model to predict the floor horizontal responses of real frame structures. But, to the best of the authors' knowledge, little research have been carried out to quantitatively evaluate these modelling errors, which is probably due to following reasons: to compare the modelling errors, the parameters of the shear structure model must be determined first. When identifying the structural parameters, some optimization problems are usually formulated that try to minimize the measured floor horizontal responses and those predicted by the shear model. However, it is well known that structural parameters identification is ill-conditioned process that is difficult to provide the accurate estimating results, particularly in the situation that the modelling error of the model exists. Usually the parameter identified will usually be widely scattered. Which parameter values of the shear model should be used to evaluate modelling error becomes a tough problem.

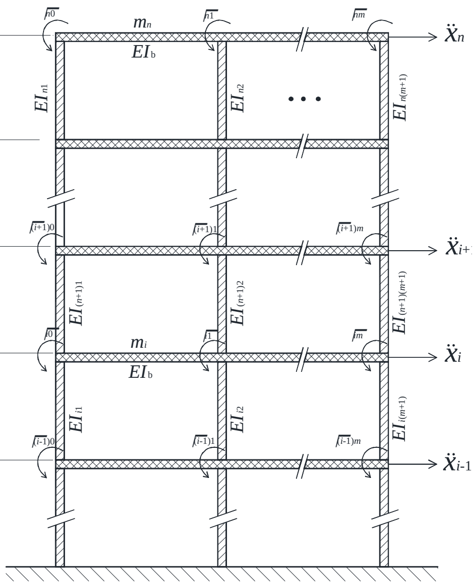

Fig. 1 a n-story frame structure

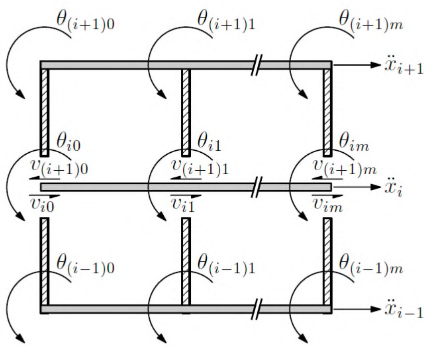

Fig. 2 the *i*-th floor substructure

Fig. 1 shows a general *n*-story and *m*-bay frame structure. According to the second Newton's law, the *i*-th floor horizontal acceleration of the structure can be represented by Eq. (1)

$$m_i \ddot{x}_i(t) = \sum_{j=0}^{m} v_{ij}(t) - \sum_{j=0}^{m} v_{(i+1)j}(t) = v_i(t) - v_{i-1}(t) \tag{1}$$

where m_i is the *i*-th floor mass of the structure; $\ddot{x}_i(t)$ is the acceleration of the *i*-th floor at time *t*; $v_{ij}(t)$ denotes the shear force of the j-th column in the i-th story. $v_i(t)$ and $v_{i-1}(t)$ denote the interstory shear forces of the *i*-th and (*i*-1)-th stories, respectively. It is assumed herein that the structural masses are concentrated on the floor levels only.

Eq. (1) indicates that if the interstory shear forces of a simple model is equal to those of the real frame building, then the *i*-th floor acceleration of the simple model will be same as that of the real structure. In other words, from the floor horizontal acceleration's point of view, the simple model is equivalent to the real building structure.

$$\begin{aligned}\int_0^{t_f} m_i^2\left[\ddot{x}_i(t)-\ddot{\tilde{x}}_i(t)\right]^2 dt &= \int_0^{t_f}\left\{[v_i(t)-\tilde{v}_i(t)]-[v_{i-1}(t)-\tilde{v}_{i-1}(t)]\right\}^2 dt \\ &= \int_0^{t_f}\left[v_i(t)-\tilde{v}_i(t)\right]^2 dt+\int_0^{t_f}\left[v_{i-1}(t)-\tilde{v}_{i-1}(t)\right]^2 dt \\ &\quad -2\int_0^{t_f}\left[v_i(t)-\tilde{v}_i(t)\right]\left[v_{i-1}(t)-\tilde{v}_{i-1}(t)\right]dt \\ &\le 2\int_0^{t_f}\left[v_i(t)-\tilde{v}_i(t)\right]^2 dt+2\int_0^{t_f}\left[v_{i-1}(t)-\tilde{v}_{i-1}(t)\right]^2 dt\end{aligned} \tag{2}$$

For the *n*-story and *m*-bay plane frame structure shown in Figure 1, the elastic restoring force $v_i(t)$ of the i^{th} story can be expressed as

$$v_i=\sum_{j=0}^{m}\left[-\frac{12EI_{ij}}{H_i^3}(x_i-x_{i-1})-\frac{6EI_{ij}}{H_i^2}(\theta_{ij}+\theta_{(i-1)j})\right] \tag{3}$$

where EI_{ij} (i = 1,⋯,n; j = 0,⋯,m) is the flexural rigidity of the *j*-th column in the *i*-th story; θ_{ij} is the rotation of the *j*-th beam-column joint in the *i*-th floor; H_i is the height of the *i*-th story.

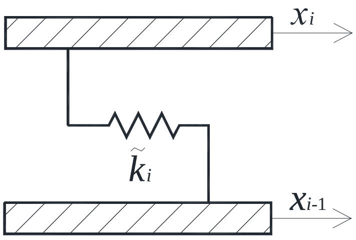

Fig. 3 Shear structure model for the *i*-th story

If a shear structure is used to model the frame structure, the *i*-th story structure will be modeled as a spring element as shown in Fig. 3. Then, the *i*-th interstory shear force in the shear structure can be represented as

$$\tilde{v}_i=k_i[x_i-x_{i-1}] \tag{4}$$

Comparing Eq. (2) and (3), it can be seen that shear structure does not include the effects of beam-column joint rotations on structural interstory shear forces, which is the root for the modelling of using a shear structure to model the dynamic responses of a frame building structure.

In the next section, a novel model is proposed, which can reflect the effects of beam-column joint rotations on structural interstory shear forces and, thus, can greatly reduce the modelling error.

3. A NOVEL REDUCE MODEL FOR PLANE FRAME STRUCTURE

For simplicity of illustration, the following variables are defined

$$k_i = \frac{12}{H_i^3} \sum_{j=0}^{m} EI_{ij} \quad (i = 1, \cdots, n) \tag{5}$$

$$\theta_i^- = \sum_{j=0}^{m} \left[\theta_{ij} EI_{ij} \Big/ \sum_{k=0}^{m} EI_{ik} \right] \quad (i = 1, \cdots, n) \tag{6}$$

$$\theta_i^+ = \sum_{j=0}^{m} \left[\theta_{ij} EI_{(i+1)j} \Big/ \sum_{k=0}^{m} EI_{(i+1)k} \right] \quad (i = 1, \cdots, n-1) \tag{7}$$

where k_i can be considered the equivalent stiffness of the *i*-th story which is the story stiffness when the floor beams are perfectly rigid and the frame vibrates as a true shear structure (i.e., the rotational responses of all joints in the frame are zeros); θ_i^- and θ_i^+ are the weighted average rotational responses of all joints in the ith floor weighted by the relative flexural stiffness of columns in the story below (the ith story)and the story above (the (i+1)-th story), respectively. Note that, for the top (nth) floor, the second rotational response θ_n^+ does not exist. Note also that $\theta_i^-(t) \equiv \theta_i^+(t)$ if $EI_{ij} \Big/ \sum_k EI_{ik} = EI_{(i+1)j} \Big/ \sum_k EI_{(i+1)k}$, which occurs if the distribution of stiffness across the columns is consistent on floors *i* and *i*+1.

Using the above defined variables, the *i*-th interstory shear force in Eq. (3) can be rewritten as

$$\begin{aligned} v_i &= k_i [x_i - x_{i-1} + (\theta_i^- + \theta_{i-1}^+) H_i / 2] \\ &= k_i [x_i - x_{i-1} + \Delta_i] \end{aligned} \tag{8}$$

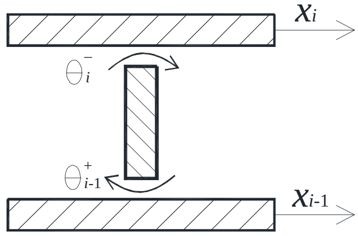

Fig. 4 New model for the *i*-th story

where $\Delta_i = (\theta_i^- + \theta_{i-1}^+) H_i / 2$, which reflects the contribution of beam-column joint rotation on the structural interstory shear force.

A new model of frame structure, the *i*-th interstory shear force of which can be represented by Eq. (8), is shown in Figure 4. Since the model has the same interstory shear force as frame building structures, it should have no modelling errors according the principle of equivalent interstory shear force. It is worth pointing out that the contribution of the interstory shear force by structural damping is not considered in the new model; thus, if damping force is considered, the new model may still have some modelling errors.

However, to use the new reduced model, Δ_i are need to be known, which requires measuring all joint rotations in the frame, very difficult to achieve in practice.

To solve this problem, a method is proposed to estimate these average rotational responses from the measured floor translational responses. Rearranging the order of the translational and rotational DOFs of the frame, the dynamic equation of the frame can be written (neglecting structural damping for simplicity) in block form as

$$\begin{bmatrix} \mathbf{M}_{\mathrm{tt}} & \mathbf{M}_{\mathrm{tr}} \\ \mathbf{M}_{\mathrm{rt}} & \mathbf{M}_{\mathrm{rr}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_{\mathrm{t}} \\ \ddot{\mathbf{x}}_{\mathrm{r}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{tt}} & \mathbf{K}_{\mathrm{tr}} \\ \mathbf{K}_{\mathrm{rt}} & \mathbf{K}_{\mathrm{rr}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{t}} \\ \mathbf{x}_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\mathrm{t}} \\ \mathbf{F}_{\mathrm{r}} \end{bmatrix} \tag{9}$$

where subscripts “t” and “r” denote the parts of the structural parameters and responses associated with the translational and rotational DOFs, respectively. It is assumed herein that the structural mass related to rotational DOF are negligible and no external moments are applied directly on the joints of the frame. Let $\mathbf{\Delta} = \begin{bmatrix} \Delta_1 & \Delta_2 & \cdots & \Delta_n \end{bmatrix}^{\mathrm{T}}$ be the average rotation response vector needed to be estimated, then

$$\mathbf{\Delta} = \mathbf{T}\mathbf{x}_{\mathrm{r}} = -\mathbf{T}\mathbf{K}_{\mathrm{rr}}^{-1}\mathbf{K}_{\mathrm{rt}}\mathbf{x}_{\mathrm{t}} = \mathbf{H}\mathbf{x}_{\mathrm{t}} \tag{10}$$

where T is the linear transformation matrix from the joint rotations $\mathbf{x}_{\mathrm{r}}$ to $\mathbf{\Delta}$. Eq. (10) demonstrates that the contribution $\mathbf{\Delta}$ of joint rotations to the structural interstory shear force can be represented by a linear combination of structural floor horizontal responses. If the stiffness matrix of the structure is known, the matrix H of combination factors can be easily computed. However, the stiffness matrix of the structure is usually unknown. Therefore, in the next section, an optimization method will be proposed to estimate the combination factors in matrix H by using the measured structural floor horizontal acceleration responses.

4. Estimating Combination Factors in Matrix H

The dynamic equation of the top n-th floor can be written in Eq. (11)

$$v_n = m_n \ddot{x}_n = k_n [x_n - x_{n-1} + \mathbf{a_n X}] \tag{11}$$

where $\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^{\mathrm{T}}$ is the vector of structural displacement responses, $\mathbf{a_n}$ is the *n*-th row in matrix H. Taking a Fourier transform of Eq. (11), the following optimization method can be formulated to estimation vector $\mathbf{a_n}$.

$$\underset{\mathbf{a_n}}{\arg\min} \quad J(\mathbf{a_n}) = \sum_{l=1}^{N} \left| m_n \ddot{X}_n(\omega_l) - \frac{k_n}{(j\omega_l)} \left[\ddot{X}_n(\omega_l) - \ddot{X}_{n-1}(\omega_l) + \mathbf{a_n} \ddot{\mathbf{X}}(\omega_l) \right] \right| \tag{12}$$

where $\ddot{X}_i(\omega_l)$ is the Fourier transform of structural response $\ddot{x}_i$ at frequency ω_l, $\ddot{\mathbf{X}}(\omega_l)$ is the Fourier transform of structural acceleration vector $\ddot{\mathbf{X}} = \begin{bmatrix} \ddot{x}_1 & \ddot{x}_2 & \cdots & \ddot{x}_n \end{bmatrix}^{\mathrm{T}}$ at frequency ω_l. After the *n*-th row of matrix H is estimated, other rows of matrix H can be obtained as follows.

The dynamic equation of any i-th non-top floor can be written as

$$\begin{aligned} v_i &= m_i \ddot{x}_i - k_{i+1}[x_{i+1} - x_i + \mathbf{a}_{i+1}\mathbf{X}] \\ &= k_i [x_i - x_{i-1} + \mathbf{a}_i \mathbf{X}] \end{aligned} \tag{13}$$

Assuming that vector $\mathbf{a}_{i+1}$ is known, the following optimization problem can be established to estimate vector $\mathbf{a}_i$

$$\arg\min_{\mathbf{a_n}} \quad J(\mathbf{a_n}) = \sum_{l=1}^{N} \left| \begin{aligned} & m_i \ddot{X}_i(\omega_l) - \frac{k_{i+1}}{(j\omega_l)} \left[\ddot{X}_{i+1}(\omega_l) - \ddot{X}_i(\omega_l) + \mathbf{a}_{i+1}\ddot{\mathbf{X}}(\omega_l) \right] \\ & + \frac{k_i}{(j\omega_l)} \left[\ddot{X}_i(\omega_l) - \ddot{X}_{i-1}(\omega_l) + \mathbf{a}_i \ddot{\mathbf{X}}(\omega_l) \right] \end{aligned} \right| \tag{14}$$

Since the *n*-th row vector $\mathbf{a_n}$ of matrix H has been identified in Eq. (13), by using Eq. (14) other rows of matrix H can be estimated iteratively.

5. A Numerical Example

To demonstrate the effectiveness of newly proposed model to reduce the modelling error of frame building structures, compared with shear model, an 8-story and 2-bay frame structure subject to ground excitation, as shown in Fig. 5, is used in this section. Flexibility rigidity of all columns and beams in the structure is same and equal to 9×10^7 N·m^2. The mass of each floor is equal to 200kg.

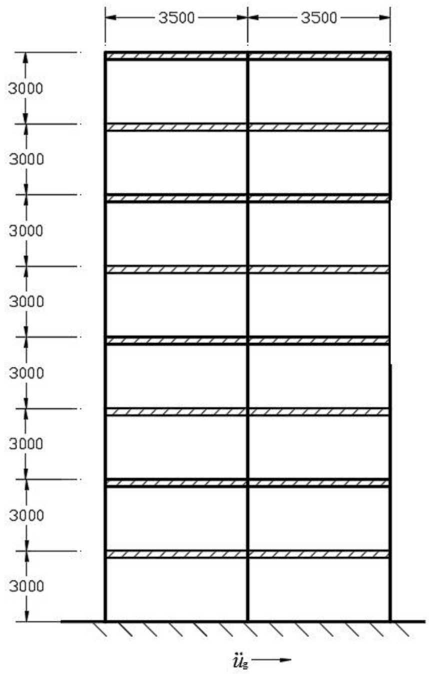

Fig. 5 Numerical example of an 8-story and 2-bay Frame Structure

To quantify the modelling error, the rms values of relative difference of interstory shear force between the model (shear structure model and newly proposed model) and the frame are used. Table 1 shows the comparison of the above values for both shear structure model and newly proposed model, which verifies that the new model can significantly reduce the modelling errors of structural interstory shear force, compared with shear structure model.

Moreover, the flexibility rigidity of beams are changed to change beam-column stiffness ratio of the structure, the above analysis was redone to check how the new model performs for frame structures with different beam-column stiffness ratio. Fig.6 shows the relative root-mean-square error (rmse) of interstory shear force for both shear structure model and the new model. It can be clearly seen that the modelling error of interstory shear force for shear structure reduces with increase of beam-column stiffness ratio, while the rms errors of the new model are very small and keep almost unchanged for the structures of different beam-column stiffness ratios.

Table 1. the comparison of rms errors of the 1st interstory shear forces

Floor #	1	2	3	4	5	6	7	8
New model	0.044	0.043	0.042	0.043	0.044	0.047	0.055	0.068
Shear Structure	0.718	1.503	1.582	1.589	1.583	1.578	1.565	1.640

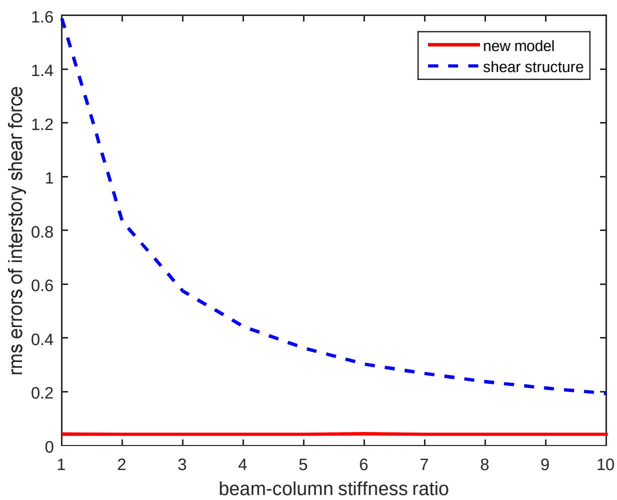

Fig. 6 Relative rmse of interstory shear force for both shear structure model and the new model

6. CONCLUSIONS

In this paper, a new model for frame is proposed which can reflects the effects of joint rotations of structural interstory shear force, therefore, the model should be able to reduce the modelling errors, compared with shear structure models. A numerical example of a 8-story and 2-bay frame structure verifies that the new model did greatly reduce the modelling error.

7. ACCKNOWLEDGEMENT

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