Bayesian Mode Identifiability investigation of a Cable-stayed Bridge

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ABSTRACT

Modal identification based on ambient vibration data has attracted extensive attention in the past few decades. In ambient vibration tests, the excitation is mainly from the environmental effects such as wind and traffic loading and no artificial excitation is applied, the signal to noise (s/n) ratio of the data acquired plays an important role in mode identifiability. Under ambient vibration conditions, certain modes may not be identifiable due to a low s/n ratio. This paper presents a study on the mode identifiability of an instrumented cable-stayed bridge with the use of acceleration response data measured by a long-term structural health monitoring system. A recently developed fast Bayesian FFT method is utilized to perform modal identification. In addition to identifying the most probable values (MPVs) of modal parameters, the associated posterior uncertainties can be obtained by this method. The data utilized in this study is 10 data sets including six collected under normal wind conditions and four collected during typhoons. A couple of fundamental modes are identified, including the ones in the vertical and transverse directions respectively and coupled in both directions. The uncertainty and s/n ratio of the deficient mode are investigated and discussed. A critical value of the modal s/n ratio is suggested to evaluate the mode identifiability of the deficient mode. The work presented in this paper could provide a base for the vibration-based condition assessment in future.

Keywords: Mode identifiability, cable-stayed bridge, ambient vibration, fast Bayesian FFT method, uncertainty

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1. INTRODUCTION

Structural health monitoring (SHM) has attracted increasing attention in the past two decades with the purpose of monitoring structural performance and conducting condition assessment of objective structures (Brownjohn et al., 2005; Ko and Ni, 2005; Li et al., 2006; Ni et al., 2011; Koo et al., 2013). For large-scale bridges, the SHM system is commonly designed to collect various structural responses including acceleration. With the collected acceleration data, modal identification is usually the first step to identify the modal parameters (Brownjohn et al., 2010), which can be further used for model updating and vibration-based damage detection (Lam et al., 2014). A fast Bayesian FFT method has recently been developed (Au, 2011; Au, 2012a,b; Zhang et al. 2015). One important merit of this method is that it can calculate the posterior uncertainty of the modal parameters analytically, making it possible to evaluate the accuracy and reliability of the identified modal parameters (Au and Zhang, 2012; Au et al. 2012a,b; Au et al. 2013; Zhang et al. 2016b; Ni et al. 2016).

In ambient modal identification, mode identifiability is a key issue worth investigating (Ni et al., 2015). Signal to noise (s/n) ratio is an important factor affecting the identifiability. If the s/n ratio for a specific mode is too low, the identification of this mode will be of high uncertainty, and even being unidentifiable. The measurement noise usually stems from the deployed sensors, signal cables, and data acquisition systems. One efficient way of improving the s/n ratio is to lower the noise corruption. On the other hand, increasing the excitation intensity is equally efficient. For bridge structures, the excitation level during a typhoon or earthquake is usually much higher than ambient vibration excitations. As a result, some deficient modes that cannot be identified under normal wind conditions may become identifiable during typhoons. In this study, a benchmark problem on the mode identifiability of a cable-stayed bridge is addressed. More details of this study can be found in Zhang et al. 2016a.

2. TING KAU BRIDGE

The benchmark study is based on the field monitoring data from the cable-stayed Ting Kau Bridge, which is 1170 m long in total with two main spans of 448 m and 475 m, respectively. A long-term structural health monitoring system has been deployed on the bridge to monitor the structural performance under in-service conditions. Sensors permanently instrumented on the bridge include accelerometers, anemometers, displacement transducers, temperature sensors, strain gauges, GPS and a weigh-in-motion sensing system. Accelerometers were installed at different locations of the bridge including the deck of the two main spans and two side spans, the longitudinal stabilizing cables, the top of the three towers, and the base of the central tower, as shown in Figure 1.

The data utilized in this study is 10 data sets including six collected under normal wind conditions and four collected during typhoons. From Table 1, it is seen that the wind speeds under typhoons are much higher than those under normal wind conditions.

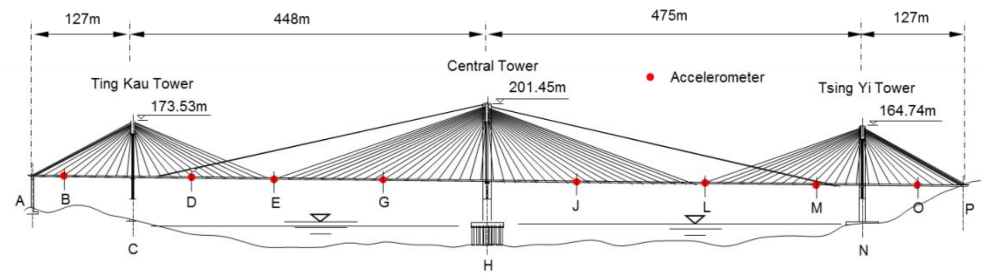


Figure 1 Layout of accelerometers on bridge deck

Table 1 Wind information for data in Group 1

Condition	Sample	Data	Mean hourly wind speed (m/s)
Normal wind conditions	Sample 1-6	Data 1-6	2.00-6.17
Typhoons	Sample 7-10	Data 7-10	12.11-21.72

3. DATA ANALYSIS

The method used in analysis is a fast Bayesian FFT method. The detailed algorithm please refer to Au 2011, Au 2012a,b, Zhang and Au 2013. Due to paper length, the first three modes are focused in this paper. Figure 2 to Figure 4 show the modal parameters identified using the data in Sample 1. The first mode identified is a vertical mode with a modal frequency of 0.161 Hz and damping ratio of 1.17%. The mode shape in the vertical direction forms a sine wave. No obvious motion is observed in the transverse direction. The modal frequency and damping ratio of the second mode identified are 0.227 Hz and 0.73% respectively. This is a coupled torsional mode in the vertical direction and lateral mode in the transverse direction. It is seen that the motion in the vertical direction is a little strange. This may be attributed to the low s/n ratio. The detailed reason will be discussed later. The third mode is a lateral mode with a modal frequency of 0.257 Hz and damping ratio of 1.32%. A sine wave is shown in the transverse direction. This is the second lateral mode of the bridge and there is almost no motion in the vertical direction. The modal parameters shown above are consistent with those obtained by a FEM analysis provided in Supplemental Document 1 of the benchmark problem and identified in a previous study by Ni et al. (2015).

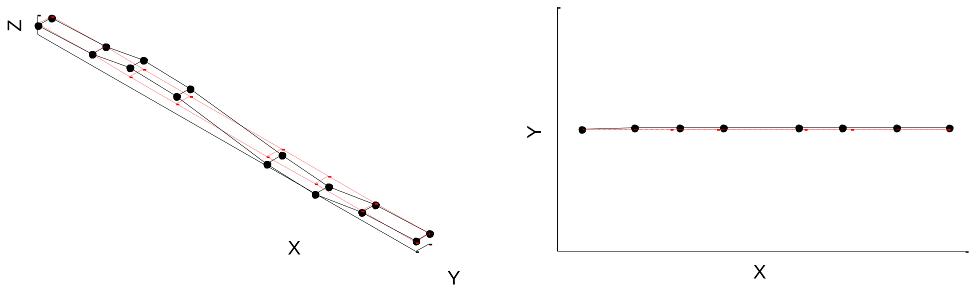


Figure 2 Mode shape of the 1st mode from Sample 1

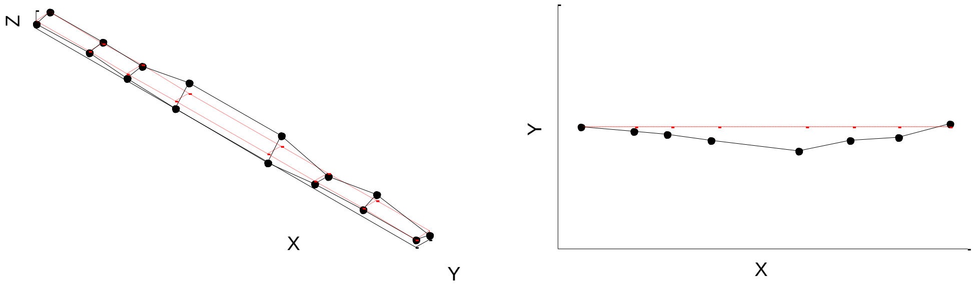


Figure 3 Mode shape of the 2nd mode from Sample 1

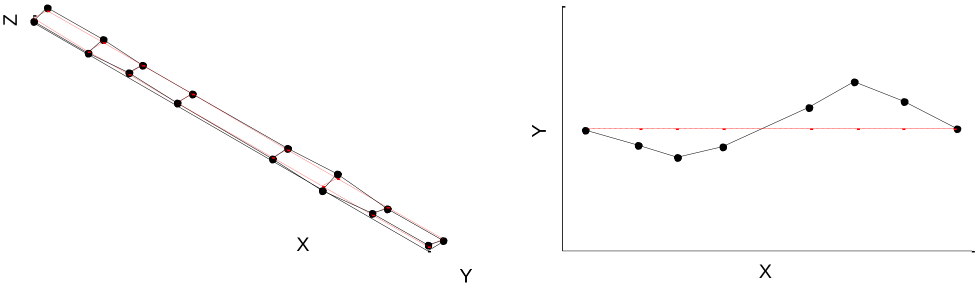
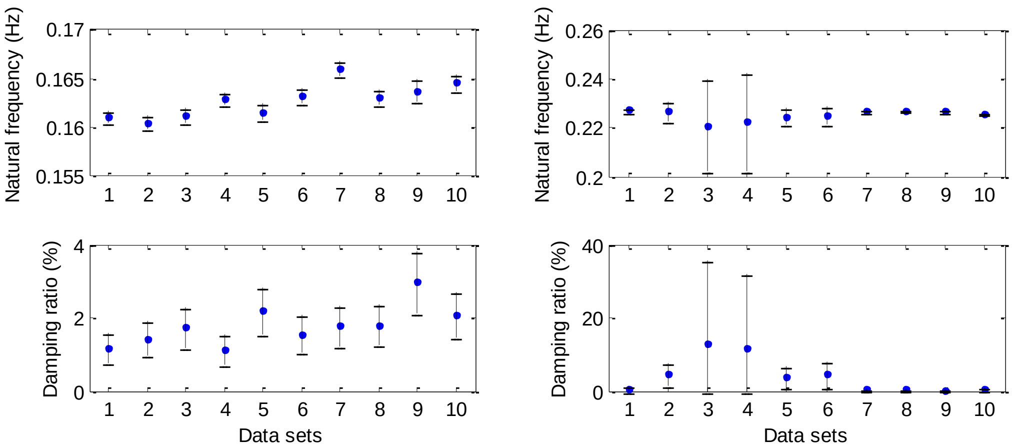
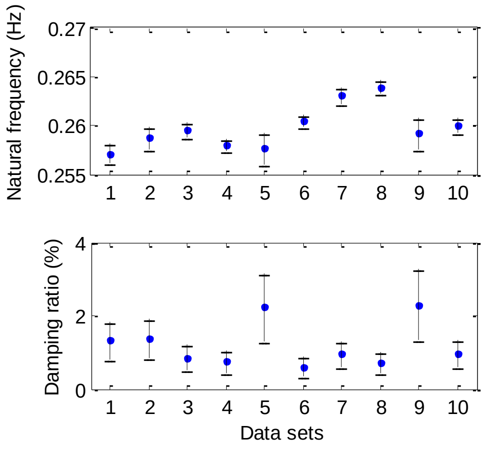


Figure 4 Mode shape of the 3rd mode from Sample 1

Figure 5 shows the posterior uncertainties of the identified modal frequencies and damping ratios from the 10 data sets. The identified results from each data set are shown with a dot at the posterior MPV and an error bar covering two posterior standard deviations. It is seen that the posterior uncertainties of modal frequencies are much smaller than those of damping ratios, implying that the identification of the former quantity is more accurate. For the first and third modes, the variation of the identified modal parameters is relatively small. The posterior uncertainty is consistent with the ensemble variability of their MPVs over different setups and therefore the Bayesian and frequentist perspectives roughly agree. The error bars for the third mode from some data sets, such as Data 5 and 9 are larger than those from other data sets, but the variations are consistent for both modal frequency and damping ratio.



a) 1st mode b) 2nd mode



c) 3rd mode

Figure 5 Identification result (±2 standard derivation error bar) of modal frequency and damping ratio

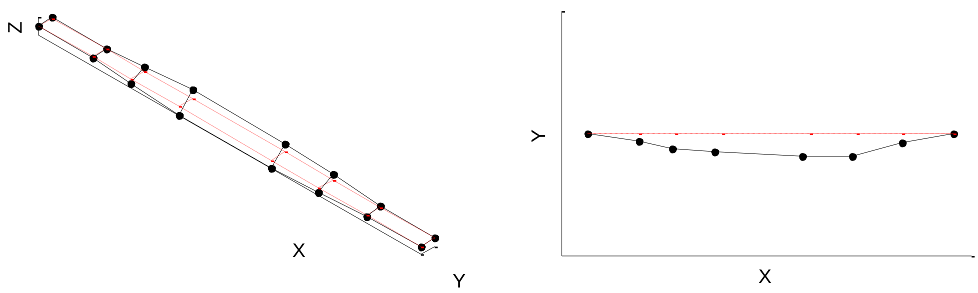


Figure 6 Mode shape of the 2nd mode from Typhoon York 1 (0.227 Hz, 0.40%)

Table 2 Signal to noise ratios in different modes and data sets

Mode \ Data	1	2	3	4	5	6	7	8	9	10
1	1194	580	607	2689	696	698	1528	1150	1096	933
2	12	6	3	3	4	4	720	603	518	551
3	46	28	77	145	23	135	247	306	48	210

It is worth mentioning that the second mode is quite different from the other modes. The error bars corresponding to Data 2, 3, 4, 5 and 6 are obviously larger than those for other data sets, especially for the results from Data 3 and 4. The MPVs of modal frequencies are still acceptable; however, this is not the fact for the damping ratio with a value higher than 10%. Recall that these five data sets correspond to the samples under normal wind conditions. The considerably large error bars mean that under normal wind conditions, the identified modal parameters for this mode are not accurate or reliable. The posterior uncertainty of modal parameters obtained from Data 1 is reasonable, but the mode shape as shown in Figure 3 is a little strange. To make a further investigation, the mode shape identified using the typhoon data is shown in Figure 6. It is observed that the identified mode shape is more reasonable in terms of the modal motion in both directions.

From the discussion above, it can be concluded that under normal wind conditions, the second mode is not identifiable. It can be well identified only when the wind speed reaches a certain value. One explanation is that the s/n ratio for the second mode under normal wind conditions is too low. Table 2 shows the s/n ratio calculated for different modes and from different data sets. It is seen that the s/n ratios obtained from the data acquired during typhoons tend to be larger than those obtained under normal wind conditions, especially for the second mode where a significant difference of the s/n ratio is observed among the data sets. The s/n ratio values under the typhoon conditions are in the order of magnitude of a few hundred while they are less than or equal to 12 under normal wind conditions. This indicates that the s/n ratio is a useful quantity to evaluate the mode identifiability. If the s/n ratio is too small, the mode would not be identifiable. Since the majority of the s/n ratio values corresponding to the identifiable cases in Table 2 are larger than 50, it is suggested to roughly take 50 as the bottom line to judge whether or not a mode is identifiable.

4. CONCLUSIONS

The mode identifiability of a cable-stayed bridge using ambient vibration data has been studied by the fast Bayesian FFT method. The acceleration response data acquired from the bridge under normal wind conditions and under typhoon conditions are used for this study. It is found that the second mode is a deficient mode which cannot be reliably identified when using the data acquired under normal wind conditions. With the identified most probable values (MPVs) and the associated posterior uncertainties of modal parameters from the fast Bayesian FFT method, the signal to noise (s/n) ratio values are derived to evaluate the mode identifiability of target modes. Based on the modal analysis results from ten data sets, a critical value equal to about 50 is suggested for the s/n ratio to examine the mode identifiability of the second mode given measured acceleration data from the bridge.

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