# A robust sequence image matching algorithm for aircraft position and heading

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# ABSTRACT

As for the translation and rotation existing between sequence images in the image matching navigation system, it is necessary to calculate the position and heading parameters simultaneously. Therefore, an algorithm for sequence image matching based on Harris corner and Mean Hausdorff Distance (MHD) was proposed. In this method, the corners are described in both polar coordinates and Cartesian coordinates. Considering the rotational invariance of radius set in polar coordinates, MHD is used to match radius sets between sequence images. Then, the rotation angle and the translation distance can be calculated. Experimental analysis has been carried out for the influence of different Gauss noise and the number of Harris corner. Results show that images can still be matched successfully with rotation angle of 0, 60 and 90 degrees under Gauss white noise with 0.01 variance, while the position error is less than one pixel, heading angle error is less than 5 degrees, and matching time is about 4 seconds. So, this algorithm can meet the requirements of real-time, robustness and accuracy of sequence image matching for navigation system.

# 1. INTRODUCTION

Image matching technology is an important technology in aircraft navigation system. The traditional task of image matching navigation is to find the location of realtime image taken by sensor in the reference image which is stored in database. In this way, the position information of the aircraft is obtained, which can be used to correct the accumulative error of Inertial Navigation System (INS) in long term work (Wu(2013)). There are two types in image matching methods: one is based on feature matching, which extracts points, lines, and other obvious features from the images, then match

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with the feature description sets of images, some feature extraction algorithms can be found in Trzcinski(2015), Shrivakshan(2012); Another is based on the gray-value-based matching, which mainly makes use of correlation by searching the template image in the reference image. However, because of the different sampling time, height, shooting angle and weather condition, there are big gray-scale variation, geometric distortion, and different noise distribution, which lead to the gray difference, and the low matching (Masatoshi(2015)). Meanwhile, the bigger size of the reference image will lead to longer search time. So, how to improve the success rate and real-time of image matching is the key to improve the performance of image matching aided navigation system (Chen(2012)).

Much work has been carried out on the image matching navigation. By calculating the correlation between real-time sequence images, YaoJun(2010) proposed the "relative" image matching between the current frame and the dynamic key frame. Because of the small difference in image-forming condition between the sequence images and the similar noise distribution, the image matching success rate is improved. However, there is not only translation but also rotation between sequence images, so how to effectively match the images having both translation and rotation is the key of sequence image matching.

Image matching based on Hausdorff distance has been widely used because of its fast calculation speed and good robustness to noise. The classical Hausdorff distance (Huttenlocher(1993)) is easily influenced by noise and occlusion. It has been improved by many scholars, such as Part Hausdorff Distance(PHD), Mean Hausdorff Distance(MHD), but they are sensitive to the rotation of the images. Leng(2006) et al. proposed Weighted Hausdorff Distance matching by making use of edge points and stable branch points (EDGE-WHD), which achieved a good performance in condition of small rotation angle. However, it is not suitable for image matching with large rotation angle, and cannot obtain the rotation the angle.

To solve these problems, we propose an algorithm based on Harris corner and MHD, which achieved a good performance with any rotation angle by using the rotation invariance of radius sets in polar coordinates. Besides, MHD between point sets of circular templates is used to calculate the rotation angle. Experimental results show that the algorithm has robustness and can meet the requirements of real-time and accuracy of sequence image matching.

# 2. PRINCIPLE OF SEQUENCE IMAGE MATCHING NAVIGATION

Fig. 1 shows that camera planted on the aircraft along the axis of body axes coordinate system takes sequence images at fixed interval  $\Delta t$ . If the plane is flying smoothly along a straight line,  $O_2$  is the center point of the current down looking image. Search for the homonymy point in the previous image, and calculate the distance between the homonymy point and center point  $O_1$  in previous image. Flight distance  $\Delta L$  can be calculated in a fixed interval  $\Delta L$  with the knowledge of flight altitude and camera parameters. During the flight, if the camera is not vertically downward, Yu(2009) have proposed a modified model.

In the condition of unstable flight, there both translation and rotation change between sequence images. In this case, the rotation angle of the images can be calculated, and we can get the change of heading angle.



Fig. 1 Relationship between the center points of sequence images

# 3. ALGORITHM OF SEQUENCE IMAGE MATCHING

#### 3.1 Feature points extraction

Sequence images include a lot of features, which are effected specifically by the translation and rotation of the images. It is common to use the corresponding relation of the sets of feature points in two frames to infer the relationship between images in image matching algorithm. In this paper the Harris corner is chosen as point feature, because of its rotation invariance and stable performance.

Harris corner detection is the feature extraction operator proposed by Harris and Stephens, specific steps can be found in Biswas(2015). Harris corner detection uses a size-fixed window moving in the image and observes the change of gray at the same time. If the window is in the flat area, the gray-scale change is very small in any direction; if in the area of edge, the gray-scale change is small in the direction of edge while the change is large in the direction perpendicular to the edge; if in the area of corner, and the gray-scale change is large in any direction. Gauss filter is used in order to improve the anti-noise ability. Gauss function is defined as:

$$\omega(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2 + y^2)/2\sigma^2}$$
(1)

x, y are the coordinates of the current point relative to the center point. This

function ensures that the high weight is assigned to the pixels near the center point. It will reduce the effect of noise.

The gray-scale change is defined as:

$$E(x, y) \cong \begin{bmatrix} x, y \end{bmatrix} M \begin{bmatrix} x \\ y \end{bmatrix}$$
(2)

The matrix *M* is:

$$M = \omega(x, y) \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$
(3)

 $I_x$ ,  $I_y$  is gradient in the direction of x and y respectively.  $\otimes$  Indicates convolution.  $\lambda_1$ ,  $\lambda_2$  are eigenvalues of the matrix M, their relationship is described as follows:

- 1)  $\lambda_1$ ,  $\lambda_2$  are both large, the pixel is a corner point.
- 2) One of  $\lambda_1$ ,  $\lambda_2$  is large, and the other one is small, the pixel is an edge point.
- 3)  $\lambda_1$ ,  $\lambda_2$  are both small, the pixel is in the flat area.

Response function is defined as:

$$R = \det M - k(traceM)^2 \tag{4}$$

det M, *traceM* is the determinant and trace of the matrix M, k usually takes 0.04. If R is greater than the threshold, the point will be judged as a corner point.

#### 3.2 Description of feature points set

Since there is a translation and rotation between sequence images, the overlapping part of two images should be selected to match the images. As shown in Fig. 2, only the circular part of the image will be considered.

Considering the distance between feature points and the center is invariant after the rotation, polar coordinates of corner points  $P(x_i, y_i)$  in the circular region is adopted, where the radius of the circular region is R, the center is  $(x_0, y_0)$ ,.Polar coordinates transformation formula is shown as follows:



Fig. 2 Relationship between adjacent sequence images

$$\begin{cases} \rho_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \\ \tan \theta_i = \frac{y_i - y_0}{x_i - x_0} \end{cases}$$

$$0 \le \rho_i \le R, 0 \le \theta_i \le 2\pi$$
(5)

Where  $\rho_i$  is the polar radius of point is  $P(x_i, y_i)$ ,  $\theta_i$  is the polar angle of point  $P(x_i, y_i)$ . Then feature points set of the image is described by the set $(\rho_i, \theta_i)$ .

# 3.3 Translation distance between sequence images based on MHD

The Harris corner extraction is needed for two adjacent frames for matching them. The problem of image matching is transformed into the matching between two point sets.

Setting that  $I_1$  is the previous image and  $I_2$  is the second. Circular template is the circular region of image  $I_2$ , centered at point  $(x_0, y_0)$ , then polar coordinates of corner points in the circular template can be transformed into feature points set  $\{(\rho_1, \theta_1), (\rho_2, \theta_2), \cdots, (\rho_p, \theta_p)\}$ . Search the circular template on the image  $I_1$  to get feature points set covered by circular template, then we can obtain the polar coordinates set  $\{(\xi_1, \varphi_1), (\xi_2, \varphi_2), \cdots, (\xi_q, \varphi_q)\}$  centered at point  $(x_0, y_0)$ .

Because of the rotation invariance of the polar sets  $\{\rho_1, \rho_2, \dots, \rho_q\}$ ,  $\{\xi_1, \xi_2, \dots, \xi_q\}$  as shown in figure 3. MHD matching algorithm can be used.

MHD is a method to measure the degree of non-similarity between two sets. For the set  $A = \{a_1, a_2, \dots, a_n\}$ ,  $B = \{b_1, b_2, \dots, b_m\}$  MHD is defined as:

$$H_{PM}(A,B) = \max[h_{PM}(A,B), h_{PM}(B,A)]$$
(6)

Where  $h_{PM}(A,B) = \frac{1}{m} \sum_{i=1}^{m} \min_{b \in B} ||a_i - b||$  is directed MHD from set A to set B.

 $h_{PM}(B,A) = \frac{1}{n} \sum_{i=1}^{n} \min_{a \in A} \|b_i - a\| \text{ is directed MHD from set } B \text{ to set } A \cdot \|\bullet\| \text{ is distance of set}$ A and B.

 $\|\bullet\| \text{ is defined as the Euclidean distance of the element } \rho_i \text{ and } \xi_j \text{ in polar sets.}$  $h_{PM}(\rho,\xi) = \frac{1}{p} \sum_{i=1}^{p} \min_{\xi_j \in \xi} |\rho_i - \xi| \quad h_{PM}(\xi,\rho) \models \frac{1}{q} \sum_{i=1}^{q} \min_{\rho_j \in \rho} |\xi_j - \xi|.$ 

Matching degree of  $\{\rho_1, \rho_2, \dots, \rho_q\}$ ,  $\{\xi_1, \xi_2, \dots, \xi_q\}$  can be described by the set of MHD, which can be obtained by searching the circular template on the image  $I_1$ :

$$\left\{H_{PM1}(\rho,\xi),H_{PM2}(\rho,\xi),\cdots,H_{PMr}(\rho,\xi)\right\}$$

The center point  $(x_0', y_0')$  in the area covered by circular template is the matching point while the MHD is minimum.

The translation distance between two frames is  $\begin{cases} \Delta x = x_0 - x_0 \\ \Delta y = y_0 - y_0 \end{cases}$ 

# 3.4 Angle of rotation between sequence images based on MHD

There is only a rotation variation between the circular matching area obtained above and the circular template.

Rotate the feature points  $\{(\rho_1, \theta_1), (\rho_2, \theta_2), \dots, (\rho_p, \theta_p)\}$  in the circular template, rotation angle is  $\Delta\theta$ . Feature point set is transformed into  $\{(\rho_1, \theta_1 + \Delta\theta), (\rho_2, \theta_2 + \Delta\theta), \dots, (\rho_p, \theta_p + \Delta\theta)\}$ . Then feature point sets can be transformed from polar coordinates into Cartesian coordinates, where the origin of Cartesian coordinate is the center of polar coordinates. The Transform can be described as follows:

$$\begin{cases} a_{\Delta\theta i} = \rho_i \cos(\theta_i + \Delta\theta) \\ b_{\Delta\theta i} = \rho_i \sin(\theta_i + \Delta\theta) \\ 0 \le \Delta\theta < 2\pi \end{cases}$$
(7)

Transform the feature points in circular template of image  $I_1$  from polar coordinates to Cartesian coordinate without rotation. Cartesian coordinates  $\{(c_1, d_1), (c_2, d_2), \dots, (c_q, d_q)\}$  can be obtained.

At this time, the task of rotating image matching is transformed into the matching of two point sets in Cartesian coordinate system.

The distance  $\|\bullet\|$  in MHD is redefined as Euclidean distance from point  $(a_{\Delta\theta i}, b_{\Delta\theta i})$  to point  $(c_i, d_i)$ :

$$\begin{split} l_{ij} = \left\| (a_{\Delta\theta i}, b_{\Delta\theta i}) - (c_j, d_j) \right\| = \sqrt{(a_{\Delta\theta i} - c_j)^2 + (b_{\Delta\theta i} - d_j)^2} \,. \end{split}$$
 Now, the MHD is expressed as:

$$H_{PM}(\Delta \theta i) = = \max(h_{PM}(\{(a_{\Delta \theta i}, b_{\Delta \theta i})\}, \{(c_j, d_j)\}), h_{PM}(\{(c_j, d_j)\}, \{(a_{\Delta \theta i}, b_{\Delta \theta i})\}))$$
(8)

Where

$$\begin{cases} h_{PM}(\{(a_{\Delta\theta i}, b_{\Delta\theta i})\}, \{(c_j, d_j)\}) = \frac{1}{p} \sum_{i=1}^{p} \min_{(c_j, d_j) \in \{(c_j, d_j)\}} \sqrt{(a_{\Delta\theta i} - c_j)^2 + (b_{\Delta\theta i} - d_j)^2} \\ h_{PM}(\{(c_j, d_j)\}, \{(a_{\Delta\theta i}, b_{\Delta\theta i})\}) = \frac{1}{q} \sum_{j=1}^{q} \min_{(a_{\Delta\theta i}, b_{\Delta\theta i}) \in \{(a_{\Delta\theta i}, b_{\Delta\theta i})\}} \sqrt{(c_j - a_{\Delta\theta i})^2 + (d_j - b_{\Delta\theta i})^2} \end{cases}$$

The MHD set can be achieved by adjusting the value of  $\Delta \theta \in [0, 2\pi)$ :

$$\left\{H_{PM1}(\Delta\theta 1), H_{PM2}(\Delta\theta 2), \cdots, H_{PMr}(\Delta\theta r)\right\}$$

The matched rotation angle corresponds to the minimum of the MHD, which means the rotation angle is obtained by  $\Delta\theta \min$ , when the MHD is taken to a minimum  $H_{PM\min}(\Delta\theta\min)$ .

#### 4. ANALYSIS OF EXPERIMENTAL RESULTS

In order to verify the algorithm, experiments are tested on the PC with CPU i5-4260U, frequency 1.4GHZ and 4G memory, combining with the opencv2.4.10 open source computer vision library.

Experiments are divided into two parts, carried out mainly from two aspects: robustness and real-time. In the first part of the experiment, the relationship between two sequence images are rotation and translation, and the robustness of the algorithm is verified by adding noise. In the second part of the experiment, different numbers of corner points are obtained by adjusting the threshold to compare the influence of the corner points number on the real-time performance.

#### 4.1 Feasibility and robustness of the algorithm

In the first part of the experiment, the size of the sequence images are 255 \* 255. There are different translation and degrees of rotation between images (as shown in Fig. a, Fig. b), and Gauss noise with different variance is also added. The algorithm extracts the Harris corner of two images, and the circular template of image 2 (Fig. c). Matching area in image 1 is demonstrated in Figure d. Specific experimental results are shown in Tab.1. The unit of distance error is pixel, angle unit is degree.

As illustrated in Tab.1, the proposed algorithm can realize the matching of sequence images with different rotation angles and Gauss white noise added. The algorithm of this paper can guarantee the real-time performance without noise, when the accuracy of the rotation angle is 0.1 degree. Images can still be matched with rotation angle of 0, 60, and 90 degrees while Gauss white noise with different variance added. However, the distance error (The distance error is the Euclidean distance between the calculated distance and the real distance) is easy to get 1 pixel, because the algorithm is calculated based on the pixel.

For the real-time performance of the experiment, it is mainly affected by the number of Harris corners, the size of images, and the size of the circular template. In reality, the size of images and the circular templates is generally unchanged, which means running time is mainly affected by the number of Harris corner points. As shown in Tab.1, the number of Harris corner points is relatively stable without noise, which makes running time is keeping around 4s. With the noise interference, running time fluctuates because the number of Harris corner points is affected by noise.



Fig. a Corner distribution of image 1



Fig. c Circular region of image 2



Fig. b Corner distribution of image 2



Fig. d Matching area in image 1

Fig. 3 Result of image matching

	Tab.1	Results	of ex	periment	s
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		Noise variance	Results of experiments			
Image size 255*255	Real rotation angle		Distance error	Rotation angle	Running time(s)	
	0.0		0.0	0.0	4.63	
	60.0	0.000	0.0	60.0	4.65	
	90.0		0.0	89.9	5.41	
	0.0	0.005	0.0	0.2	3.48	
		0.010	0.0	1.4	3.72	
		0.050	4.1	64.7	1.20	
	60.0	0.005	1.0	59.2	5.21	
		0.010	1.0	59.1	3.89	
		0.050	21.2	71.2	3.98	
	90.0	0.005	0.0	89.9	3.35	
		0.010	1.0	86.3	2.03	
		0.050	7.1	79.8	2.74	

Traditional Hausdorff Distance matching algorithm (Leng(2006)), such as Weighted Hausdorff Distance algorithm based on edge and branch points (EDGE-WHD), Mean Hausdorff Distance algorithm based on edge (EDGE-MHD) cannot effectively cannot effectively match the rotation images or calculate the rotation angle. The comparison results of the matching algorithms are shown in Tab.2. Image size is 255\*255.

Noise	Rotation	proposed algorithm		EDGE-WHD		EDGE-MHD		
variance	angle	Distance	Angle	Time	Distance	Time	Distance	Time
		error	error	(S)	error	(s)	error	(S)
0.000	0.0	0.0	0.0	5.59	0.0	15.82	0.0	16.53
	1.0	0.0	0.3	3.06	0.0	15.65	1.0	18.49
	3.0	1.0	0.8	4.20	fail		fail	
0.005	0.0	0.0	1.0	5.85	0.0	19.46	0.0	19.55
0.010	0.0	0.0	3.3	2.48	fail		0.0	24.79

Tab.2 Comparison of different algorithms for image matching

From Tab.2, we can see that the proposed algorithm based on feature points is better than EDGE-WHD, EDGE-MHD in real-time. EDGE-WHD, EDGE-MHD can only supports small angle rotation between images, and have been ineffective under 3 degrees. In the aspect of anti-noise, compared to EDGE-WHD and EDGE-MHD, this algorithm is also more robust.

#### 4.2 The influence of Harris corner threshold on the real-time and error

The number of Harris corner points can be adjusted by tuning the threshold used in the detection. The second part of the experiment shows the relationship between running time and the threshold of Harris corner, which is described in Tab.3 with 255\*255 size of images. Fig. 4 and Fig. 5 shows the relationship between the experimental error and the threshold value (The distance error is the Euclidean distance between the calculated distance and the real distance, and the data is the average value of several experimental results with different rotation angles).

Threshold	The number of corner points in image1	The number of corner points in image2	Running time(s)
210	6	6	0.20
200	7	6	0.24
190	10	9	0.49
180	15	16	0.94
170	17	18	1.33
160	20	20	2.23
150	25	25	3.03
140	30	28	4.86
130	36	34	7.77
120	49	45	13.74
110	57	54	18.98
100	74	68	32.48

#### Tab.3 Relationship between running time and the threshold



Fig. 4 Relationship between distance error and threshold



Fig. 5 Relationship between angle error and threshold

In the second part of the experiment, running time is closely related to the number of corner points. At the threshold of 140 (about 30 points), the algorithm can ensure real-time performance, the distance matching error is basically less than 1 pixel, and the angle error is less than 1 degree. With the increase of the threshold, the number of corner points decreases, the running time also decreases while the error increases. At the threshold above 190, there are few corner points, and the algorithm is invalid.

For the traditional image matching algorithm, matching time greatly increases with the increase of the size of the image. Algorithm in this paper can adjust the running time of the program by tuning the threshold, which extend the application of this algorithm.

#### 5. CONCLUSIONS

In this paper, the MHD matching is performed by using the Harris corner set of sequence images. Sequence images can be successfully matched while the relationship between images have both translation and rotation, and the translation distance and rotation angle can also be obtained. The simulation results show that the algorithm is robust to noise. Then the relationship between the threshold and the running time of the algorithm was also explained. However, strict mathematical model have not been built. More research work need to study about determining the appropriate threshold value to adjust the running time and the accuracy in different sizes of sequence images in the future.

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