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Post-buckling responses of elastoplastic FGM beams on nonlinear elastic foundation

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Abstract. The elastoplastic response of functionally graded material (FGM) beams resting on a nonlinear elastic foundation to an eccentric axial load is investigated by using the finite element method. The FGM is assumed to be formed from ceramic and metal phases with their volume fraction vary in the thickness direction by a power-law function. A bilinear elastoplastic behavior is assumed for the metallic phase, and the effective elastoplastic properties of the FGM are evaluated by Tamura-Tomota-Ozawa (TTO) model. Based on the classical beam theory, a nonlinear finite beam element taking the shift in the neutral axis position into account is formulated and employed in the investigation. An incremental-iterative procedure in combination with the arc-length control method is employed in computing the equilibrium paths of the beams. The validation of the formulated element is confirmed by comparing the equilibrium paths obtained by using the present element and the one available in the literature. The numerical results show that the elastoplastic post-buckling of the FGM beams is unstable, and the post-buckling strength is higher for the beams associated with a higher ceramic content. Different from homogeneous beams, yielding in the FGM beam occurs in the layer near the ceramic layer before in the layer near metal surface. A parametric study is carried out to highlight the effect of the material distribution, foundation support and eccentric ratio on the elastoplastic response of the beams.

Keywords: FGM beam; elastoplastic behavior; nonlinear elastic foundation; eccentric axial load; nonlinear finite element analysis

1. Introduction

Analysis of beams resting on elastic foundation is an important topic in the field of structural mechanics, and it has been drawn much attention by many researchers for a long time. A large number of studies of beams on elastic foundation are referred to in the excellent monograph by

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Hetényi (1946). Recent contributions that are most relevant to the present topic are briefly discussed herein. Zhaohua and Cook (1983) studied the bending problem of beams on a two-parameter elastic foundation by using the exact interpolation in the derivation of the stiffness matrix. Chaht *et al.* (2015) addressed theoretical bending and buckling behaviors of size-dependent nanobeams made of FGM on the basis of the nonlocal elastic continuum model. Razaqpur and Shah (1991) derived the exact stiffness matrix and nodal force vector for assessing the deflection and internal forces of beams resting on a two-parameter elastic foundation. Chegenizadeh *et al.* (2014) investigated FGM beams on the elastic and plastic soil mediums subjected to dynamic and static loadings by using commercial finite element software ABAQUS. Budkowska and Szymczak (1997) used a simple finite element model in studying the post-buckling behavior of beams partially embedded in a Winkler foundation. Kounadis *et al.* (2006) have shown that the post-buckling behavior of elastic beams resting on a Winkler foundation is stable. The equilibrium paths, computed by Patel *et al.* (1999), Nguyen *et al.* (2004) by using the finite element method, have also confirmed the stable behavior in the post-buckling region of axially loaded beams resting on a two-parameter elastic foundation. However, due to the increase of the critical load by the elastic foundation support, the stress in beams on an elastic foundation may exceed yield stress when the deflection is still very small, even before buckling. Thus, the effect of plastic deformation is an important factor for the buckling behavior of beams resting on the elastic foundation. In this line of work, based on Hill's variational principle Cheb and Neal (1984) developed a finite element procedure for investigating the buckling and post-buckling behavior of elastic-plastic beams resting on a nonlinear elastic foundation. They have then shown that the post-buckling behavior of the elastic-plastic beams on the foundation is unstable, and the maximum load that the beams can withstand is sensitive to the imperfection and the foundation stiffness. Also using a finite element procedure, Nguyen and his co-workers have confirmed that the post-buckling behavior of beams on elastic foundation subjected to an eccentric axial load is unstable, and the post-buckling strength, measured in term of a ratio between the axial load and the critical load, increases with an increment in the foundation stiffness.

The new type of composite developed recently, namely functionally graded material (FGM) has high potential to use as a structural material. This composite, usually formed from metals and ceramics, has no thermal stress concentration and delaminating problems as often met in conventional composites. Many investigations have been reported on the behavior of FGM structures subjected to static or dynamic loadings. Concerning to analysis of FGM beams, Chakraborty *et al.* (2003) derived a first-order shear deformable beam element for investigating the thermoelastic behavior of FGM beams. Based on the third-order shear deformation beam theory, Kadoli *et al.* (2008) derived a beam element for studying the static behavior of FGM beams under ambient temperature. Singh and Li (2009) proposed a model for computing buckling loads of non-uniform axially FGM columns by approximating the column by another one with piecewise uniform geometric and material properties. Kang and Li (2009, 2010) took the shift in the neutral axis position into account in their derivation of the expressions for tip response of a cantilever FMG Euler-Bernoulli beam subjected to a transverse end force or an end moment. Nguyen (2013, 2014) derived the co-rotational beam elements for large displacement analysis of tapered beams made of axially or transversely FGM. Also using the finite element method, Nguyen and Gan (2014), Nguyen *et al.* (2014) investigated the geometrically nonlinear behavior of beams and frames made of transversely FGM.

Analysis of elastoplastic FGM structures in general and FGM beam, in particular, has not been carried out sufficiently by researchers. Only a few recent publications on this topic can be found in

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