The 2016 World Congress on
The 2016 Structures Congress (Structures16)
Jeju Island, Korea, August 28-September 1, 2016

The Effect of Viscoelastic Material Parameter Regarding Standard Model on the Conical Helicoidal Rod

*Merve Ermis[1)], Akif Kutlu[2)], Nihal Eratlı[3)] and Mehmet H. Omurtag[4)]

[1), 2), 3), 4)] *Faculty of Civil Engineering, Istanbul Technical University, Istanbul, Turkey*
[1)] *ermism@itu.edu.tr*

ABSTRACT

The scope of this study is to investigate the dynamic viscoelastic response of a linear viscoelastic conical type helical rod having a circular cross-section via mixed finite element formulation in regards to the effects of viscoelastic material parameter. For this purpose, the proposed viscoelastic model exhibits a standard type of distortional behavior while having elastic Poisson's ratio. The material properties are accounted through the use of the correspondence principle in the formulation. Field equations are based on Timoshenko beam theory and the curvatures and arc length of helix geometry is directly taken into account in the finite element algorithm. The analysis is carried out in the Laplace space and the results are transformed back to time space numerically by Modified Durbin's algorithm. The viscoelastic conical type helical rod having fixed from both ends is subjected to the step type of uniformly distributed dynamic load. The effects of viscosity parameter on the dynamic analysis of linear viscoelastic conical type helical rod are investigated.

1. INTRODUCTION

The viscoelastic behavior of the material has become more popular with the advances in technology. Viscoelastic materials are encountered in the application as automobile bumpers, dampers, and road construction, carbon nanotubes (CNTs), fiber, polymer multiscale composite beams Lewandowski and Lasecka-Plura (2016), viscoelastic soft dielectric elastomer generators Bortot et al. (2016). There are remarkable theoretical studies on viscoelastic materials such as Fung (1965), Flügge (1975), Christensen (1982).

There are numerous studies about the viscoelastic behaviors of beams (Chen and Lin 1982, Chen 1995, Wang et al. 1997, Kocatürk and Şimşek 2006a-b, Payette and

[1)] Research Assistant
[2)] Assistant Professor
[3)] Associate Professor
[4)] Professor

Reddy 2010, Martin 2016), however, a few studies about the viscoelastic behavior of helicoidal rods exist, in the literature. Temel (2004) and Temel et al. (2004) studied quasi-static and dynamic analysis of viscoelastic cylindrical helicoidal rods in the Laplace space, by using the complementary functions method and the ordinary differential equations based on the Timoshenko beam theory. By using the mixed finite element method, the dynamic behavior of viscoelastic helices based on the Timoshenko beam theory are investigated by Eratlı et al. (2014), Ermis (2015), Ermis et al. (2016). The curvatures and arc length are satisfied exactly at the nodal points and linearly interpolated through the element (Eratlı et al. 2014), and, the study of Eratlı et al. (2014) are improved by considering the exact curvatures and arc length directly through the finite element formulation (Ermis 2015, Ermis et al. 2016).

In this study, the dynamic behavior of linear viscoelastic conical helicoidal rod with clamped at both ends subjected step type of uniformly distributed dynamic load is investigated by employing the mixed finite element method proposed in Ermis (2015) and Ermis et al. (2016). The field equations of the helicoidal rod are based on the Timoshenko assumptions. The viscoelastic material exhibits the standard type of distortional behavior while having elastic Poisson's ratio. The viscoelastic material constants are accounted by using the correspondence principle (Shames and Cozarelli 1997), which states that the equations for viscoelastic case in the Laplace space may be obtained from those for elastic case by replacing the elastic constants by complex moduli according to the chosen viscoelastic model. The solution of the structural problem is carried out in the Laplace space. The results of the dynamic problem obtained in the Laplace space and are transformed back to the time domain numerically by means of the Modified Durbin's transformation algorithm (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). In order to investigate the influence of viscosity parameter on the dynamic behavior of linear viscoelastic conical helicoidal rod having circular cross-section is handled and discussed in detail as a benchmark example.

2. FORMULATION

2.1 The Helix Geometry

In the Cartesian coordinate system, the parameters of helical geometry can be given as: $x = R(\varphi)\cos\varphi$, $y = R(\varphi)\sin\varphi$ and $z = p(\varphi)\varphi$, where $p(\varphi) = R(\varphi)\tan\alpha$, horizontal angle φ and the pitch angle α . $p(\varphi)$ defines the step for unit angle of the helix. With $c(\varphi)=\sqrt{R^2(\varphi)+p^2(\varphi)}$, the infinitesimal arc length becomes $\mathrm{d}s = c(\varphi)\mathrm{d}\varphi$. The centerline radius of conical helix is defined $R(\varphi) = R_{\max}+(R_{\min}-R_{\max})\varphi/2n\pi$ where $R_{\max}$, $R_{\min}$ are the bottom and top radii of the helix, and n is the number of active turns, respectively.

2.2 The Field Equations in the Laplace Space

In the Laplace space, the field equations based on the Timoshenko beam theory can be given as follows (Eratlı et al. 2014):

$$
\begin{aligned}
-\bar{\mathbf{T}}_{,s} - \bar{\mathbf{q}} + \rho A z^2\, \bar{\mathbf{u}} &= \mathbf{0} \\
-\bar{\mathbf{M}}_{,s} - \mathbf{t}\times\bar{\mathbf{T}} - \bar{\mathbf{m}} + \rho \mathbf{I}\, z^2\, \bar{\mathbf{\Omega}} &= \mathbf{0} \\
\bar{\mathbf{u}}_{,s} + \mathbf{t}\times\bar{\mathbf{\Omega}} - \bar{\mathbf{C}}_{\gamma}\bar{\mathbf{T}} &= \mathbf{0} \\
\bar{\mathbf{\Omega}}_{,s} - \bar{\mathbf{C}}_{\kappa}\bar{\mathbf{M}} &= \mathbf{0}
\end{aligned}
\qquad (1)
$$

where comma as a subscript under the variable designates the differentiation with respect to s, z is the Laplace transformation parameter, and, the Laplace transformed variables are denoted by the over bars. Using the Frenet coordinate system in Eq.(1), the displacement vector, the rotation vector, the force vector and the the moment vector are given $\bar{\mathbf{u}} = \bar{u}_t\,\mathbf{t} + \bar{u}_n\,\mathbf{n} + \bar{u}_b\,\mathbf{b}$, $\bar{\mathbf{\Omega}} = \bar{\Omega}_t\,\mathbf{t} + \bar{\Omega}_n\,\mathbf{n} + \bar{\Omega}_b\,\mathbf{b}$, $\bar{\mathbf{T}} = \bar{T}_t\,\mathbf{t} + \bar{T}_n\,\mathbf{n} + \bar{T}_b\,\mathbf{b}$, $\bar{\mathbf{M}} = \bar{M}_t\,\mathbf{t} + \bar{M}_n\,\mathbf{n} + \bar{M}_b\,\mathbf{b}$, respectively. $\bar{\mathbf{q}}$ and $\bar{\mathbf{m}}$ are the distributed external force and moment vectors, $\bar{\mathbf{C}}_{\gamma}$ and $\bar{\mathbf{C}}_{\kappa}$ are the compliance matrices. ρ is the density of material, A is the area of the cross section, $\mathbf{I} = I_t\,\mathbf{t} + I_n\,\mathbf{n} + I_b\,\mathbf{b}$ is the moment of inertia of the cross section (Eratlı et al. 2014).

2.3 The Functional in the Laplace Space

The field equations in Eq. (1) are written in operator form $\mathbf{Q} = \mathbf{Ly} - \mathbf{f}$. After proving this operator to be potential, the functional of the structural problem is obtained in the Laplace space as follows:

$$
\begin{aligned}
\mathbf{I}(\bar{\mathbf{y}}) = &-[\bar{\mathbf{u}}, \bar{\mathbf{T}}_{,s}] + [\mathbf{t}\times\bar{\mathbf{\Omega}}, \bar{\mathbf{T}}] - [\bar{\mathbf{M}}_{,s}, \bar{\mathbf{\Omega}}] - \tfrac{1}{2}[\mathbf{C}_{\kappa}\bar{\mathbf{M}}, \bar{\mathbf{M}}] - \tfrac{1}{2}[\mathbf{C}_{\gamma}\bar{\mathbf{T}}, \bar{\mathbf{T}}] \\
&+ \tfrac{1}{2}\rho A z^2 \left[\bar{\mathbf{u}}, \bar{\mathbf{u}}\right] + \tfrac{1}{2}\rho\, z^2 \left[\mathbf{I}\bar{\mathbf{\Omega}}, \bar{\mathbf{\Omega}}\right] - \left[\bar{\mathbf{q}}, \bar{\mathbf{u}}\right] - \left[\bar{\mathbf{m}}, \bar{\mathbf{\Omega}}\right] \\
&+ [(\bar{\mathbf{T}} - \hat{\bar{\mathbf{T}}}), \bar{\mathbf{u}}]_{\sigma} + [(\bar{\mathbf{M}} - \hat{\bar{\mathbf{M}}}), \bar{\mathbf{\Omega}}]_{\sigma} + [\hat{\bar{\mathbf{u}}}, \bar{\mathbf{T}}]_{\varepsilon} + [\hat{\bar{\mathbf{\Omega}}}, \bar{\mathbf{M}}]_{\varepsilon}
\end{aligned}
\qquad (2)
$$

In Eq. (2), the subscripts ε and σ, represent the geometric and dynamic boundary conditions, respectively, and, the terms with hats in Eq. (2) define the known values on the boundary. The details of the variational formulation and functional exist in Eratlı et al. (2014).

2.4 Mixed Finite Element Formulation

A two-noded curvilinear elements is used to discretize the helicoidal rods. Linear shape functions $\phi_i = (\varphi_j - \varphi)/\Delta\varphi$ and $\phi_j = (\varphi - \varphi_i)/\Delta\varphi$ are employed where $\Delta\varphi = (\varphi_j - \varphi_i)$ and the curvature $\chi(\varphi)$, the torsion $\tau(\varphi)$ and arc length $c(\varphi)$ of helicoidal geometry is directly taken into account in the mixed finite element formulation (Ermis 2015, Ermis et al. 2016). Each node has 12 degrees of freedom namely, $\{\bar{\mathbf{u}}, \bar{\mathbf{\Omega}}, \bar{\mathbf{T}}, \bar{\mathbf{M}}\}$.

2.5 The Standard Model

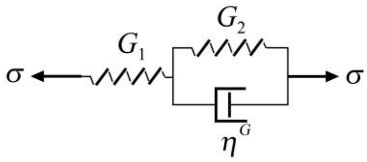


Fig. 1 Standard model

When the viscoelastic material exhibits the standard type of distortional behavior (see Fig. 1), then the complex shear modulus can be expressed in the following form (Mengi and Argeso 2006, Baranoğlu and Mengi 2006)

$$\bar{G} = G\left[\frac{1+\beta^G \tau_r^G z}{1+\tau_r^G z}\right] \tag{3}$$

where

$$G = \frac{G_1 G_2}{(G_1+G_2)} \tag{4}$$

$$\tau_r^G = \frac{\eta^G}{G_1+G_2} \tag{5}$$

$$\beta^G = \frac{G_1+G_2}{G_2} > 1 \tag{6}$$

τ_r^G is the retardation time of relaxation function associated with the shear modulus G, the damping parameter η^G, and the ratio of the instantaneous value of relaxation function to the equilibrium value of relaxation function β^G (Eratlı et al. 2014). By using Eqns (4) and (6) can be defined as below:

$$G_1+G_2 = \frac{(\beta^G)^2}{\beta^G-1} G \tag{7}$$

$$G_1 = \left(\beta^G - 1\right) G_2 \tag{8}$$

The graphical representation of the variation of $(\beta^G)^2 / \beta^G - 1$ coefficient versus β^G is obtained as shown in Fig. 2.

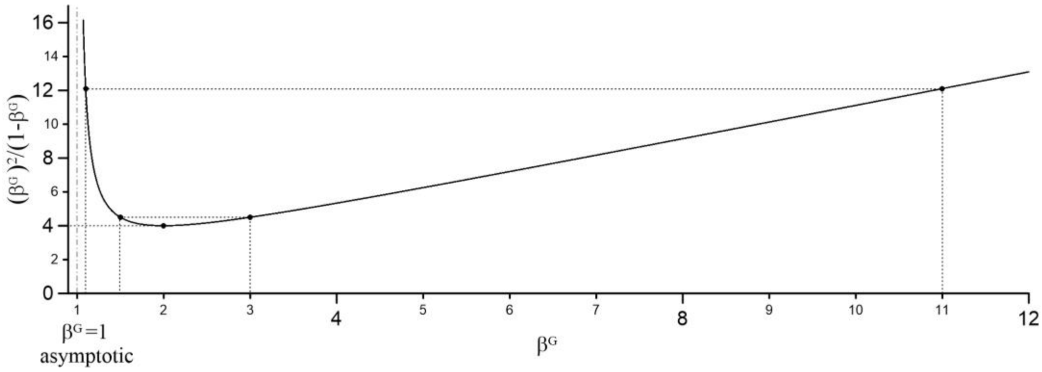


Fig. 2 $(\beta^G)^2/\beta^G-1$ coefficient

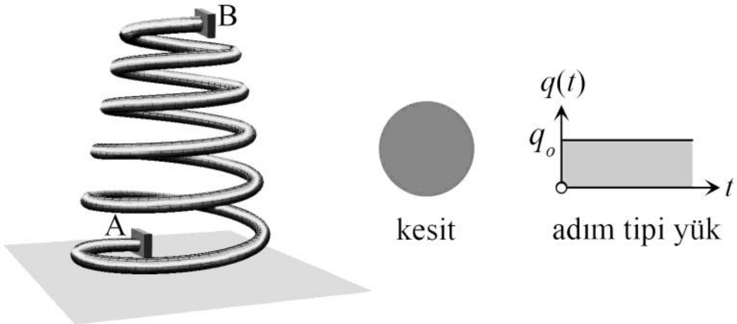
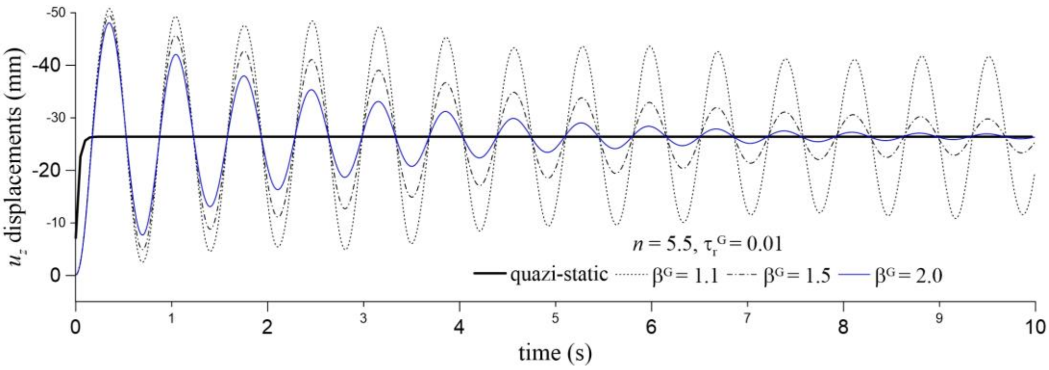


Fig. 3 The conical helicoidal rod having circular cross-section.

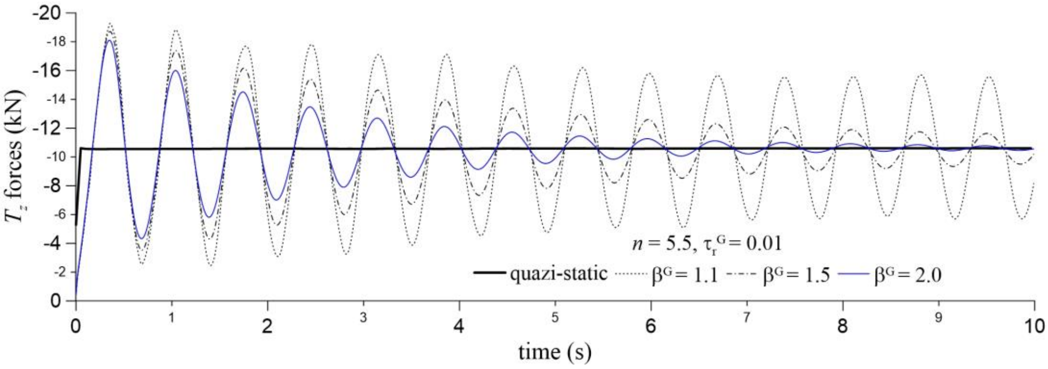
3. NUMERICAL EXAMPLES

A viscoelastic conical helicoidal rod with fixed-fixed boundary condition is solved (see Fig. 3). The helix geometry has $n=5.5$ number of active turns, the height of the rod is $H=5\,\mathrm{m}$ and the minimum radius of helix to maximum of helix ratios $R_{\min}/R_{\max}=0.5$ where $R_{\max}=2\,\mathrm{m}$. The circular cross-sectional diameter of the rod is $D=20\,\mathrm{cm}$. The viscoelastic material exhibits the standard type of distortional behavior while having elastic Poisson's ratio $\bar{\upsilon}=\upsilon=0.3$. The material parameters are $G=80\,\mathrm{GPa}$, $\tau_r^G=0.01\mathrm{s}$, $\beta^G=1.1, 1.5, 2.0, 3.0, 11.0$ and the density of material $\rho=7850\,\mathrm{kg/m^3}$. The form of the complex shear modulus can be obtained by using Eq. (3). The rod is subjected to a dynamic rectangular impulsive type of uniformly distributed load $q=q_z(t)$, where $q_o=500\,\mathrm{N/m}$ (see Fig. 3). The quasi-static and dynamic responses of the rod are determined within $0\le t\le 10\,\mathrm{s}$. The analyses are carried out in the Laplace space and the results are transformed back to the time space numerically by modified Durbin's algorithms (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The parameters which are used in the analysis for inverse Laplace transformation algorithm are $N=2^{11}$ and $aT=6$ which are verified by Eratlı et al. (2014).

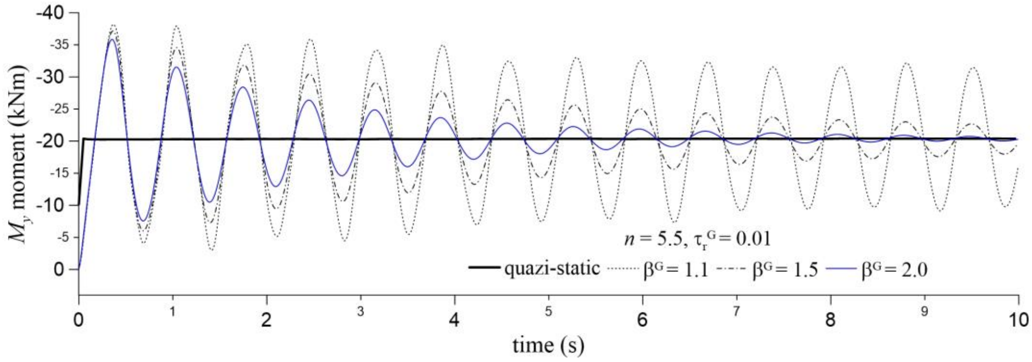
The vertical displacements u_z at the middle point of the helicoidal rod, the force T_z the moments M_y at the clamped end A (see Fig. 1) are determined using 100 finite elements through the analysis. The time variation curves of u_z , T_z , and M_y are depicted for $\beta^G = 1.1, 1.5, 2.0$ and $\beta^G = 2.0, 3.0, 11.0$ ratios in Figs. 4-5, respectively. The aim of these figures is to investigate the influence of viscoelastic material parameter β^G on the dynamic response of viscoelastic conical helicoidal rod. For the same values of τ_r^G retardation time, viscosity parameter η^G , and shear modulus G , there exist two different β^G ratios, except $\beta^G = 2$ that is a transition value ($G_1 = G_2$). One of these β^G ratios is between $1 < \beta^G < 2$ ($G_1 < G_2$) and the another one is $\beta^G > 2$ ($G_1 > G_2$) (see Fig. 2). G_1, G_2 values are related to viscosity part and elastic part of the standard model type viscoelastic material. In Figs. 4-5, the dashed, the dot and the solid lines are used for $\beta^G = 1.5, 3.0$, $\beta^G = 1.1, 11.0$ and $\beta^G = 2$ ratios, respectively. The responses curves of $\beta^G = 2$ values is lowest contour $\beta^G = 1.1, 1.5$ (see Fig. 4), whereas is highest contour $\beta^G = 3.0, 11.0$ (see Fig. 5). The amplitudes of u_z, T_z, and M_y (see Figs.4-5) decrease with the increasing β^G ratios, and approach to the quasi static case. When the amplitude of curves are compared Fig. 5 with respect to Fig. 4, it is observed that the amplitudes for $\beta^G > 2$ values dissipate more rapidly.



(a) u_z

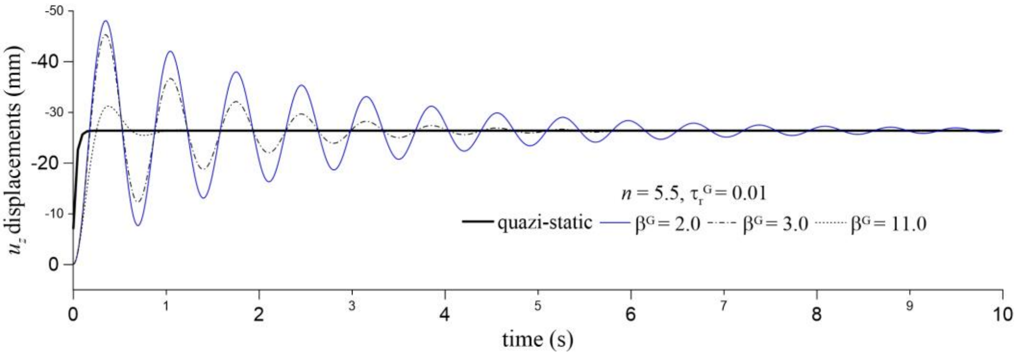


(b) T_z

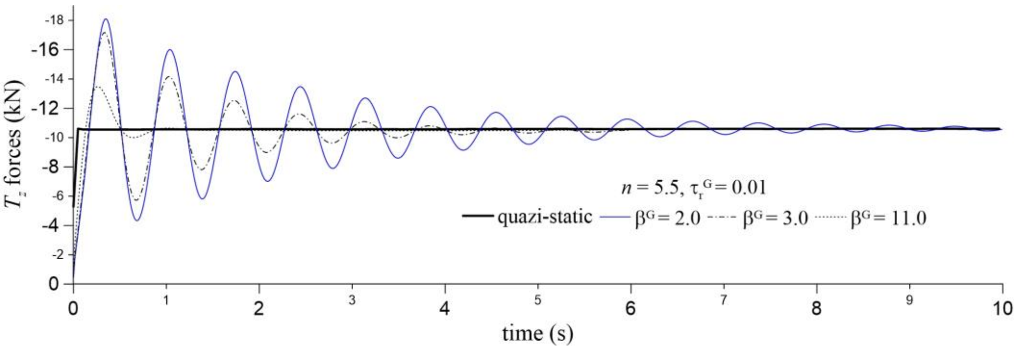


(c) M_y

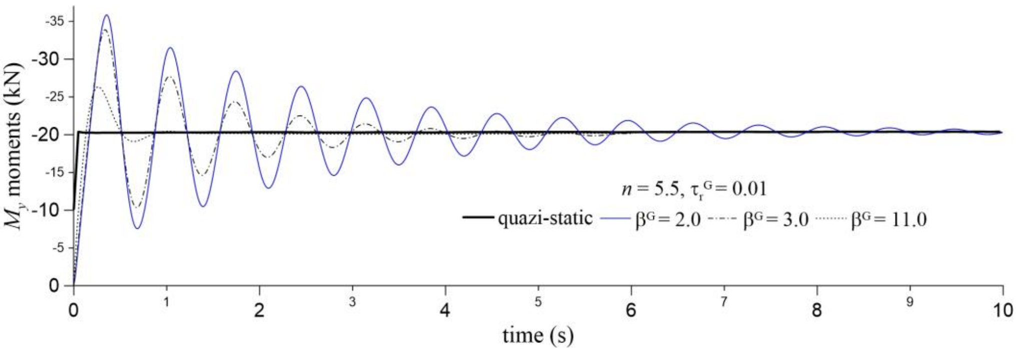
Fig. 4 Time histories of viscoelastic conical helicoidal rod for $\beta^G = 1.1, 1.5, 2.0$



(a) u_z



(b) T_z



(c) M_y

Fig. 5 Time histories of viscoelastic conical helicoidal rod for $\beta^G = 2.0, 3.0, 11.0$

4. CONCLUSION

The dynamic viscoelastic behavior under the step type of uniformly distributed dynamic load of conical helicoidal rod is examined for different viscosity parameter β^G ratio via the mixed finite element method. For this purpose, the viscoelastic material behavior is simulated by using the standard model while having elastic Poisson's ratio and the viscoelastic properties are accounted using the correspondence principle. The finite element solutions are carried out in the Laplace space. The results obtained in the Laplace space are transformed back to time space via modified Durbin's algorithm. The effects of the viscoelastic material parameter β^G ratios on the dynamic behavior of the conical helix are discussed extensively.

ACKNOWLEDGMENT

This research is supported by The Scientific and Technological Research Council of Turkey under project no 111M308. These supports are gratefully acknowledged by the authors.

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