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Damage detection of a thin plate using pseudo local flexibility method

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Abstract. The virtual forces of the original local flexibility method are restricted to inducing stress on the local parts of a structure. To circumvent this restriction, we developed a pseudo local flexibility (PLFM) method that can successfully detect damage to hyperstatic beam structures using fewer modes. For this study, we further developed the PLFM so that it could detect damage in plate structures. We also devised the theoretical background for the PLFM with non-local virtual forces for plate structures, and both the lateral and rotary degree of freedom (DOF) measurements were considered separately. This study investigates the effects of the number of modes, the actual location that sustained damage, multiple damage locations, and noise in modal parameters for the damage detection results obtained from damaged numerical plates. The results revealed that the PLFM can be used for damage detection, localization, and quantification for plate structures, regardless of the use of the lateral DOF and/or rotary DOF.

Keywords: pseudo local flexibility method; rotary DOF; long gauge; plate; damage detection

1. Introduction

Plate structures are widely used as important structural components in many engineering fields, including civil, mechanical, aerospace, and automotive engineering. Structural condition assessments of in-service plate structures play a critical role in global structural health monitoring. In recent years, the research community has paid particular attention to vibration-based structural damage detection techniques that can be used to perform damage diagnosis based on modal parameters (e.g., Doebling *et al*. 1996, Salawu 1997, Caicedo 2003, Giraldo 2006).

Among studies on vibration-based structural damage detection methods, those that focus on their application to 2D plate-type structures are relatively limited. Cawley and Adams (1979) were probably the first to detect damage on a rectangular plate using frequency shifts. Many other methods for detecting damage in plate-like structures have also been proposed based on finite element models (e.g., Dos Santos *et al*. 2005, Ge and Lui 2005, Fu *et al*. 2013).

However, a number of proposed vibration-based methods are capable of detecting damage to plate-like structures without requiring any information from a finite element model. For instance, to locate the damaged area of a plate, Cornwell *et al*. (1999) used a damage index based on the fractional strain energy calculated from measured mode shapes with many points. Moreover, Bayissa and Haritos (2007) proposed using the spectral strain energy derived from a moment-curvature response to detect damage to a plate-like structure. Thus, damage locations can be identified using non-mass-normalized mode shapes and natural frequencies, without requiring a finite element model.

Fan and Qiao (2009) applied a 2D continuous wavelet transform algorithm on mode shapes with dense grids that were identified with a roving excitation test. They compared the proposed algorithm against the 2D gapped smoothing method and the 2D strain energy method, and concluded that the proposed method outperformed its counterparts. Zhang *et al*. (2013) applied a modal-frequency-based method to a steel plate by using the 2D gapped smoothing method to calculate the residual values. To detect and localize damage in plate-like structures through vibration testing, the method employs variations of modal frequency data as a roving mass traverses to various locations on a plate.

Ng (2015) proposed using a dual-stage imaging approach for quantitative damage inspection in metallic plates by using the fundamental antisymmetric mode of the Lamb wave. He employed a number of transducers to transmit and receive Lamb wave pulses, thereby sequentially scanning the plate structures before and after the occurrence of damage. Torkzadeh *et al*. (2016) proposed a novel two-stage methodology for damage detection to flexural plates by using an optimized artificial neural network. Their study investigated the location of damaged areas in the plates by using curvature-moment and curvature-moment derivative concepts in the first stage. Afterward, using a properly trained cascade feed-forward neural network as a surrogate model, they evaluated an index of the multiple-damage location assurance criterion based on the frequency change vector of the structures.

Reynders and De Roeck (2010) recently developed the local flexibility method (LFM); its theoretical foundations

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are robust, and the method can be used to determine not only the damage location but also the extent of damage sustained. The general procedure of the LFM involves using flexibility matrices and designated virtual forces that generate locally restricted stress fields in the structure to perform damage localization and quantification.

The structural modal parameters identified from the ambient vibration signals both before and after damage can be used to construct the flexibility matrices, and are key data for the LFM. Thus, the LFM does not require a finite element model of the structure. The general theory of the LFM has been applied to beam structures for damage detection, and few modes are typically necessary.

Especially for simple cases such as those involving a simply supported beam, there are conditions where the first mode alone suffices. However, for a hyperstatic beam or other, more complex structures, the number of modes required for damage estimation can increase significantly. This reduces the feasibility of the LFM because, in practice, only the first few modes can be identified accurately using ambient vibration signals.

Moreover, application of the LFM to other structures has not been achieved, mainly because of the difficulty in identifying virtual forces guaranteed to limit the existence of the induced stress to the local region of another structure (e.g., a plate structure).

Hsu *et al*. (2014) developed the pseudo local flexibility method (PLFM), which successfully detects damage to hyperstatic beam structures using fewer modes. The PLFM eliminates the limitation of virtual forces inducing stress only to the local part of a structure, as is the case with the LFM.

In this manner, the non-local virtual forces that generate concentrated stresses in a local part, and relatively small stresses in other parts of a structure, can be employed. Most importantly, removing this limitation enables the identification of suitable virtual forces for plate structures.

Therefore, this study proposes employing the PLFM for damage detection in plate structures. First, we lay out the theoretical basis for the PLFM (using non-local virtual forces for plate structures). We then investigate the effects of the number of modes, the damage location, multiple damage locations, and the noise in the modal parameters on the damage detection results for the numerical plates. Both lateral and rotational degree of freedom (DOF) measurements are considered separately. The results indicate that both the damage locations and the extent of the damage sustained can be estimated using a few modal parameters identified from the measured vibration signals.

2. Pseudo local flexibility method for thin plates

The PLFM considers a structure with volume, Ω, and boundary, Γ, that is subjected to Dirichlet boundary conditions along a part of the boundary. A first load configuration, f^1, is applied at a limited number of r DOFs, where the response can be measured. The first load configuration for the PLFM is chosen such that the induced stress field, σ^1, consists of concentrated stresses in the local volume, Ω_p, and a small stress outside Ω_p (i.e., Ω_q), as shown in Fig. 1. The first load configuration f^1 is assumed to only cause non-zero stress within Ω_p for the LFM. Based on the virtual work principle with the body force neglected

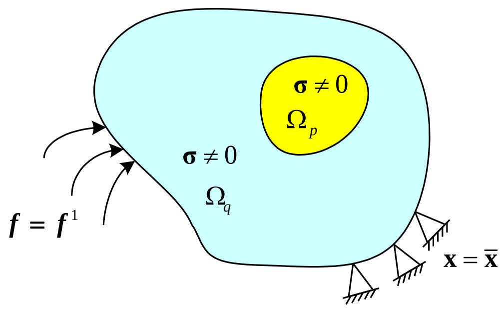

Fig. 1 Structure subjected to the first load configuration, f^1, causing concentrated stress within the local region Ω_p, and relatively small stress outside the local region

$$\int_{\Gamma} t^T \delta x d\Gamma = \int_{\Omega} \sigma^T \delta\varepsilon d\Omega \tag{1}$$

where t is the vector with applied tractions, σ represents the corresponding stress vector, δx is a virtual displacement field that obeys the Dirichlet boundary conditions, and $\delta\varepsilon$ depicts the corresponding virtual strain vector. If the virtual displacement field is chosen as that induced by f^1, but the forces and stresses are due to the second load configuration, f^2, which obeys the boundary conditions of the system, then we derive the following

$$\sum_{j=1}^{r} f_j^2 x_j^1 = \int_{\Omega_p} (\sigma_p{}^2)^T \varepsilon_p{}^1 d\Omega_p + \int_{\Omega_q} (\sigma_q{}^2)^T \varepsilon_q{}^1 d\Omega_q \tag{2}$$

where x_j^1 is the displacement at DOF j corresponding to f^1. Assume that the structure is linearly elastic, and that σ^1 is proportional to ε^1, with stiffness constant K. If the virtual work is calculated both before and after the damage has occurred, we obtain the following

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} = \frac{\int_{\Omega_p} (\sigma_p{}^2)^T \frac{\sigma_p{}^1}{K_p} d\Omega_p + \int_{\Omega_q} (\sigma_q{}^2)^T \frac{\sigma_q{}^1}{K_q} d\Omega_q}{\int_{\Omega_p} (\sigma_{pd}{}^2)^T \frac{\sigma_{pd}{}^1}{K_p+\Delta K_p} d\Omega_p + \int_{\Omega_q} (\sigma_{qd}{}^2)^T \frac{\sigma_{qd}{}^1}{K_q+\Delta K_q} d\Omega_q} \tag{3}$$

where subscript d represents the parameter of the structure in a damaged state. Assume that stresses σ^1 and σ^2 are concentrated within the local volume Ω_p, and σ^1 and σ^2 outside the local volume are small; hence, the strain energy outside the local volume is much smaller than that within the local volume. We can neglect the strain energy outside the local volume, and obtain

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} \cong \frac{\int_{\Omega_p} (\sigma_p{}^2)^T \frac{\sigma_p{}^1}{K_p} d\Omega_p}{\int_{\Omega_p} (\sigma_{pd}{}^2)^T \frac{\sigma_{pd}{}^1}{K_p+\Delta K_p} d\Omega_p} \tag{4}$$

Assume that K and ΔK are constant within Ω_p, which means that only the lump estimation of K and ΔK within Ω_p can be achieved. They can then be moved outside the