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Nonlinear analysis of multi-layered composite beams including partial interaction effect

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ABSTRACT

This paper presents a non-linear beam formulation for the analysis of multi-layered composite beams with partial interaction effect. The present beam formulation is directly degenerated from an assemblage of 3D solid elements, and thus the beam can accurately account for complex geometries and mechanical behaviors under various loading and boundary conditions. The most notable feature is that its embedded cross-sectional discretization provides an individual modeling of the cross-section of each beam layer. This feature makes it easy to model complicated layered beam geometries including arbitrary number of layers, varying and composite cross-sections, various interlayer stiffnesses, and eccentricities. Geometrical and material nonlinearities can be considered. A numerical example is presented for demonstration.

1. Introduction

Multi-layered composite beam structures are broadly used in many engineering fields. Since the multi-layered beams can be composed of several sub-components that have different structural and material properties, they exhibit improved mechanical properties such as high specific stiffness and strength as well as practical manufacturing process depending on the various design requirement (Kim et. al. 2019).

However, the loss of stiffness due to partial interaction is what matters most in the performance of the layered beam structures. The degree of partial interaction depends mainly on stiffness of interlayer shear connection used, but complete shear connection can hardly be obtained in general. Due to the deformable shear connection, there exist interlayer slips between the adjacent layers of the layered beam. The classical beam models not concerning partial interaction are often used due to simplicity, but they may give significant underestimation. In addition, the partial interaction often leads to geometrically nonlinear behaviors. Consequently, it is important to consider partial interaction and interlayer slips appropriately for accurate analysis of the multi-layered composite beam structures. In this paper, nonlinear finite element formulations for the analysis of multi-layered composite beams are briefly introduced and their numerical

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examples are presented.

2. Nonlinear finite element formulations for the multi-layered composite beams

We here briefly introduce the nonlinear finite element formulation for the analysis of the multi-layered composite beams. The detailed formulation of the continuum mechanics based beam element is well described in Yoon et. al. 2012 and Yoon and Lee 2014.

The geometry interpolation of the multi-layered beam including relative displacements between the layers is obtained by a material position of sub-beam m in layer n, see Fig. 1 (Kim et. al. 2019),

$$ {}^{t}\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\,{}^{t}\mathbf{x}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)(n)}\,{}^{t}\mathbf{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)(n)}\,{}^{t}\mathbf{V}_{\bar{z}}^{k} + \sum_{k=1}^{q} h_k(r)\,{}^{t}\varphi_k^{(n)}\,{}^{t}\mathbf{V}_{s}^{k} \quad (1) $$

with $\bar{y}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{y}_k^{j(m)}$ and $\bar{z}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{z}_k^{j(m)}$. (2)

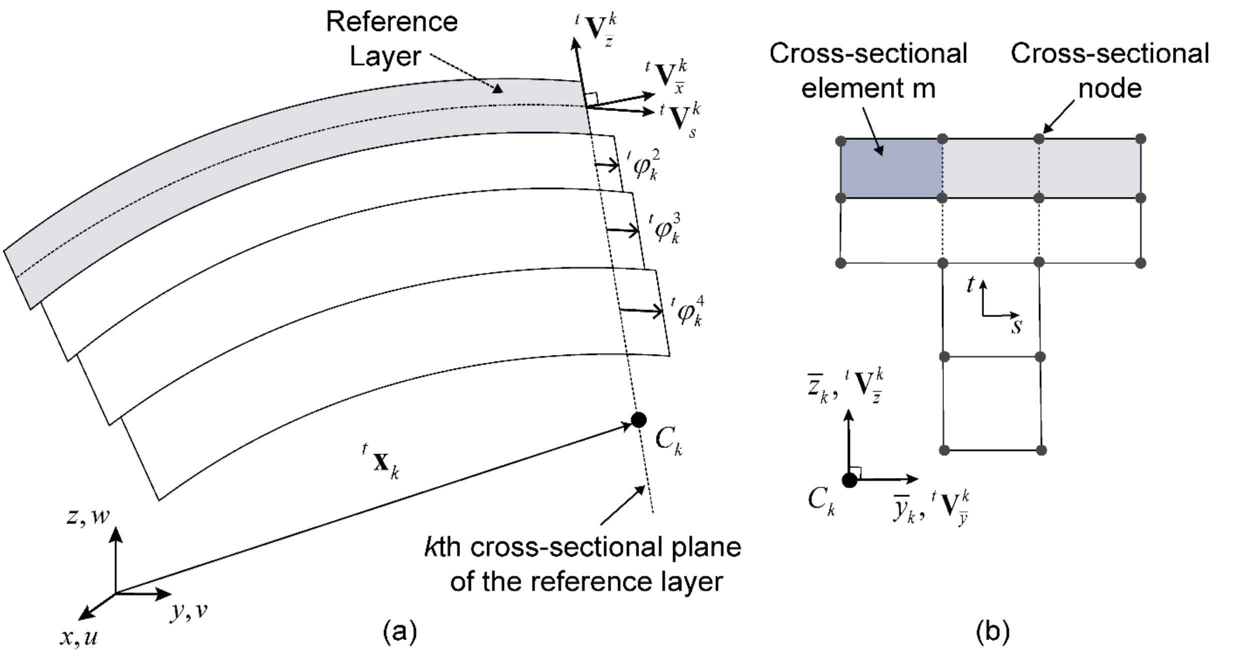

Fig. 1 (a) The concept of the continuum mechanics based beam finite element for multi-layered beam including interlayer slips and (b) cross-sectional discretization.

In Eq. (1), $\bar{y}_k^{(m)(n)}$ and $\bar{z}_k^{(m)(n)}$ denote the position in the cross-section of sub-beam m of layer n, ${}^t\varphi_k^{(n)}$ is the layer DOF given to each layer, ${}^t\mathbf{V}_{\bar{y}}^k$ and ${}^t\mathbf{V}_{\bar{z}}^k$ are director vectors orthonormal to each other, and ${}^t\mathbf{V}_s^k$ is the slip director vector that indicates the longitudinal direction of the beam.

From the geometry interpolation in Eq. (1), the incremental displacements between the configurations at times $t+\Delta t$ and t of sub-beam m in layer n is obtained as

$$
{}_0\mathbf{u}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\,{}_0\mathbf{u}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)(n)}\left({}^{t+\Delta t}\mathbf{V}_{\bar{y}}^k - {}^t\mathbf{V}_{\bar{y}}^k\right) + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)(n)}\left({}^{t+\Delta t}\mathbf{V}_{\bar{z}}^k - {}^t\mathbf{V}_{\bar{z}}^k\right)
$$
$$
+\sum_{k=1}^{q} h_k(r)\left({}^{t+\Delta t}\varphi_k^{(n)}\,{}^{t+\Delta t}\mathbf{V}_s^k - {}^t\varphi_k^{(n)}\,{}^t\mathbf{V}_s^k\right) \tag{3}
$$

with ${}_0\mathbf{u}_k = \begin{bmatrix} {}_0u_k & {}_0v_k & {}_0w_k \end{bmatrix}^{\mathrm{T}}$ and $\boldsymbol{\theta}_k = \begin{bmatrix} \theta_x^k & \theta_y^k & \theta_z^k \end{bmatrix}^{\mathrm{T}}$ (4)

3. Numerical example

In this section, a bending problem of a four-layer cantilever beam with three interlayers is considered.

Consider the four-layer cantilever beam that consists of two different linear elastic materials connected with three deformable interlayers. The first and second layers from the bottom are composed of Material 1 with Young's modulus $E_1 = 35\times10^3$ and the other two layers are composed of Material 2 with $E_2 = 80\times10^3$. The connection stiffnesses of three interlayers are zero. That is, frictionless tangential slips are allowed. The beam is subjected to a tip bending moment M_y at $x = \mathrm{L}$ as shown in Fig. 2.

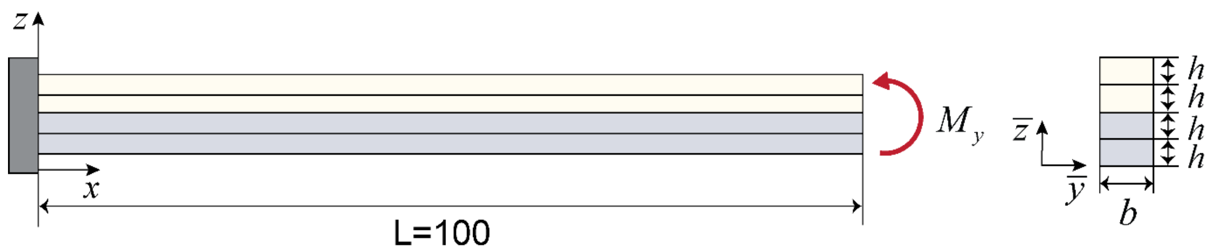

Fig. 2 Four-layer cantilever beam subjected to tip bending moment M_y. ($b = 1$, $h = 0.5$)

The beam is modeled using 20 beam elements, and the cross-section is discretized by four 4-node cross-sectional elements. Fig. 3 shows the calculated load-displacement curves for the tip x- and z- directional displacements (u and w) compared with the theoretical solutions in Krawczyk et. al. 2007 and Battini et. al. 2009. The present beam model shows good agreements even when the beam is subjected to a large rotation.

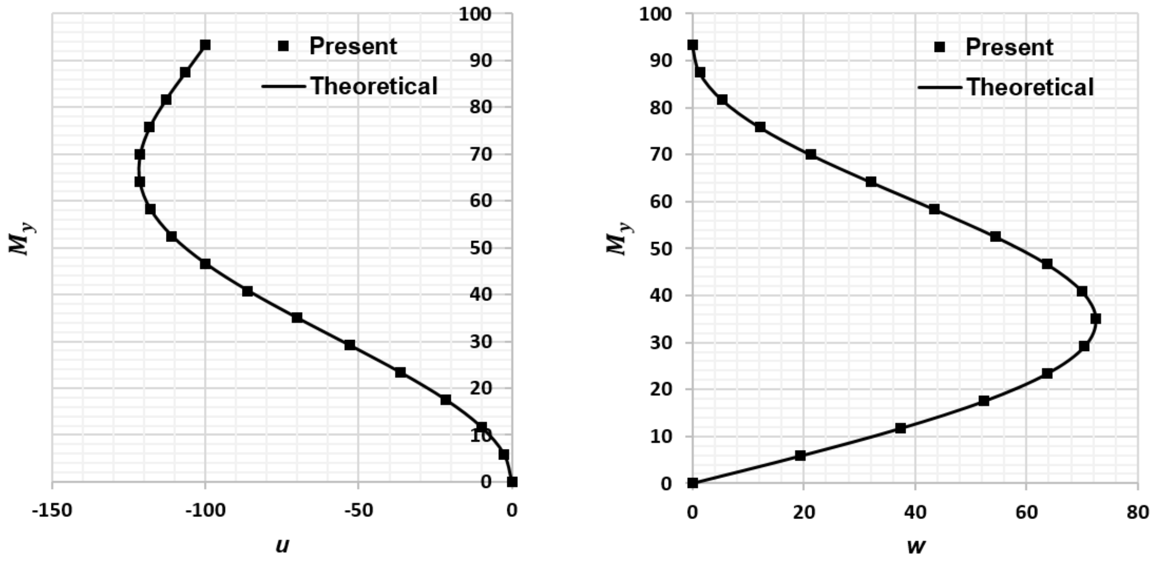

Fig. 3 Load-displacement curves for the u- and z-directional displacements at the free tip in the four-layer cantilever beam problem.

4. Conclusions

In this paper, the non-linear beam formulation for analysis of multi-layered composite beams is introduced. Since the present beam formulation is degenerated from an assemblage of 3D solid elements, it exhibits great modeling capabilities including complex multi-layered beam geometries, different interlayer stiffnesses, and various loading and boundary conditions. A simple verification of the present beam formulation is performed with theoretical solutions of geometrical nonlinear behavior of four-layer cantilever beam subjected to large rotations, and good agreement is observed. While only the geometrical nonlinear beam problems with no interlayer connections is illustrated in this paper, the present beam formulation is successfully applicable to the analysis of multi-layered beams with partial interactions considering geometrical and material nonlinearities. The idea proposed for beams can be extended for finite element analysis of plates and shells (Lee et. al. 2010, Lee et. al. 2014, Jeon et. al. 2015, and Ko et. al.2016)

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