The 2019 World Congress on
Advances in Structural Engineering and Mechanics (ASEM19)
Jeju Island, Korea, September 17 - 21, 2019

Graded metamaterials for vibration attenuation in wide frequency range

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ABSTRACT

We study linear wave propagations in 1D chains consisting of slender cylinders with gradually varying lengths. Slender cylinders have low frequency bending vibration modes which are coupled with propagating waves along the chain and work as local resonances in the chain. The local resonance of the cylinder induces negative mass density in a specific frequency range and efficiently attenuates propagating waves. We demonstrate that gradually varying local resonances in the cylinder chain are superposed and it efficiently suppresses vibrations in a wide frequency range.

1. INTRODUCTION

Recently mechanical metamaterials attract great attentions to control mechanical waves such as sound and elastic waves. Mechanical waves propagate through media having elasticity and inertia, thus their characteristics significantly depend on the effective mechanical properties of the media. Therefore if one control the effective mechanical properties of media by designing their internal structure, the wave can be controlled to be localized (Campbell 2004), dispersed (Kim 2015, 2017, 2019), focused (Spadoni 2010), accelerated or decelerated (Chaunsali 2018) and etc. The mechanical metamaterials may categorized into two types. One is continuum based structure having complex internal geometries, and the other is granular type structure consisting of discrete granules having relatively simple geometry. In granular structures, waves propagate through contacts of each granules and contact behavior between each granular significantly affects the wave propagation. Contact behavior between two granule having curvature at contact surface is nonlinear, which is well presented in Hertzian contact (Johnson 1985). This inherent nonlinearity in the contact behavior enables abundant wave dynamics in the granular system from nearly linear to highly nonlinear regime.

Here we study linear elastic wave propagations in a single chain of slender cylinders having low frequency bending vibration modes. A bending vibration mode of the cylinder coupled with propagating waves along the chain makes negative effective mass density in a specific frequency range. This is because the vibration mode of a

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cylinder works as local resonance in the chain. Slender cylinders have countless bending vibration modes and their corresponding natural frequencies. Thus, within an interested frequency regime we can locate multiple local resonances and vary their frequencies by designing geometry or material properties of the cylinder. Multiple local resonances aligned in series in the chain are superposed and make wide frequency band gaps. Here we numerically investigate how the local resonances are superposed and create wide frequency band gaps in the cylinder chain.

2. NUMERICAL MODEL

We first investigate frequency band structures of two homogeneous chains consisting of twenty number of 80 mm cylinders and twenty number of 50 mm cylinders, respectively. Then we compare the frequency band structures with that of combined chain having ten number of 80 mm cylinders and ten number of 50 mm cylinders aligned in series (Fig. 1(a)) to investigate superposition of the frequency band structures. Fig. 1(a) shows a schematic diagram of the combined chain. Young's modulus and Poisson's ratio of the cylinder is 70 GPa and 0.23, respectively, and its diameter is 5 mm. We assumed that the chain is compressed with 7.5 N. The cylinders bending vibration modes and their natural frequencies are analyzed with a finite element method (Fig. 1(b)). Based on the modal analysis data we create a simplified discrete element model (DEM) consisting of point masses connected with linear springs for efficient numerical analysis.

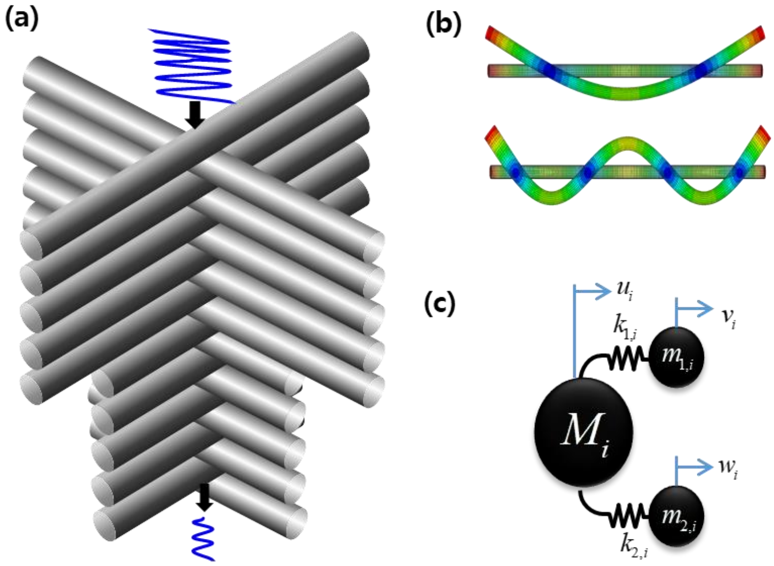

Fig. 1 (a) schematic diagram of the combined cylinder chain, (b) 1st and 3rd bending vibration mode of a slender cylinder, and (c) discrete element model (DEM) of a cylinder taking into account two vibration modes.

First, we create an equivalent DEM for a single cylinder that can take into account two bending vibration modes (the 1st and 3rd bending mode as shown in Fig. 1(b)). It is note that even numbered bending vibration modes (asymmetric modes) have a nodal point at center of the cylinder and they are not coupled with the propagating waves when the contact position is at center of the cylinders. Then we connect the

DEMs with springs representing linearized contact stiffness (β) for the cylinder chain. The equation of motions of the i^{th} cylinder model is expressed as follows:

$$\begin{aligned} \ddot{u}_i &= \frac{\beta}{M_i}\left(u_{i-1} - 2u_i + u_{i+1}\right) + k_{1,i}\left(v_i - u_i\right) + k_{2,i}\left(w_i - u_i\right) \\ \ddot{v}_i &= \frac{k_{1,i}}{m_1}\left(u_i - v_i\right) \\ \ddot{w}_i &= \frac{k_{2,i}}{m_2}\left(u_i - w_i\right) \end{aligned} \tag{1}$$

3. RESULTS

We analyze frequency response functions of the three chain models and compare them in Fig. 2 to understand how the frequency band structures of the 80 mm chain and 50 mm chain are superposed in the combined chain model. The 80 mm homogeneous chain shows band gaps in 3.2 kHz ~ 4.0 kHz and 11.6 kHz ~ 20.5 kHz, and 50 mm chain shows band gaps in 5.7 kHz ~ 10.1 kHz and 24.2 kHz ~ 49.0 kHz. In the combined chain, the wave amplitude in the band gap frequencies of the 80 mm chain and 50 mm chain is significantly reduced. It shows clear pass bands only for the frequency regimes where both of the chain models have pass band. This is because the frequency band structures of the 80 mm homogeneous chain and 50 mm homogeneous chain are supposed in the combined chain.

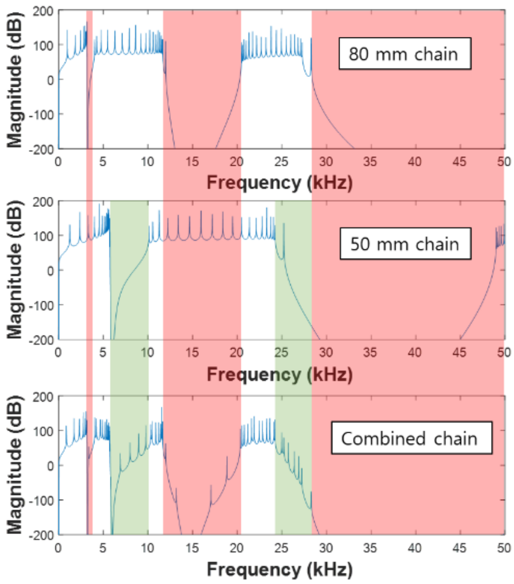

Fig. 2 Frequency response function (FRF) of a 80 mm chain model (top), 50 mm chain model (middle), and combined chain model (bottom)

To create wide frequency regime showing significant wave attenuation we make a combined chain using six different homogeneous chains consisting of 100 mm, 90 mm, 80 mm, 70 mm, 60 mm, 50 mm cylinders, respectively, as shown in the inset of Fig. 3. Here each homogeneous chain consists of 4 cylinders. The magnitude plot of the FRF of this combined chain shows that a clear pass band appears only a narrow frequency region (0 kHz ~ 2.2 kHz). Except this frequency pass band wave attenuates due to the superposed band gaps of the six homogeneous chains.

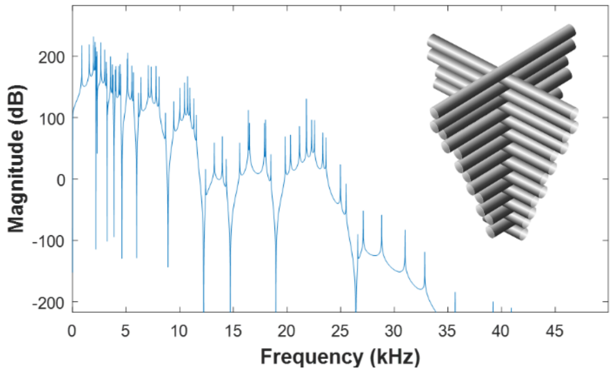

Fig. 3 Frequency response function (FRF) of the combined chain of six different homogeneous chains consisting of 100 mm, 90 mm, 80 mm, 70 mm, 60 mm, 50 mm cylinders, respectively. The inset show schematic diagram of the combined chain.

3. CONCLUSIONS

We numerically demonstrate that a cylinder chain consisting of slender cylinders with various lengths can attenuate waves in a wide frequency range. This is because slender cylinders have low frequency bending vibration modes creating resonance band gaps and they are superposed in the combined chain. This superposition of resonance band gaps can be used for various applications for vibration suppression.

ACKNOWLEDGEMENT

This work is supported from the National Research Foundation of Korea, NRF-2017R1C1B5018136 .

REFERENCES

Campbell, D.K., Flach, S., and Kivshar, Y.S. (2004), “Localizing energy through nonlinearity and discreteness,” Physics Today, 57(43).

Chaunsali, R., Kim, E., Yang, J. (2018) “Demonstration of accelerating and decelerating nonlinear impulse waves in functionally graded granular chains,” Phil. Trans. R. Soc. A, 376, 20170136.

Johnson K.L. (1985) “Contact Mechanics”, Cambridge University Press.

Kim, E., Yang, J., Hwang, H.Y., Shul, C.W. (2017) “Impact and blast mitigation using locally resonant woodpile metamaterials,” Int. J. Impact Eng., 101, 24-31.

Kim, E., Yang, J., (2019) “Review: Wave propagation in granular metamaterials,” Funct. Compos. Struct., 1, 012002.

Kim, E., Kim, Y.H.N., Yang, J. (2015) “Nonlinear stress wave propagation in 3D woodpile elastic metamaterials,” Int. J. Solid Struct., 58, 128-135.

Spadoni A., and Daraio C. (2010) “Generation and control of sound bullets with a nonlinear acoustic lens,” PNAS, 107(16), 7230-7234.