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Monitoring data based modeling of temperature-frequency correlation

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ABSTRACT

Numerous investigations have indicated that varying environmental and operational conditions significantly affected structural modal parameters. This paper investigates the dependency of modal frequencies on environmental factors which includes temperature and wind velocity based on the structural health monitoring records on a single tower cable-stayed bridge. The Principal Component Analysis (PCA) is first employed as a signal pre-processing tool to distinguish temperature and wind effects on structural modal parameters from other environmental factors. The pre-processed dataset by PCA implies the relationship between modal parameters and temperature as well as wind velocity. Consequently, the Gaussian Processing Regression (GPR) technique is employed to model the relationship between the pre-processed modal parameters and environmental factors, with a confidence interval that indicates statistical characteristics. Numerical results indicate that pre-processed modal parameters by PCA can retain the most features of original signals. Furthermore, the trend of the pre-processed modal frequency is dramatically affected by temperature and wind velocity. With a confidence interval, the GPR regression models are capable of mapping relationship of environmental factors and modal frequency as well as the uncertainty on modeling and predicting. However, environmental effects on modal damping ratio and the entire modal shapes needs further investigation.

1. INTRODUCTION

Many researchers have carried out investigations for the influence of environmental factors on dynamical characteristics via field measurements and dynamic tests, including (Sohn 1999; Ni 2005; Xia 2006). Their investigations have indicated that temperature causes the most of variation in modal frequencies. However, the investigation of wind velocity and humidity effect on modal parameters of bridges is

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insufficient. (Hua 2007) have proposed a modeling method that utilized principal component analysis (PCA) for extracting predominant feature vectors and support vector regression (SVR) for databased statistical learning. (Li 2009) have proposed an approach that combined nonlinear principal component analysis (NLPCA) with artificial neural network (ANN) to investigate the effects of temperature and wind velocity on modal parameters of cable-stayed bridges.

In this paper, a investigation for the dependency of modal frequencies on environmental factors which includes temperature and wind velocity based on the structural health monitoring records on a single tower cable-stayed bridge. PCA is first employed as a signal pre-processing tool to distinguish temperature and wind effects on structural modal parameters from other environmental factors. The pre-processed dataset by PCA implies the relationship between modal parameters and temperature as well as wind velocity. Consequently, the Gaussian Processing Regression (GPR) technique is employed to model the relationship between the pre-processed modal parameters and environmental factors, with a confidence interval that indicates statistical characteristics.

2. METHOD OF MODAL IDENTIFICATION BASED ON PCA AND GPR

2.1 Principal component analysis

Large-scale bridges such as suspension bridges and cable-stayed bridges are usually instrumented with long-term monitoring systems. These systems include temperature sensors or other types of sensors which are usually installed at different structural parts. Due to these numerous sensors, a monitoring data matrix composites with large dimensions, which makes computing very inefficient. As well known, PCA works as a powerful mathematical tool to extract predominant feature vectors, such as the temperature principal components.

Principal component analysis is introduced briefly in this paper, more details can be learnt in (Hua 2007). Given the j th measurement data set $\{\mathrm{x}\}_j = (\mathrm{x}_{j1}, \mathrm{x}_{j2}, \dots, \mathrm{x}_{jn})^T, j = 1,2,\dots,m$, where *T* denotes transposition and m is the total number of measurements, the $n \times n$ dimension covariance matrix C is formed as

$$\mathrm{C} = \sum_{j=1}^{m} \{\mathrm{x}\}_j \{\mathrm{x}\}_j^T \tag{1}$$

The singular value decomposition on the covariance matrix is conducted as

$$\mathrm{C} = \mathrm{U}\Lambda\mathrm{U}^T \tag{2}$$

where U= orthogonal eigenvector matrix with $\mathrm{U}^T\mathrm{U}=\mathrm{I}$; and Λ= eigenvalue or singular value matrix which has the form of

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \vdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \tag{3}$$

with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$. The transformation to the principal components is then applied as

$$\{z\}_j = U^T \left[\{x\}_j - \{\bar{x}\} \right] \tag{4}$$

where $\{\bar{x}\}$ is the vector of means of x-data. It can be shown that the covariance matrix of $\{z\}_j$ $(j = 1, 2, \ldots, m)$ has the expression

$$C_z = \sum_{j=1}^{m} \{z\}_j \{z\}_j^T = [\Lambda] = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \vdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \tag{5}$$

which implies that the vectors $\{z\}_j$ $(j = 1, 2, \ldots, m)$ are uncorrelated.

The proportion of variance explained by the first *p* eigenvalues is defined as

$$\text{Prop}_p = \frac{\sum_{i=1}^{p} \lambda_i}{\sum_{j=1}^{p} \lambda_j} \tag{6}$$

then the transformation to the first p principal components is

$$\{\bar{z}\}_j = U_{n \times p}^T \left[\{x\}_j - \{\bar{x}\} \right] \tag{7}$$

2.2 Gaussian processing regression model

GPR model provides both mean values and confidence ranges (standard deviations) on its predictions, which could be very helpful if the prediction is used for damage detection.

Similar to most regression problems, the system is modeled as

$$y = f(X) + \varepsilon \tag{8}$$

where $X \in R^{n \times d}$ is the input, n is the number of data sets, d is the dimension of different input features, $y \in R^{n}$ is the output and ε is process noise, which is modeled as Gaussian distribution with zero mean and a standard deviation of σ.

$$\varepsilon \sim N(0, \sigma) \tag{9}$$

The distribution of the function f is a Gaussian process,

$$f = GP(m, k) \tag{10}$$

where m is the function mean and k is the function covariance. Assuming the mean is zero for the given input x,

$$f \sim N(0, K) \tag{11}$$

where K is the covariance function. Then for $f = \{ f(\mathrm{x}) : \mathrm{x} \in X \}$, $f^{*} = \{ f(\mathrm{x}) : \mathrm{x} \in X^{*} \}$,

$$\left[\begin{array}{c} f \\ f^{*} \end{array} \right] \Bigg| X, X^{*} \sim N \left(0, \left[\begin{array}{cc} K(X,X) & K(X,X^{*}) \\ K(X^{*},X) & K(X^{*},X^{*}) \end{array} \right] \right) \tag{12}$$

After conditioning a joint Gaussian distribution of $\left[\begin{array}{c} y \\ y^{*} \end{array} \right]$,

$$y^{*} \sim N(\mu^{*}, (\sigma^{*})^{2}) \tag{13}$$

the mean and covariance are then defined as,

$$\mu^{*} = K(X^{*}, X)(K(X,X) + \sigma^{2} I)^{-1} y \tag{14}$$

$$(\sigma^{*})^{2} = K(X^{*}, X^{*}) + \sigma^{2} I - K(X^{*}, X)(K(X,X) + \sigma^{2} I)^{-1} K(X, X^{*}) \tag{15}$$

Also, more details on GPR can be found in other researches, such as (Rasmussen 2006).

When taking the first principal components $\{\overline{z}\}_{j}$ as the input and the temperature measurements y as the output, a training pair can be denoted as $(\{\overline{z}\}_{j}, y)$ and then put into Eq. (8) to train the GPR model for regression and prediction. The results of prediction of PCA-GPR and GPR models in simulation are illustrated in Fig. 1.

3. CONCLUSIONS

The first principal component after PCA procedure contains the predominant feature vector of enviromental monitoring data, such as temperature. Using the principal component and the modal frequency as the train set of the Gaussian Processing Regression, a method is proposed to model the relationship between the pre-processed modal parameters and environmental factors, with a confidence interval that indicates statistical characteristics. The results of numerical simulation implies that the PCA-GPR method has the capacity of modeling the dependency of modal frequencies on environmental factors.

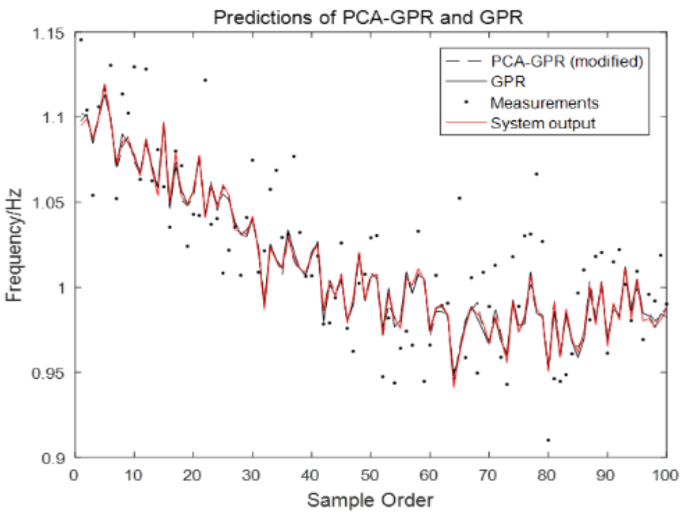

Fig. 1 Prediction of PCA-GPR and GPR models in Simulation

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