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Selection of sensor location for structural system identification using degree of freedom-based reduction method

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ABSTRACT

Structural system identification method matches numerical model and real model with experimental sensor data. In this study, measured modal data is applied for finite element mode-based system identification. Due to lack of number of sensors, degree of freedom-reduction method is used to revise full model. When revising finite element model by restored responses, selection of sensor location is important. Method for selection of sensor location will be introduced and verified. Numerical examples demonstrate that the proposed method improves the accuracy and efficiency when solving structural system identification problems.

1. INTRODUCTION

Numerical analysis techniques have increased geometrically along the expansion of calculation ability. Modern computers have enabled the construction of sophisticated numerical models such as finite element method. Finite element method is one of the most widespread approach for numerical modelling in engineering design. As numerical analysis techniques have increased, numerical models became more complicated and requires more calculation time. Therefore, reduction methods were proposed and widely used (Guyan, 1965; O'Callahan *et al*, 1989; Friswell *et al.*, 1995). Structural system identification method matches numerical model and real model with experimental sensor data. In this study, measured modal data is applied for finite element model based system identification. Dynamic measurements are carried out on a limited number of accessible nodes, therefore degree of freedom-based reduction method was applied (Cho *et al.*, 2006; Chang *et al.*, 2015).

The experimental results and numerical predictions conspire to disagree frequently. These matters arise from inaccuracy in the model and inexactitude of information in the measurements. Thus, it is unaffected to use the well tested results to

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update numerical model. The importance of the selection of sensor location arises when updating numerical model with tested results.

Method for selection of sensor location will be introduced and verify with numerical examples. Numerical examples demonstrate that the proposed method improves the accuracy and efficiency in structural system identification.

2. METHODOLOGY

2.1 System identification on continuum model

Modern computers have enabled the construction of sophisticated numerical modeling in engineering modeling such as finite element model. Finite element method is widely spread approach when considering numerical model. A method for identifying the element properties of a truss structures is developed and applied (Liu, 1995; Hajela *et al.*, 1989). Applying structural system identification on continuum model had difficulties of handling geometrically increasing number of parameters.

2.2 Degree of freedom based-reduction method

Difficulties when identifying continuum model arises when considering huge number of parameters. Calculation time increases significantly and receiving mode shape data from each degree of freedom is also impossible. Degree of freedom based-reduction method calculates full finite element model by considering only chosen primary degree of freedom information which are the data received from sensor. Eigenvector of real model (φ') is a perturbed value of eigenvector of numerical model (φ) and can be described as

$$\varphi' = \begin{Bmatrix} \varphi'_p \\ \varphi'_s \end{Bmatrix} = \begin{bmatrix} I \\ t' \end{bmatrix} \varphi'_p = T'\varphi'_p \quad . \tag{1}$$

Limitary measured data gained from selected sensor placement is to expand the measured mode shape vectors defined as φ'_p and subscript p relates to the primary coordinates respectively which can also be written as master. Measured data from sensors are to estimate the data at unmeasured locations defined as φ'_s and subscript s relates to the secondary coordinates respectively which can also be written as slave. Expansion of mode shape is the reverse of model reduction, transformation matrices of reduction method are used. Due to approximations used when solving transformation matrices, keeping accuracy quality of transformation matrices is necessary. In this study, IIRS (Iterative Improved Reduced System) method is used to improve accuracy of transformation method (Friswell *et al*, 1995).

2.3 Inverse perturbation

In this study, identification of structural system can be solved by inverse perturbation method. Inverse perturbation method is used to find out the differences between numerical model and real model in this problem. Residual error $\mathbf{R}$ is defined as eigenvalue problem as on Eq. (2a) and can be solved by primary degrees of freedom vector as on Eq. (2b). Purpose of using inverse perturbation method is to make residual error minimum and define thickness change of stiffness matrix and mass matrix.

$$\mathbf{R} \equiv \mathbf{K}'\boldsymbol{\varphi}' - \mathbf{M}'\boldsymbol{\varphi}'\boldsymbol{\lambda}' \tag{2a}$$

$$\mathbf{R} \cong \left[\mathbf{K} - \boldsymbol{\lambda}'\mathbf{M} + \sum_{i=1}^{NE} \left(\Delta K_i(\alpha_i) - \lambda' \Delta \mathrm{M}_i(\alpha_i) \right) \right] \begin{bmatrix} \mathbf{I} \\ t'(\alpha) \end{bmatrix} \boldsymbol{\varphi}'_{\mathbf{p}} \tag{2b}$$

2.4 Selection of sensor location

Mode shapes from structural dynamics are often very significant, and expanding unmeasured data also needs delicate approach. Since selecting sensor location is a consequence matter, a method for selecting sensor placement is proposed and also validated by numerical model problems.

As considered above, primary degrees of freedom are used as recommended sensor location in structural system identification. Method for selecting primary degrees of freedom, there are two steps for this method. First step is to select degree of freedom by using the concept from kinetic energy estimation method (Kim *et al.*, 2000). Restrictively, Ritz vector is necessary in reduction method since there is no information about stiffness matrix and mass matrix (Kim *et al*, 2006). However, in structural system identification problem full information of matrices are provided.

In general, position of sensor should be located at the point where displacement of corresponding degree of freedom which is quite simple concept that was considered on first step. Trouble is that, however, maximum displacement point changes in each mode shapes, and occasionally maximum displacement point of one mode shape tend to be minimum displacement in other mode shape. In this study, this point has been considered as nodal point and conferred as critical point.

3. NUMERICAL PROBLEMS

Consideration of nodal point, in this study, is the most concerned issue when selecting sensor location. Since experimental data can be gathered only from selected location, reliable of unmeasured data depends on measured data. Measurement error influence when restoring full matrices of model from nodal point can be easily validated. Simple one dimension bar will be confirmed as numerical model.

Low frequency modes are considered from simple bar problem. Figure 1 shows 1st mode shape. Blue colored circle is the first sensor position, which is not on nodal point. Mode shape restored by corresponding data, blue line, shows high quality of accuracy. Red colored circle is the sensor position on nodal point. Unfortunately, corresponding data restores different mode shape.

$$\mathrm{MAC}\left(\boldsymbol{\varphi}, \boldsymbol{\varphi}^{\mathrm{Measured}}\right) = \frac{\left(\boldsymbol{\varphi} \cdot \boldsymbol{\varphi}^{\mathrm{Measured}}\right)}{\left(\boldsymbol{\varphi}^T \cdot \boldsymbol{\varphi}\right)\left(\left(\boldsymbol{\varphi}^{\mathrm{Measured}}\right)^T \cdot \boldsymbol{\varphi}^{\mathrm{Measured}}\right)} \tag{3}$$

Influence of measurement error can be validated by simple beam problem. To validate eigenvector error of two different data gathered from each point. The comparison between eigenvector $\boldsymbol{\varphi}$ and measured vector $\boldsymbol{\varphi}^{\text{Measured}}$ are considered as MAC (modal assurance criterion) value and has been shown in Eq. (3). MAC value gives 1 as an answer when two vectors are same. Having MAC value as 1 is eigenvector at blue colored circle from figure 1. However MAC value gives deviated value on 0.02 measurement error which is red colored circle case.

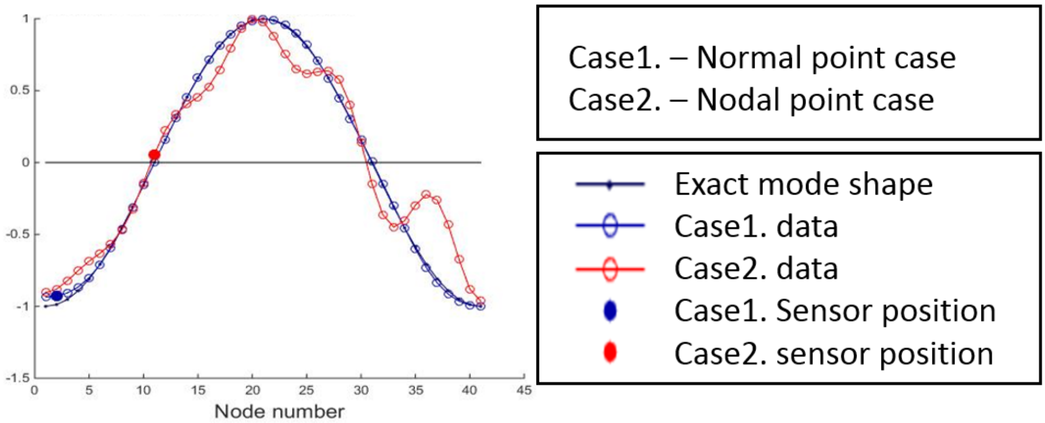

Fig. 1 Measurement error on nodal points for 1^{st} mode shape

4. CONCLUSIONS

In this study, method for selecting sensor location has been proposed. The efficiency of proposed method appears in structural system identification, applied in continuum model. Difficulties in handling numerous number of parameters were solved by transformation matrix from degree of freedom-based reduction method. Since primary degrees of freedom represents full model, problem of selecting sensor location became important when receiving data from real model. Primary degrees of freedom from degree of freedom-reduction method brought the concept of proposed method to select sensor location. Numerical examples demonstrate that the proposed method improves the accuracy and efficiency when solving structural system identification problems.

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