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Performance of the enhanced Craig-Bampton method

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ABSTRACT

In this work, we introduce a new component mode synthesis (CMS), an enhanced Craig-Bampton (CB) method, recently developed for precise finite element (FE) model reduction (Kim and Lee, 2015). Its key to success is the consideration of the residual substructural modes, while those are only neglected in the original CB method. Due to this consideration, the reduced models in the enhanced CB method are much precisely constructed. To obtain the insight into characteristics of the enhanced CB method, we investigate its original formulation in the component matrix level, and test its performance with numerical examples varying the number of interface DOFs.

1. INTRODUCTION

Component mode synthesis (CMS) is a reduced order modeling (ROM) technique using partitioning and assembling strategies (Craig and Bampton, 1965, Hurty, 1965, MacNeal, 1971, Park and Park, 2004, Rixen, 2004, Bennighof and Lehoucq, 2004). The Craig-Bampton (CB) is the most popular CMS method, and the AMLS (Automated multilevel substructuring) method, a CB method with the multilevel substructuring technique, has been widely used for efficiently solving eigenvalue problems of large structural systems.

Recently, we developed two enhanced CMS methods for more accurate ROM (Kim, Boo and Lee, 2015, Kim and Lee, 2015). The new methods consider the effect of residual substructural modes only truncated in the original methods, and consequently we could construct much more accurate reduced models. In the previous works, their accuracy and computational efficiency were verified using various numerical examples. However, there still exist lots of potential research issues to improve its performance as follows: interface reduction techniques, improving their computational efficiency, considering the damping system, and mode selection and error estimating techniques

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(Park, Kim and Lee, 2012, Kim, Lee and Lee, 2014, Kim and Lee, 2014, Kim, Lee and Park, 2015).

In this study, we newly derive the enhanced CB method in the component matrix level. Using the numerical examples, we compare the eigenvalue solution accuracy between the original and enhanced CB methods. In particular, we study the effect of the number of interface DOFs that is a significant roll of solution accuracy of the reduced system.

2. ENHANCED CRAIG-BAMPTON METHOD

In structural dynamics, the equations of motions with free vibration are presented

$$\mathbf{M}_g \ddot{\mathbf{u}}_g + \mathbf{K}_g \mathbf{u}_g = \mathbf{0}, \quad (1)$$

$$\mathbf{M}_g = \begin{bmatrix} \mathbf{M}_s & \mathbf{M}_c \\ \mathbf{M}_c^T & \mathbf{M}_b \end{bmatrix}, \ \mathbf{K}_g = \begin{bmatrix} \mathbf{K}_s & \mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{K}_b \end{bmatrix}, \ \mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix}, \quad (2)$$

where $\mathbf{M}$ and $\mathbf{K}$ are mass and stiffness matrices, respectively, and $\mathbf{u}$ is a displacement vector. The subscripts g, s and b denote the global, substructural and interface boundary quantities, respectively, and the subscript c denotes the coupled quantities. Here, N_g denotes the size of the global matrices.

In the CB method, the global displacement vector is defined as

$$\mathbf{u}_g = \mathbf{T}_0 \mathbf{u} \ \text{ with } \ \mathbf{T}_0 = \begin{bmatrix} \mathbf{\Phi}_s & \mathbf{\Psi}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}, \ \mathbf{\Phi}_s = [\mathbf{\Phi}_s^d \quad \mathbf{\Phi}_s^r], \ \mathbf{\Psi}_c = -\mathbf{K}_s^{-1}\mathbf{K}_c, \ \mathbf{u} = \begin{bmatrix} \mathbf{q}_s^d \\ \mathbf{q}_s^r \\ \mathbf{u}_b \end{bmatrix}, \quad (3)$$

where $\mathbf{T}_0$ is the CB transformation matrix, which is constructed by combination of the fixed-interface normal modes $\mathbf{\Phi}_s$ and interface-constraint modes $\mathbf{\Psi}_c$, $\mathbf{I}_b$ is an identity matrix, and $\mathbf{q}_s$ is the generalized coordinate vector corresponding to $\mathbf{\Phi}_s$. The superscripts d and r denote the dominant and residual quantities, respectively. Note that $\mathbf{\Phi}_s$ is calculated from the substructural eigenvalue problems.

After pre-multiplying $\mathbf{T}_0^T$ in Eq. (1), and condensing $\mathbf{q}_s^r$ with $d^2(\ )/dt^2 = -\omega^2$, the global displacement vector $\mathbf{u}_g$ is approximated as

Table 1. Comparison of the original and enhanced CB methods

	CB	Enhanced CB
Transformation matrix	$\overline{\mathbf{T}}_0$	$\overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_a \mathbf{H}$
Reduced mass matrix	$\overline{\mathbf{M}}$	$\overline{\mathbf{M}} + \mathbf{R}\mathbf{H} + \mathbf{R}^T\mathbf{H}^T + \mathbf{H}^T\overline{\mathbf{T}}_a^T\mathbf{M}_g\overline{\mathbf{T}}_a\mathbf{H}$
Reduced stiffness matrix	$\overline{\mathbf{K}}$	$\overline{\mathbf{K}} + \mathbf{H}^T\mathbf{R}\mathbf{H}$
Size of the reduced matrices	$\overline{N}$	$\overline{N}$

$$\mathbf{u}_g \approx \overline{\mathbf{u}}_g = \overline{\mathbf{T}}_1 \overline{\mathbf{u}} \quad \text{with} \quad \overline{\mathbf{T}}_1 = \overline{\mathbf{T}}_0 + \omega^2 \overline{\mathbf{T}}_a , \quad \overline{\mathbf{u}} = \begin{bmatrix} \mathbf{q}_s^d \\ \mathbf{u}_b \end{bmatrix}, \tag{4}$$

with

$$\overline{\mathbf{T}}_0 = \begin{bmatrix} \mathbf{\Phi}_s^d & \mathbf{\Psi}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}, \quad \overline{\mathbf{T}}_a = \begin{bmatrix} \mathbf{0} & \mathbf{F}_{rs}\hat{\mathbf{M}}_c \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{M}}_c = \mathbf{M}_c + \mathbf{M}_s \mathbf{\Psi}_c , \quad \mathbf{F}_{rs} = \mathbf{K}_s^{-1} - (\mathbf{\Phi}_s^d)(\mathbf{\Lambda}_s^d)^{-1}(\mathbf{\Phi}_s^d) . \tag{5}$$

where $\mathbf{F}_{rs}$ is the residual flexibility.

Neglecting $\omega^2 \overline{\mathbf{T}}_a$ in $\overline{\mathbf{T}}_1$, the reduced matrices of the CB method are obtained as

$$\overline{\mathbf{M}} = \overline{\mathbf{T}}_0^T \mathbf{M}_g \overline{\mathbf{T}}_0 , \quad \overline{\mathbf{K}} = \overline{\mathbf{T}}_0^T \mathbf{K}_g \overline{\mathbf{T}}_0 . \tag{6}$$

The reduced matrix size $\overline{N}$ is much smaller than the global matrix size ($\overline{N} << N_g$).

Then, using $\overline{\mathbf{T}}_1$, the reduced matrices of the enhanced CB method are newly derived

$$\widetilde{\mathbf{M}} = \overline{\mathbf{M}} + \mathbf{R}\mathbf{H} + \mathbf{R}^T \mathbf{H}^T + \mathbf{H}^T \overline{\mathbf{T}}_a^T \mathbf{M}_g \overline{\mathbf{T}}_a \mathbf{H} , \quad \widetilde{\mathbf{K}} = \overline{\mathbf{K}} + \mathbf{H}^T \mathbf{R} \mathbf{H} , \tag{7}$$

with

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{M}}_c^T \mathbf{F}_{rs} \hat{\mathbf{M}}_c \end{bmatrix}, \quad \overline{\mathbf{T}}_a^T \mathbf{M}_g \overline{\mathbf{T}}_a = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{M}}_c^T \mathbf{F}_{rs} \mathbf{M}_s \mathbf{F}_{rs} \hat{\mathbf{M}}_c \end{bmatrix}, \quad \mathbf{H} = \overline{\mathbf{M}}^{-1} \overline{\mathbf{K}} . \tag{8}$$

Note that Eq. (7) might lead to much accurate reduced models without adding the substructural modes due to compensation of the residual mode effect in $\mathbf{F}_{rs}$. We present the comparison between the original and enhanced CB methods in Table 1.

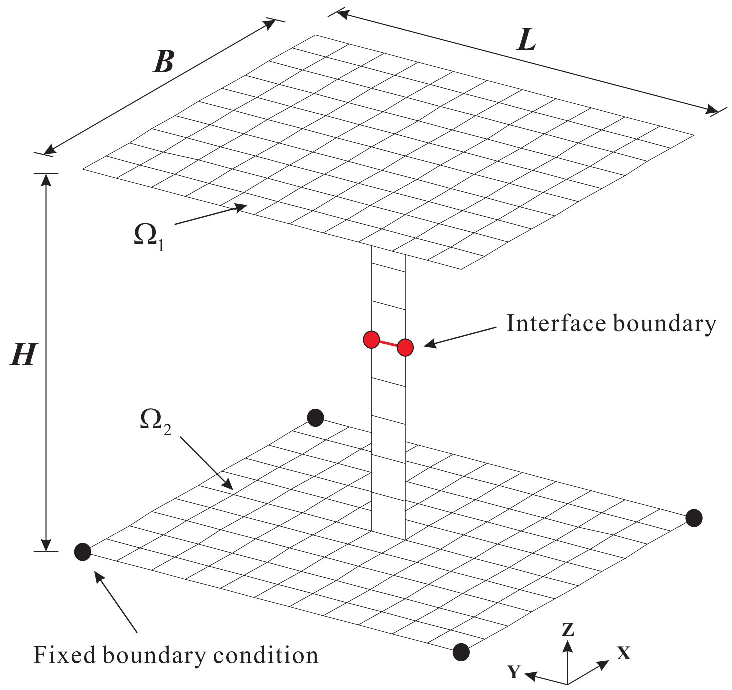

Fig. 1 A structural model.

Table 2. Numerical cases

DOFs	Displacement			Rotation	
	u	v	w	α	β
Case 1	○	○	○	○	○
Case 2	○	○	○	×	×
Case 3	○	×	×	×	×

○: free, ×: fixed

In addition, although the residual flexibility $\mathbf{F}_{rs}$ in $\mathbf{R}$ and $\overline{\mathbf{T}}_a^T \mathbf{M}_g \overline{\mathbf{T}}_a$ is only related with the boundary interface (see Eq. (8)), it affects the total reduce model because of multiplying $\mathbf{H}$. Therefore, the small interface boundary DOFs is insignificant to the accuracy of the reduced system in the enhanced CB method. This is numerically tested in Section 3. However, for better computational efficiency, a fully populated matrix $\mathbf{H}$ might be handled in near future.

3. NUMERICAL STUDY

We here compare the solution accuracy of the approximated eigenvalues between the original and enhanced CB methods using the relative eigenvalue error. We consider a structural model that is two simple plates connected by a slender column. Length L and width B of two plates are 11 *m* and 10 *m*, column height H is 10 *m*, thickness t is 0.05 *m*. Young's modulus E is 206 *GPa*, Poisson's ratio υ is 0.3, and density ρ is 7,850 *kg/m*^{3}. Two plates and column are modeled by 11$\times$10 and 10$\times$1 shell elements, respectively, and the four edges of the bottom plate are fixed, see Fig. 1.

The structural model is partitioned by two substructures at the middle of the column, and, to investigate the effect of the small interface DOFs, we consider three numerical cases varying the number of interface DOFs, see Table 2. Ten substructural modes, selected by the frequency cut-off rule, are used to construct the reduced models. Fig. 2 clearly shows that the enhanced CB method leads to better solution accuracy than the original CB method in the numerical cases considered here.

4. CONCLUSIONS

We introduce the enhanced CB method and its performance. Its original formulation is newly derived in the component matrix level, and then the more efficient formulation is proposed here. Due to the residual mode effect, the enhanced CB method leads to much better precise reduced model than the original CB method without increasing the number of substructural modes. In addition, the enhanced CB method shows good performance despite the small interface boundary DOFs.

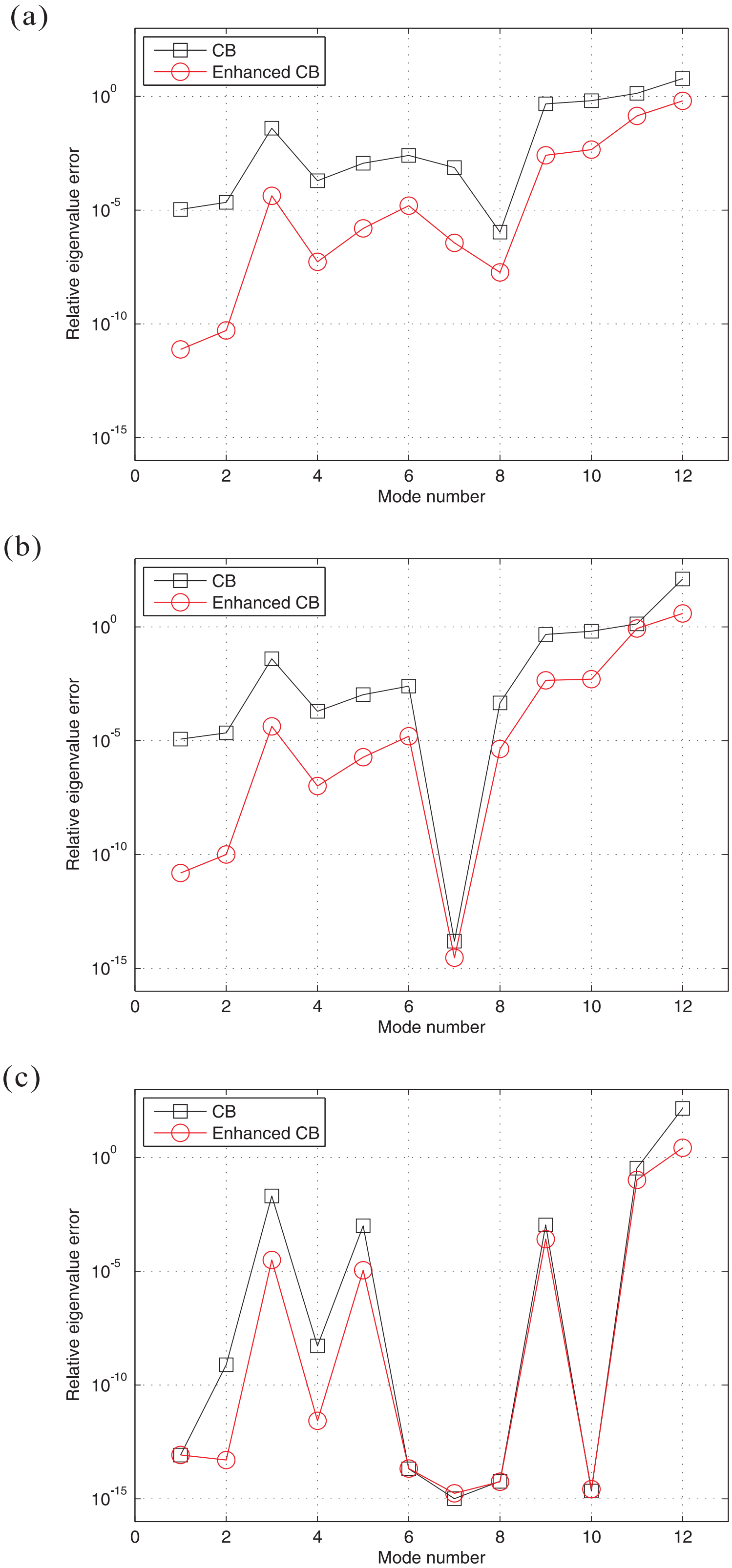

Fig. 2 Relative eigenvalue errors, 10 substructural modes selected. (a) Case 1, (b) Case 2 and (c) Case 3.

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