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Plastic behavior of circular discs with temperature-dependent properties containing a solid inclusion

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ABSTRACT

Plastic behaviors, based on the von Mises yield criterion, of circular discs containing a circular rigid inclusion are studied with the 3D finite element analysis. The material properties of the matrix are assumed to be temperature dependent. The inclusion is assumed temperature independent and not able to be yielded. It is found that pileup around the matrix-inclusion interface is significant due to the competition on the thermal expansion between the inclusion and matrix. Since yield stress is reduced for the temperature dependent case, the magnitude of its stress is significantly smaller than that of the temperature independent case. However, the plastic behaviors near the matrix-inclusion interface are quite different for the temperature dependent and temperature independent case. The elastic-irreversible temperature and plastic collapse temperature of the composite disc are numerically determined.

1. INTRODUCTION

The in-plane behaviors of discs under thermal loading have attracted much attention in the past years due to their industrial importance. In addition to vase analytical results on solid discs, annular discs under the plane strain or plane stress condition have also been widely studied (Lubliner 1990). However, analysis on the disc problems with the considerations of temperature dependent properties is limited, even though they are of technical importance. Although some analytical results exist in the literature for the annular disc with temperature dependent properties, analytical solutions of the composite disc with temperature-dependent properties for their Young's modulus, thermal expansion coefficient and yield stress are still lacking, except some papers on rotating discs for the rigid inclusion case (Guven 1997). In view of this, we numerically study the plastic behavior of the composite disc with the 3D finite element method to provide some numerical data for our future analytical solutions on such problem.

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2. MODELING AND RESULTS

The outer rim of the composite disc is fixed in the finite element model. The von Mises criterion requires stresses satisfy the following equation for the 2D problem with the tensile yield stress σ_Y.

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_\theta \sigma_r = \sigma_Y^2 \tag{1}$$

Our finite element calculations are fully three dimensional, and the above equation can be generalized to 3D straightforwardly. Following Aegeso and Eraslan (2008), the temperature functions, $f_E(T)$, $f_g(T)$, and $f_s(T)$, for a high-strength low-alloy steel the temperature difference is in the range $0 \leq T < 400^0 C$ can be expressed as follows.

$$f_E(T) = 1 + \frac{T}{2000\ln(T/1100)}, \quad f_s(T) = 1 + \frac{T}{600\ln(T/1630)},$$
$$f_g(T) = 1 + 2.56 \times 10^{-4} T + 2.14 \times 10^{-7} T^2 \tag{2}$$

where T is in ˚C and the initial temperature is 20˚C. Also, $E_0 = 200 GPa$, $\sigma_0 = 410 MPa$, $\nu = 0.3$, and $\gamma_0 = 11.7 \times 10^{-6}$ per ˚C.

Since it is impossible to set the inclusion to be rigid in the finite element calculations, it is needed to numerically test the effects of the setting on the inclusion's yield strength. Fig. 1 show the results on two different values for the inclusion's yield stress. They were set to be 1000 and 10000 GPa. Since the chosen yield strengths are large enough, the two cases show identical Mises stress distribution along the radius for a given temperature difference T. These results ensure that the inclusion is in the purely elastic state.

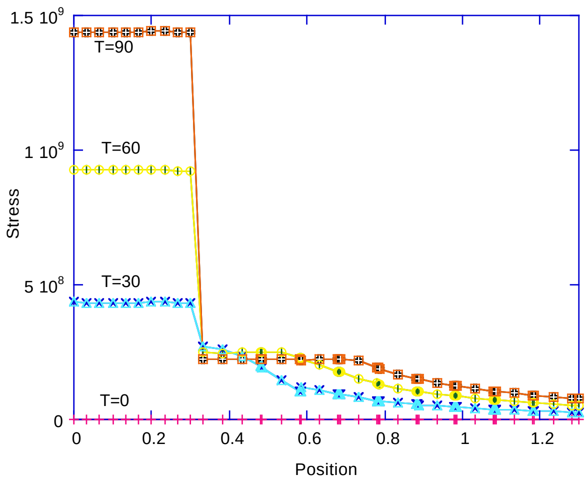

Fig. 1 Effects of inclusion modulus

With the chosen parameters used in the finite element calculations, Fig. 2 (a) shows the Mises stress distribution along the radius for the temperature-dependent and temperature-independent material properties under a given temperature difference loading. Since the yield stress in the temperature dependent case is largely reduced at high temperature, the Mises stress is expected to be smaller than that for the temperature independent case. In other words, analysis based on the temperature independent model always overestimates the capacity of the materials. Fig. 2 (b) shows a representative deformation shape of the composite disc. Color indicates the Mises stress. The stress in the inclusion is uniform, i.e. not a function of radius, as expected from the Eshelby 's uniformity theorem. Under the thermal loading, the disc expands along the Z direction, i.e. thickness direction, significantly.

Fig. 3 shows changes of the Mises stress in the composite disc under different temperature loading magnitudes, in terms of temperature differences. With the chosen set of parameters, the outer disc reaches plastic collapse at temperature difference equal to 60. When the normalized radius reaches 1, i.e. near the rim of the disc, our 3D finite element solutions show large reduction in the stress. The elastic-irreversible temperature difference is about 20. The yield stress of the inclusion was set to a large number so that the inclusion would not yield under the temperature loading. In addition, the stress distribution inside the inclusion, near the matrix-inclusion interface, shows a change a pattern around temperature difference being 40 and 50, when the matrix is partially yielded. The interface is assumed to be perfect during this study.

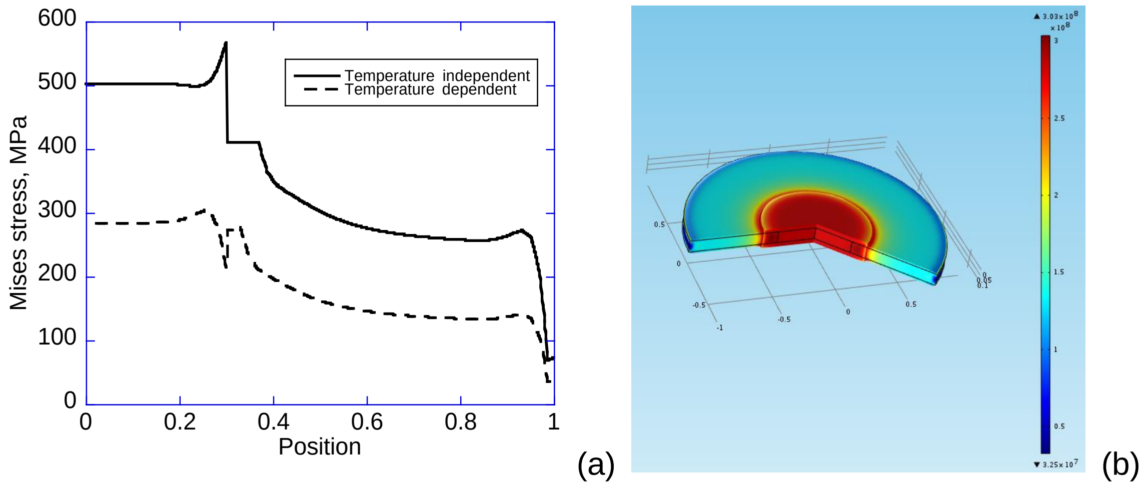

Fig. 2 (a) Mises stress vs. radial position at the middle plane of the thin disc, (b) Mises stress contour and deformation

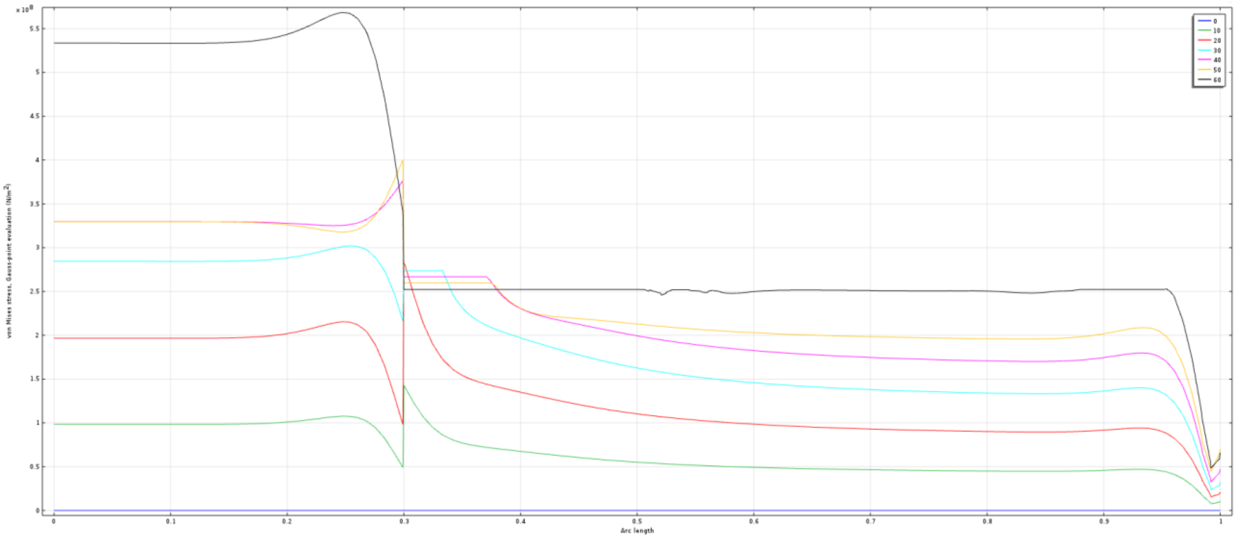

Fig. 3 Mises stress distribution along the radius under various temperature differences

3. CONCLUSIONS

The composite disc, with the temperature dependent properties for the matrix, is studied with the finite element method. Its elastic-irreversible temperature and plastic collapse temperature are numerically determined. The competitions between the inclusion and matrix depend on their mechanical and thermal properties. Our 3D finite element solutions can be used to compared with 2D analytical solutions, and may shed light on the possible difficulties in analyzing the problem within the 2D analytical framework, either under the plane stress or plane strain conditions, due to the missing information on the third dimension.

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