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# Analytical Model for Pressure Maintenance of Vacuum Tube Structures

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## ABSTRACT

A vacuum tube structure is an essential part of emerging innovative transportation systems such as the super-speed tube train or pneumatic freight conveying systems. One of the basic technical requirements of a vacuum tube structure for realization of these systems is maintaining the reduced pressure inside the tube for a certain period of time. In this study an analytical model for description of the pressure change inside the tube structures is presented. Two different cases are considered for developing the analytical model depending on what kind of fluid is intruding through the vacuum tube. One is an air-intrusion model, which reproduces the case where the tube structure is installed on the surface, and the other is a water-intrusion model for submerged vacuum tubes. Formulas for determining the flow rate of the fluid (either air or water) movement caused by the pressure difference inside and outside the tube structure are derived on the basis of Darcy's law. The derivation is performed for the cases with both compressible and incompressible fluid.

## 1. INTRODUCTION

A vacuum tube structure is an essential part of an emerging innovative transportation system called super-speed tube train (SSTT), where trains can operate with a practical speed as high as 700 km/h on maglev lines through vacuum tubes, either underground or elevated, or even under water (Cassat et al., 2003; Korea Railroad Research Institute, 2009).

From the viewpoint of the infrastructure, the basic technical requirements for the realization of an SSTT system would be (1) reducing the pressure inside the tube within a limited time and (2) maintaining this reduced pressure for a certain period of time. The pressure inside the tube is controlled by a series of vacuum pumps that are installed at regular intervals along the longitudinal direction of the tube. Once the

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internal pressure in the tube is reduced, its structure should be able to slow the infiltration of outside air as much as possible to minimize the pump operations needed to maintain the low pressure. If the tube allows excessive leakage or intrusion of air, the cost for maintaining the pressure becomes very high, as a higher pump capacity and a higher number of pumps would be necessary. The importance of the airtightness in the tube structures has also been empirically demonstrated by a practical application of an atmospheric railway, which has been partially installed in the nineteenth century (Buchanan 1992). It is therefore desirable to make the tube structure as airtight as possible, unless it significantly affects the overall construction cost.

The formula for expressing the fluid inflow or pressure rise inside a closed structure with a partial vacuum is derived on the basis of Darcy's law. The derivation is performed for the cases with both compressible and incompressible fluid.

## 2. DERIVATION OF ANALYTICAL MODELING FOR FLUID INTRUSION IN CLOSED STRUCTURES

The permeability of a porous material is its inherent ability to transmit fluids, which could be represented by Darcy's law (Mehta and Monteiro 2005) as follows.

$$\frac{dq}{dt} = \frac{kA}{\mu}\frac{\Delta P}{L} \tag{1}$$

where dq/dt is the flow rate ($m^3/s$); A, the cross-sectional area of the porous media ($m^2$); $\Delta P$, the pressure drop through the porous media (Pa); k, the intrinsic permeability ($m^2$), μ, the viscosity of the fluid (kg/m·s); and L, the length of the flow over the porous media (m). If k, A, L, μ, and $\Delta P$ are constant, then the flow rate, dq/dt, is also constant. However, for the intrusion of a gas into a closed hollow structure with a constant thickness and an internal pressure that is less than the outside pressure (Fig. 1), $\Delta P$ is not constant because the internal pressure increases as the fluid flows into the structure.

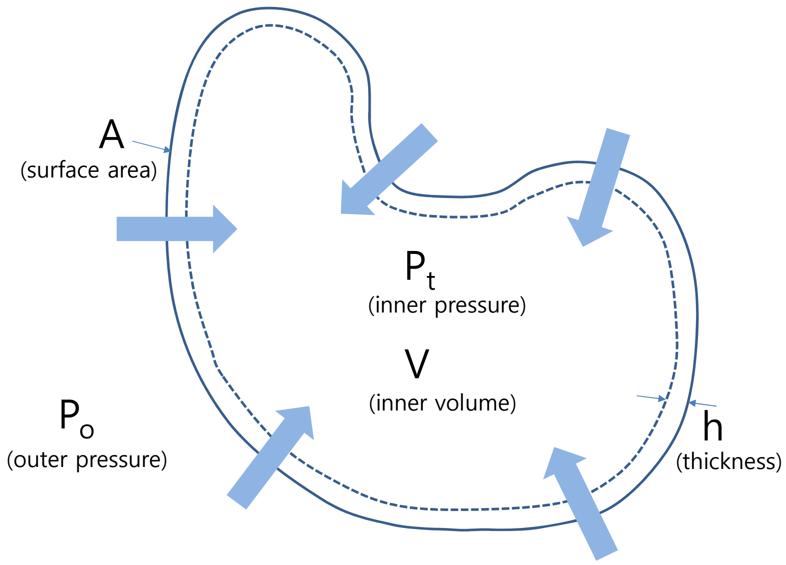

Fig. 1 Flow of fluid into a closed structure due to pressure drop

According to the conceptual design of the SSTT system (Cassat et al. 2003, Korea Railroad Research Institute 2009, Lee et al. 2008), the intended internal pressure of the tube is 0.1 atm, or 10 kPa. In this case, the air outside the tube structure at atmospheric pressure would flow into the tube as depicted in Fig. 2.

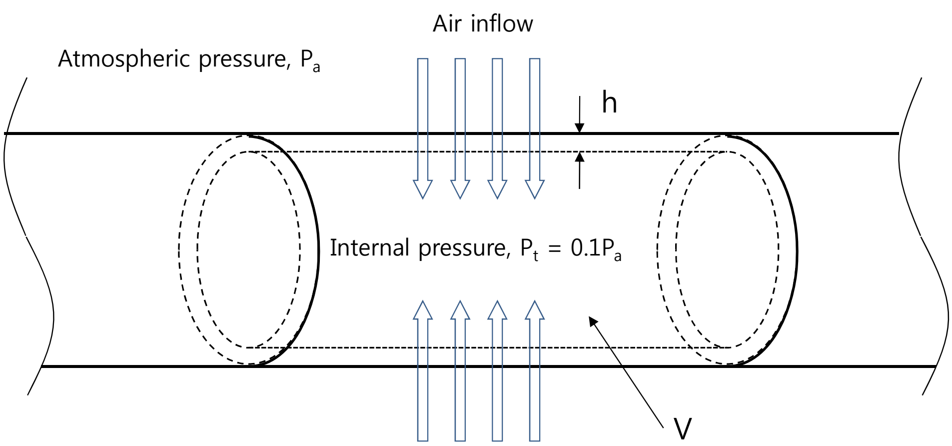

Fig. 2 Inflow of air into tube structure for SSTT

### *2.1 Inflow of compressible fluid*

For compressible fluids such as air, Eq. (1) should be altered to consider the pressure at the point where the flow is measured (Mehta and Monteiro 2005). The flow rate of the air measured at the outside surface of the tube structure is

$$\frac{dq}{dt} = \frac{kA(P_o^2 - P_t^2)}{2P_o\mu h} \qquad (2)$$

where h and A are the constant thickness and surface area per unit length of the tube, respectively; Po is the atmospheric pressure measured outside the tube; and Pt is the pressure inside the tube, which varies with time. If the temperatures inside and outside the tube are equal, the pressure rise inside the structure will be proportional to the air inflow, i.e., $P_o \frac{dq}{dt} = \frac{dP_t}{dt} V$, where V denotes the internal volume of the tube per unit length. Therefore, Eq. (2) becomes

$$\frac{dP_t}{dt} = \frac{kA(P_o^2 - P_t^2)}{2V\mu h} = \frac{kA{P_o}^2}{2V\mu h} - \frac{kA}{2V\mu h} \cdot {P_t}^2 \qquad (3)$$

On solving the differential equation (with a form of $y' = b - a \cdot y^2$), the pressure inside the tube can be expressed as

$$P_t = P_o \cdot \frac{1 + C_1 \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right)}{1 - C_1 \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right)} \qquad (4)$$

where C1 is a constant. Under the initial condition that the pressure inside the tube is 0.1 times the outside pressure, i.e., Pt at t = 0 is 0.1 Po, C1 is estimated to be -0.8182. The pressure change can then be expressed as

$$P_t = P_o \cdot \left[\frac{1 - 0.8182 \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right)}{1 + 0.8182 \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right)}\right] \qquad (5)$$

In case the flow rate of the compressible fluid is measured at the inside surface of the tube structure,

$$\frac{dq}{dt} = \frac{kA(P_o^2 - P_t^2)}{2P_t\mu h} \qquad (6)$$

From $P_o \frac{dq}{dt} = \frac{dP_t}{dt} V$, Eq(6) becomes

$$\frac{dP_t}{dt} = \frac{kA}{\mu h} \cdot \left[ \frac{P_o^2}{2\left(\frac{P_t}{V} P_o\right)} - \frac{\left(\frac{P_t}{V} P_o\right)}{2} \right] = \frac{kAP_o}{2\mu h} \left( \frac{V}{P_t} - \frac{P_t}{V} \right) = -\frac{kAP_o}{2\mu hV} \cdot P_t + \frac{kA{P_o}^3}{2\mu hV} \cdot \frac{1}{P_t} \qquad (7)$$

On solving the differential equation (with a form of $y' = ay + \frac{b}{y}$, the pressure inside the tube can be expressed as

$$P_t = \sqrt{C \cdot \exp\left(-\frac{kAP_o}{\mu hV} t\right) + {P_o}^2} \qquad (8)$$

where C is a constant. Under the initial condition that the pressure inside the tube is 0.1 times the outside pressure, i.e., $P_t$ at t = 0 is 0.1 $P_o$, C is estimated to be $-0.99{P_o}^2$. The pressure change can then be expressed as

$$P_t = \sqrt{-0.99{P_o}^2 \cdot \exp\left(\frac{kAP_o}{\mu hV} \cdot t\right) + {P_o}^2} = P_o \sqrt{1 - 0.99 \cdot \exp\left(\frac{kAP_o}{\mu hV} \cdot t\right)} \qquad (9)$$

### *2.2 Inflow of incompressible fluid*

For incompressible fluids such as water, the flow rate of the air measured at the outside surface of the tube structure is

$$Q = \frac{kA}{\mu} \frac{P_o - P_t}{h} \qquad (10)$$

From $P_o \frac{dq}{dt} = \frac{dP_t}{dt} V$, Eq(10) becomes

$$\frac{dP_t}{dt} = \frac{kA}{\mu}\frac{P_o - P_t}{h} = \frac{kA}{\mu h} \cdot P_o\left(1 - \frac{P_t}{V}\right) \tag{11}$$

On solving the differential equation (with a form of $y' = ay + \frac{b}{y}$, the pressure inside the tube can be expressed as

$$y = C \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right) + P_o \tag{12}$$

where C is a constant. Under the initial condition that the pressure inside the tube is 0.1 times the outside pressure, i.e., $P_t$ at t = 0 is 0.1 $P_o$, C is estimated to be $-0.9P_o$. The pressure change can then be expressed as

$$y = -0.9P_o \cdot \exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right) + P_o = P_o\left[1 - 0.9\exp\left(-\frac{kAP_o}{\mu hV} \cdot t\right)\right] \tag{13}$$

## 3. SUMMARY

Formulas for determining the flow rate of the fluid (either air or water) movement caused by the pressure difference inside and outside the tube structure are derived on the basis of Darcy's law. The derivation is performed for the cases with both compressible and incompressible fluid. If the intrinsic air permeability (k) of the material is known, the internal pressure change in a tube structure over time could be anticipated mathematically. It should be noted that Eq. (5), (9), and (13) only express the pressure change due only to the air intrusion through the surface of the tube. Decrease in the airtightness performance of a tube structures due to discontinuous regions such as construction joints should be considered separately from this study.


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