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Numerical solution of hypersonic flow along the stagnation streamline of blunt body

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ABSTRACT

The hypersonic flow near the stagnation streamline of a blunt body is analyzed with the quasi one-dimensional Navier-Stokes equations approximated by adopting the local similarity to the two-dimensional Navier-Stokes equations. The governing equations are solved with finite volume method. The computational domain is confined from the stagnation point to the shock wave, and shock fitting method is used to find the shock position. The shock wave angle in the vicinity of the stagnation streamline is used for the boundary conditions at the shock wave. Numerical computations are performed for the hypersonic flow in perfect gas about a cylinder. Along the stagnation streamline, the profiles of flow quantities computed by the quasi one-dimensional Navier-Stokes code are comparable with those by the two-dimensional Navier-Stokes code. For the shock standoff distance, the quasi one-dimensional code predicts shorter than the two-dimensional code.

1. INTRODUCTION

Flowfield analysis around the nose-tip and leading edge of the hypersonic vehicle has been much interested because the largest heat flux occurs at these regions. After the success of numerical analysis of hypersonic blunt body flows by Moretti (1966) computational algorithms have been progressed rapidly. However the fundamental frame of newly developed computational methods is not much different from that of Moretti. The computational domain should cover the subsonic region in the shock layer to make flow at the outflow boundary is supersonic (Fig. 1). Therefore we have to solve the two-dimensional domain though the heat flux at the stagnation point is needed only. On the other hand if it is possible to solve the flow equations at the stagnation streamline, the computational time could be reduced greatly. Many researchers have tried to solve the flow near the stagnation streamline of blunt bodies in hypersonic flow.

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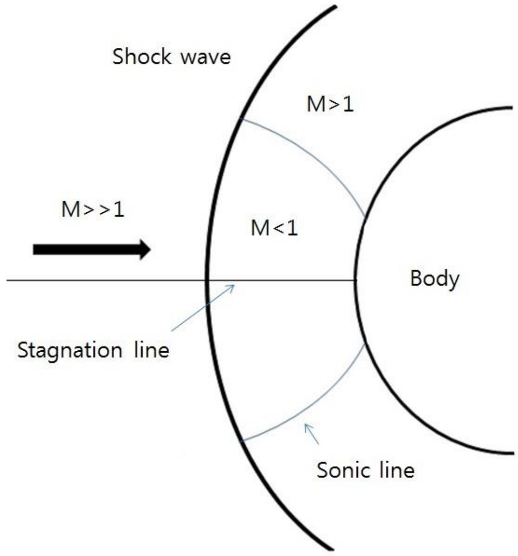

Fig. 1 Hypersonic flow abound blunt body

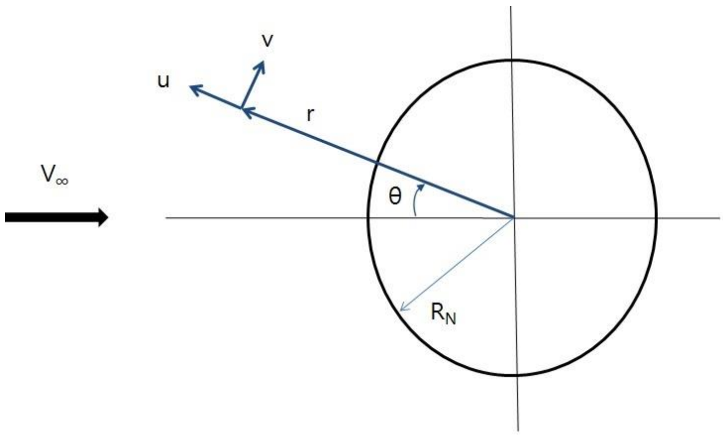

Fig. 2 Coordinate system

Kao (1964) considers the flow analysis near the stagnation streamline of a blunt body by applying the local similarity to the Navier-Stokes equations in the rarefied flow. He claims the assumption of local similarity is very accurate for predicting flow quantities near the stagnation streamline of a blunt body. Jain (1970) investigates the flow structure of the merged layer near the stagnation point of a blunt body for flights in the rarefied atmosphere by solving the Navier-Stokes equations adopting the concept of local similarity. Klomfass (1996,1997) conducts computations for the stagnation streamline of a sphere in hypersonic flows with dimensionally reduced Navier-Stokes equations approximated by the local similarity. Implicit finite volume method is applied to solve the equations, and results from the reduced equations are compared with those from the two-dimensional Navier-Stokes equations. In the result the shock standoff distance is always underestimated by the reduced Navier-Stokes equations. William (1997) conducts computations for the stagnation streamline of sphere with similar governing equations used by Klomfass (1997). He adopts shock capturing method applying upwind schemes for computational algorithm. As a result, the shock standoff distance predicted by the reduced governing equations shows similar behavior to Klomfass (1997).

In this study, we suggest a computational procedure to solve the flow quantities along the stagnation streamline of a blunt body in hypersonic flow. The dimensionally reduced quasi one-dimensional Navier-Stokes equations are solved by applying implicit finite volume method. The computational domain is covered from the stagnation point to the shock wave, and shock fitting method is used to find the shock position. The shock wave angle in the vicinity of the stagnation streamline is determined by using the shock wave shape correlation by Billig (1967) in the limit of small distance along the body. And the shock boundary conditions are given with this shock wave angle. Numerical computations are performed for the hypersonic flow in perfect gas about a cylinder. Results such as shock standoff distance and stagnation point heat flux produced by the present procedures are compared with those by two-dimensional Navier-Stokes code and other sources.

2. GOVERNING EQUATIONS

The cylindrical coordinate is considered and the free-stream is in the direction of $\theta = 0$ as illustrated in Fig. 2. The two-dimensional Navier-Stokes equations are simplified by reducing the number of independent spatial variables from two to one by applying approximations used by Kao (1964) in which the flow variables are expanded about the axis of symmetry with respect to $\sin\theta$ and only first truncation is retained. For the pressure p_2 is included in the first truncation.

$$u = \bar{u}(r)\cos\theta \quad v = \bar{v}(r)\sin\theta \quad p = \bar{p}_1(r) + \bar{p}_2(r)\sin^2\theta$$
$$\rho = \bar{\rho}(r) \quad T = \bar{T}(r) \quad \mu = \bar{\mu}(r) \tag{1}$$

The flow quantities are functions of the radial coordinate alone by letting $\theta \to 0$, then the equations for the stagnation streamline can be specified. Flow quantities are made dimensionless by dividing by the freestream conditions.

The conservative form of quasi one-dimensional equations for the analysis of stagnation streamline of cylindrical body is given by Klomfass (1996) as

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial r} - \frac{\partial F_v}{\partial r} + \frac{1}{r}(G - G_v) = 0 \tag{2}$$

The difference between the present equations and equations by Klomfass is the expression of the pressure term. In this study we follow the form by Kao (1964) as above while Klomfass approximates the pressure like the surface pressure in Newtonian theory. The components of conserved variables vector and inviscid and viscous flux vectors are given as follows:

$$
Q = \begin{bmatrix} \bar{\rho} \\ \bar{\rho}\bar{u} \\ \bar{\rho}\bar{v} \\ \bar{\rho}E \end{bmatrix} \quad F = \begin{bmatrix} \bar{\rho}\bar{u} \\ \bar{\rho}\bar{u}^2 + \bar{p}_1 \\ \bar{\rho}\bar{u}\bar{v} \\ \bar{\rho}\bar{u}H \end{bmatrix} \quad F_v = \begin{bmatrix} 0 \\ \tau_{rr} \\ \tau_{r\theta} \\ \bar{u}\tau_{rr} - q_r \end{bmatrix}
$$

$$
G = \begin{bmatrix} \bar{\rho}(\bar{u}+\bar{v}) \\ \bar{\rho}\bar{u}(\bar{u}+\bar{v}) \\ 2\bar{\rho}\bar{v}(\bar{u}+\bar{v}) + 2\bar{p}_2 \\ \bar{\rho}(\bar{u}+\bar{v})H \end{bmatrix} \quad G_v = \begin{bmatrix} 0 \\ \tau_{rr} + \tau_{r\theta} - \tau_{\theta\theta} \\ 2\tau_{r\theta} - \tau_{\theta\theta} \\ \bar{u}\tau_{rr} + \bar{u}\tau_{r\theta} + \bar{v}\tau_{\theta\theta} - q_r \end{bmatrix} \tag{3}
$$

Pressure $\bar{p}_1$ is determined by equation of state, and $\bar{p}_2$ is calculated from the second set equation of radial momentum equation.

$$
\bar{p}_1 = (\gamma - 1)\bar{\rho}\,\bar{e} \tag{4}
$$

$$
\frac{d}{dr}(\bar{p}_1 + \bar{p}_2) = \frac{\bar{\rho}\bar{v}}{r}(\bar{u} + \bar{v}) \tag{5}
$$

3. NUMERICAL PROCEDURES

3.1 Computational Algorithms

Eq. (2) is transformed into a generalized coordinate η, and the computational domain is discretized using a cell centered finite volume method with unit spacing $\Delta\eta = 1$.

$$
\left(\frac{1}{J}\frac{\partial Q}{\partial t}\right)_i + \left(\tilde{F} - \tilde{F}_v\right)_{i+\frac{1}{2}} - \left(\tilde{F} - \tilde{F}_v\right)_{i-\frac{1}{2}} + \left[\frac{1}{Jr}(G - G_v)\right]_i = 0 \tag{6}
$$

A tilde denotes numerically approximated flux at the cell interface. Eq. (6) provides a set of coupled ordinary differential equations with respect to time. Application of the implicit Euler backward scheme and the time linearization of the nonlinear terms result in

$$
\left[\frac{I}{J\Delta t} + \left(\frac{\partial R}{\partial Q}\right)^n\right]\Delta Q^n = -R^n(Q) \tag{7}
$$

The residual vector is given as

$$
R(Q) = \left(\tilde{F} - \tilde{F}_v\right)_{i+\frac{1}{2}} - \left(\tilde{F} - \tilde{F}_v\right)_{i-\frac{1}{2}} + \left[\frac{1}{Jr}(G - G_v\right]_i \tag{8}
$$

For the calculation of inviscid flux, $\tilde{F}$, low-diffusion flux-splitting scheme (Edwards 1997) is used. High order accuracy in space is achieved with the MUSCL approach; primitive variables are extrapolated to the cell interface with minmod limiter function. The viscous flux is discretized by central difference. For the calculation of Jacobian matrices, $\partial R/\partial Q$ in the implicit part (left-hand side), the firsr-order Steger-Warming's flux vector splitting scheme is used for inviscid flux, and the treatment by Tysinger and Caughy (1991) is used for viscous flux. The residual vector at point (i) in the implicit part is dependent on the states of two neighboring grid points. The resulting matrix equation at point (i) can be written as

$$C_1 \Delta Q_i^n + C_2 \Delta Q_{i-1}^n + C_3 \Delta Q_{i+1}^n = -R_i^n \tag{9}$$

Coefficient matrices composed of inviscid and viscous flux Jacobians are given as

$$\begin{aligned} C_1 &= A_i^+ - A_i^- + J_{i+\frac{1}{2}} A_{v\,2_i} + J_{i-\frac{1}{2}} A_{v2_i} + \left(\frac{B}{Jr}\right)_i \\ C_2 &= -A_{i-1}^+ - J_{i-\frac{1}{2}} A_{v2_{i-1}} \\ C_3 &= A_{i+1}^+ - J_{i+\frac{1}{2}} A_{v2_{i+1}} \end{aligned} \tag{10}$$

ΔQ^n can be obtained by solving a block tridiagonal matrix equation. Then the conserved variable vector at the (n+1)th time level is finally updated as

$$Q^{n+1} = Q^n + \Delta Q^n \tag{11}$$

3.2 Shock Wave Angle

The shock wave angle in the vicinity of stagnation streamline is obtained as follows. The correlation formula for a cylinder proposed by Billig (1967) is employed to depict the shock shape as follows:

$$x_s = 1 + \delta - \frac{R_s}{\tan^2\alpha} \sqrt{1 + \frac{y_s^2 \tan^2\alpha}{R_s^2}} + \frac{R_s}{\tan^2\alpha} \tag{12}$$

where x_s, y_s are coordinates at the shock position. δ is the shock standoff distance along the stagnation line, and R_s is radius of curvature of the shock,

$$\begin{aligned} \delta &= 0.386 \exp(4.67/M_\infty^2) \\ R_s &= 1.386 \exp[1.8/(M_\infty - 1)^{0.75}] \end{aligned} \tag{13}$$

For a small value of θ_1, the line along the radial direction with a slope of $a = \tan\theta_1$ is given as

$$y_s = a\, x_s \tag{14}$$

Then we can obtain the coordinate of intersection point between Eq. (12) and Eq. (14). With geometrical relation the shock wave angle at the intersection point is

$$\theta_s = \theta_b + \tan^{-1}\left(\frac{d\delta/d\theta}{1+\delta}\right) \tag{15}$$

Where the derivative of shock standoff distance with respect to θ is approximated as follows:

$$\frac{d\delta}{d\theta} \approx \frac{\Delta\delta}{\Delta\theta} = \frac{\delta_1 - \delta}{\theta_1} \tag{16}$$

3.3 Shock Fitting Method

The shock fitting method developed by Henrick (2006) is employed, which uses the momentum equation along the shock normal direction instead of a characteristic relation. Conservation of mass across the shock wave gives

$$\rho_s(u_s - V_s) = \rho_\infty(u_\infty - V_s) \tag{17}$$

The momentum normal to the shock wave is obtained by using the Rankine-Hugoniot relations as follows:

$$\rho_s u_s = \frac{\rho_\infty(V_s - u_\infty)[\rho_\infty u_\infty \gamma(V_s - u_\infty) - 2\gamma p_\infty + \rho_\infty(2V_s^2 - 3V_s u_\infty + u_\infty^2)]}{2\gamma p_\infty + \gamma\rho_\infty(V_s - u_\infty)^2 - \rho_\infty(V_s - u_\infty)^2} \tag{18}$$

The shock acceleration is calculated as

$$\frac{dV_s}{dt} = \left[\frac{d(\rho_s u_s)}{dV_s}\right]^{-1} \frac{d(\rho_s u_s)}{dt} \tag{19}$$

The derivative of the momentum at the shock with respect to the shock velocity is obtained from Eq. (18) and the term $d(\rho_s u_s)/dt$ is computed from the momentum equation as

$$\frac{d(\rho_s u_s)}{dt} = -\left[\frac{d(\rho u^2 + p)}{dr}\right]_s - \frac{2}{r}\rho_s u_s(u_s + v_s) \tag{20}$$

4. RESULTS AND DISCUSSION

A hypersonic flow in calorically perfect gas over a two-dimensional cylinder is chosen to validate the applicability of quasi one-dimension Navier-Stokes equations. The specific flow conditions are: $R_N = 0.00615\text{m}$, $M_\infty = 5.73$, $Re_\infty = 2050$, $T_\infty = 40\text{K}$, $T_w = 210\text{K}$. Viscosity is calculated from Sutherland's law. These conditions correspond to the experimental conditions by Tewfik (1960). The Prandtl number is chosen to be 0.77 as Kopriva (1993). The number of grid points from the stagnation point to the shock is 100. Grid points are clustered to the wall to resolve large gradient. The freestream conditions are used as an initial data, and the fixed time step with the CFL number of 20 is used throughout the calculations. In Fig. 3 convergence histories of shock velocity and shock stand-off distance are shown. We can see the good convergence and robustness of computational procedure. Fig. 4 shows profiles of flow quantities along the stagnation streamline comparing results by the quasi one-dimensional Navier-Stokes code with those by two-dimensional Navier-Stokes code (Lee 2002). The results by two codes show good agreements. For the shock standoff distance the quasi one-dimensional code predicts shorter than two-dimensional code.

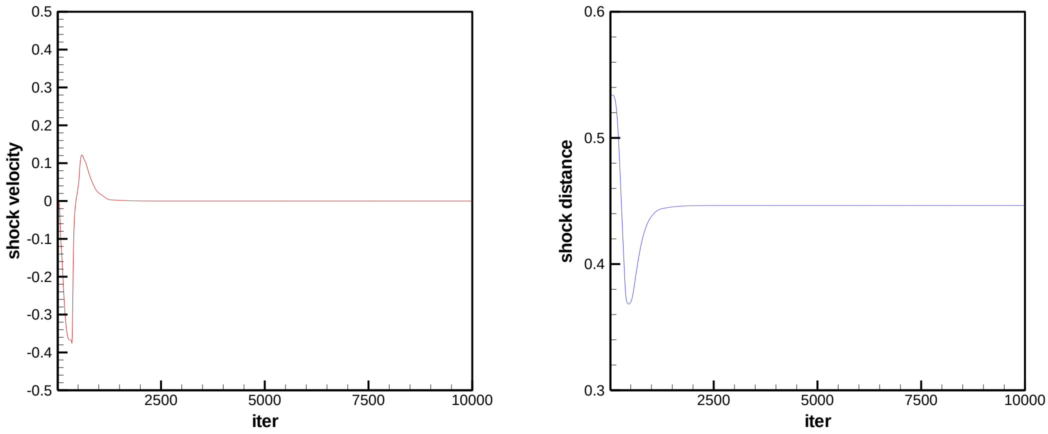

Fig. 3 Convergence histories of shock velocity and shock distance

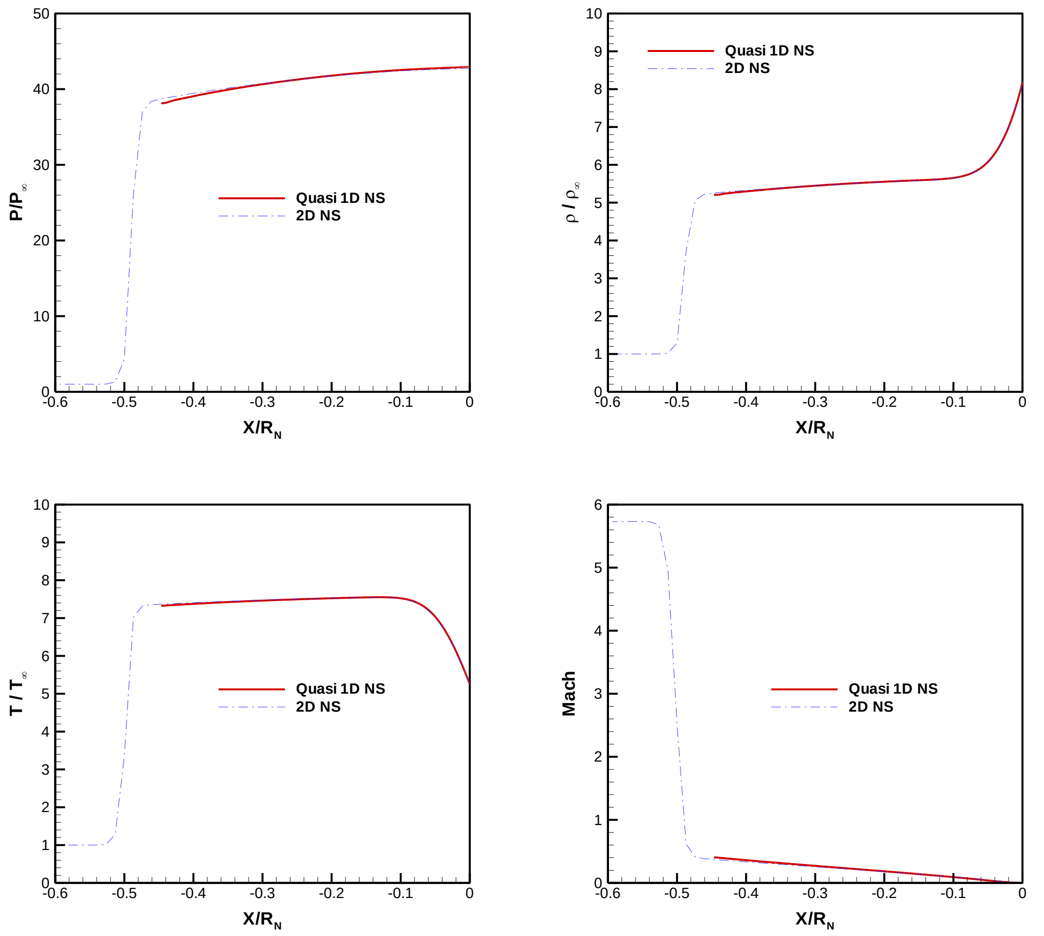

Fig. 4 Profiles of pressure, density, temperature, and Mach number along the stagnation streamline

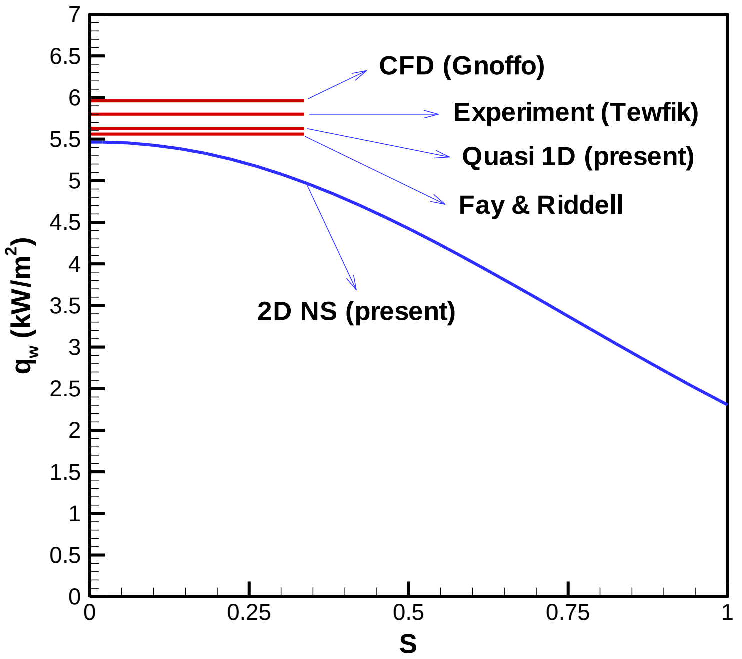

Fig. 5 Comparison of heat flux at the stagnation point

If the shock shape from spectral solution by Kopriva (1993) is determined as the reference, the quasi one-dimensional Navier-Stokes code predicts about 8% shorter. Fig. 5 shows the comparison of heat fluxes at the stagnation point with data from various sources. For the experimental data by Tewfik and Giedt (1960) as a reference value, our codes and Fay & Riddell predict slightly lower values. While result by Gnoffo (1980) shows slightly higher value.

5. CONCLUSIONS

Numerical computations are conducted for the hypersonic flow in perfect gas about a cylinder by using the quasi one-dimensional Navier-Stokes equations code. The proposed shock boundary condition improves the prediction for shock standoff distance and stagnation point heat flux in comparison with previously presented results. The shock standoff distance predicted by the quasi one-dimensional code is about 8% shorter than that predicted by spectral method. The present numerical procedure is very efficient in the aspect of computing time and can be extended to thermochemical nonequilibrium flow.

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