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PI Servo with State-D Feedback for DC Motor Control with Consideration of Thermal Effect

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ABSTRACT

This paper presents PI-servo with state-D feedback for DC motor control. The effect of winding temperature is included in the DC motor model. This temperature causes slow change in dynamic responses of the DC motor. The response of the proposed control and the classical PI control are compared and investigated. With the proposed control method, the effect of winding temperature to the DC motor control can be reduced. The control result shows that the DC motor with the proposed control give satisfactory response and provided good stability.

1. INTRODUCTION

DC motors can be used extensively in industries such as automatic control system, robotic, electrical vehicle, etc. Control of a DC motor can be used either PI controllers or other. But the increase in motor's resistance with winding temperature variation causes the motor's thermal time constant to be increased while its electrical time constant is reduced. In case of permanent magnet motors there are the reversible thermal demagnetization that reduces the motor constant presented by Richard H. Welch Jr. and George W. Younkin (2002). This thermal effect can degrade controller performance.

PI servo with state-D feedback for LTI systems has recently been reported by Sarawut, S., Witchupong, W. and Surachai, W. (2012). It is noticed that most of these results are aimed for disturbance rejection by state-D and command following control. An advantage over the conventional state feedback is that it results in smaller derivative gains.

This paper proposes PI-servo system with state-D feedback control DC motor with consideration of thermal effect by using the pole-placement approach. Section 2 of the paper explains the DC Motor with thermal effect. Our design gain PI and state D controller for DC motor are presented in Section 3. Section 4 provides results for DC motor control, while conclusions follow in Section 5.

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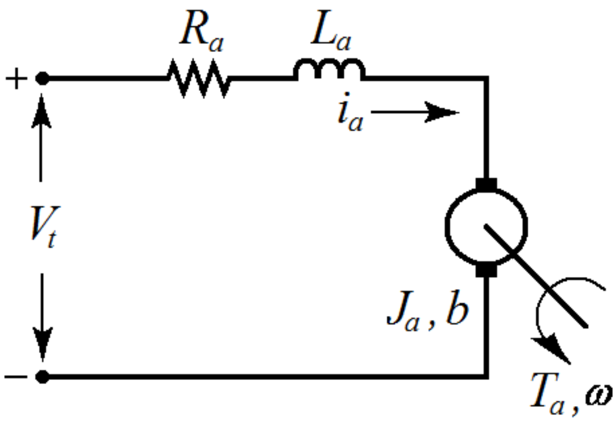


Fig. 1 Diagram representing a DC motor model

2. DC MOTOR MODEL WITH THERMAL EFFECT

DC motor model can be represents by the diagram in Fig. 1. The dynamic system can be described by a state Eq. (1).

$$\begin{bmatrix} \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} -\frac{b}{J_a} & \frac{K_t(T_0)}{J_a} \\ -\frac{K_e(T_0)}{L_a} & -\frac{R_a(T_0)}{L_a} \end{bmatrix} \begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L_a} \\ \frac{1}{J_a} & 0 \end{bmatrix} \begin{bmatrix} T_a \\ V_t \end{bmatrix} \tag{1}$$

Copper winding's resistance at a temperature change, is given by Eq. (2) and can be found in Richard H. W. Jr. and George W. Y.(2002).

$$R_a(T) = R_a(T_0)\left[1 + 0.00393(T - T_0)\right] \tag{2}$$

In permanent magnet motors $K_e(T)$ and $K_t(T)$ depend on each specific magnet material and temperature. It described by Eq. (3) and Eq. (4) respectively as given in Richard H.W. Jr. and George W.Y. (2002).

$$K_e(T) = K_e(T_0)\left[1 - B(T - T_0)\right] \tag{3}$$

$$K_t(T) = K_t(T_0)\left[1 - B(T - T_0)\right] \tag{4}$$

where

$R_a(T_0)$ = armature resistance (Ω) at specified temperature (°C)
L_a = armature inductance (H)

b = coefficient of viscous friction (Kg-m^2/s)
J_a = armature inertia (Kg-m^2)
$K_e(T_0)$ = motor voltage constant (V-s/rad) at specified temperature ($^\circ C$)
$K_t(T_0)$ = motor torque constant (Nm/A) at specified temperature ($^\circ C$)
ω = angular speed of the motor shaft (rad/s)
i_a = armature current (A).
V_t = DC voltage fed to armature circuit (V).
T_a = load torque (Nm).
T_0 = specified temperature ($^\circ C$)
T = winding's temperature ($^\circ C$)
T_{amb} = ambient temperature ($^\circ C$)
B = coefficient for each magnet (Alnico = 0.0001, SmCo = 0.00035, NdFeB = 0.001, Ferrite = 0.002)(/$^\circ C$)
R_{th} = thermal resistance ($^\circ C$/W)
τ = thermal time constant (s)

The temperature increase in DC motor can be described by Eq. (5). Eq. (6) illustrates the ODE of changed temperature as given in Nisit, K. De. and Prasanta K. Sen (2006). Power dissipation can be expressed by Eq. (5).

$$P_{Diss} = i_a^2 R_a(T) \tag{5}$$

$$T(t) = \left(\left(P_{Diss} \times R_{th}\right) + T_{Amb}\right) + \left(T_0 - \left(\left(P_{Diss} \times R_{th}\right) + T_{Amb}\right)\right)e^{-\frac{t}{\tau}} \tag{6}$$

$$\frac{dT}{dt} = -\frac{T}{\tau} + \frac{\left(\left(P_{Diss} \times R_{th}\right) + T_{Amb}\right)}{\tau} \tag{7}$$

The following parameters of the motor are used: $R_a = 0.87\ \Omega$, $\mathrm{L}_a = 0.00062\ \mathrm{H}$, $b = 0\ \text{kg-m}^2/\mathrm{s}$, $\mathrm{J}_a = 7.06667\times10^{-6}\ \text{kg-m}^2$, $\mathrm{K}_e = 0.0161\ \text{V-s/rad}$, $\mathrm{K}_t = 0.0161\ \text{Nm/A}$, $R_{th} = 6.2$ $^\circ C$ /W, $\tau = 1200$ s, $B(\text{Alnico}) = 0.0001$ / $^\circ C$, $T_a = 0.0565$ Nm, $V_t = 15$ V, $T_0 = 25$ $^\circ C$, $T_{amb} = 30\ ^\circ C$. Substitute these parameters into Eq. (1). Eq. (8) can be formed.

$$\begin{bmatrix} \dot{\omega} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} 0 & 2,278.3 \\ -25.968 & -1,404.2 \end{bmatrix} \begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 & 1,612.9 \\ 1.4151\times10^5 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ V_t \end{bmatrix} \tag{8}$$

Based on Eq. (7) temperature rise in the DC motor is shown in Fig. 2. Temperature started rising from 30 $^\circ C$ to be saturate at 166.3°C within 2 hours. Armature resistance R_a, K_e and K_t are changed accordingly by Eq. (2), Eqs. (3) and (4). Fig. 3 illustrates the armature resistance changed by the temperature. It started from 0.87Ω to 1.355Ω at the same 2 hour period. Fig. 4 exhibits the motor voltage and torque constant K_e and K_t. They are slightly reduced from 0.0161 to 0.0159.

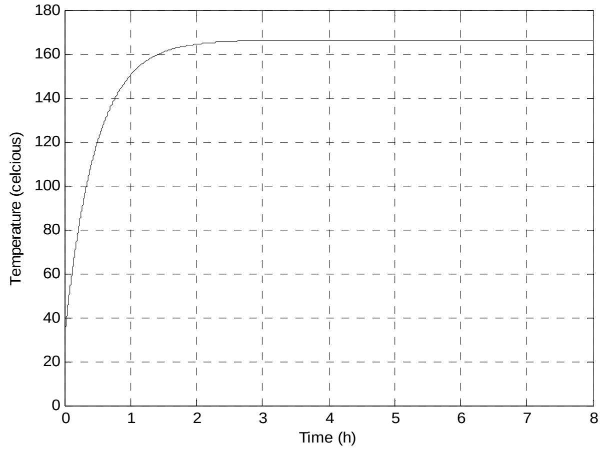


Fig. 2 Temperature rise in the DC motor

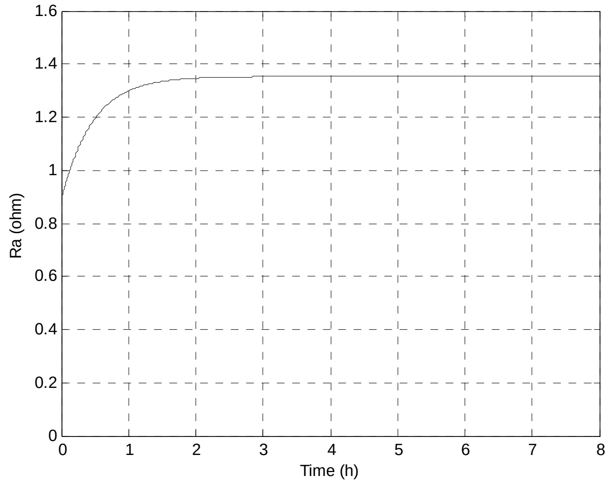


Fig. 3 R_a with the thermal effect

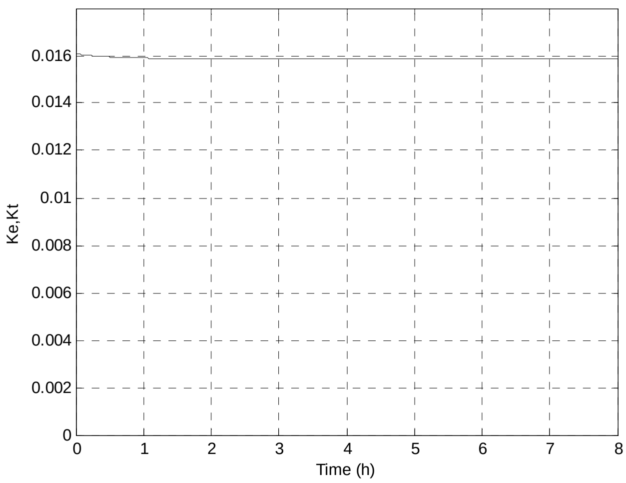
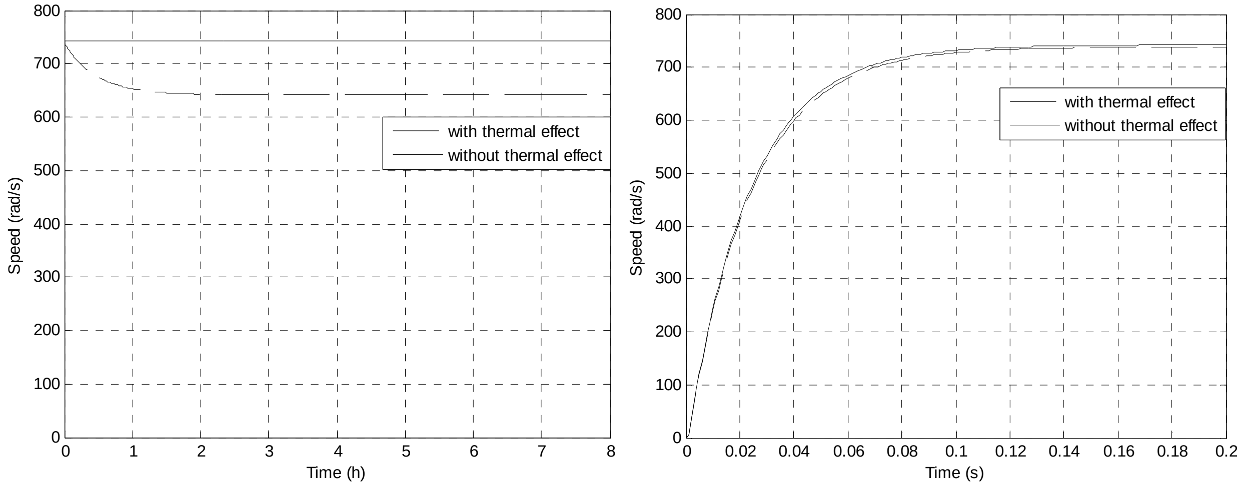


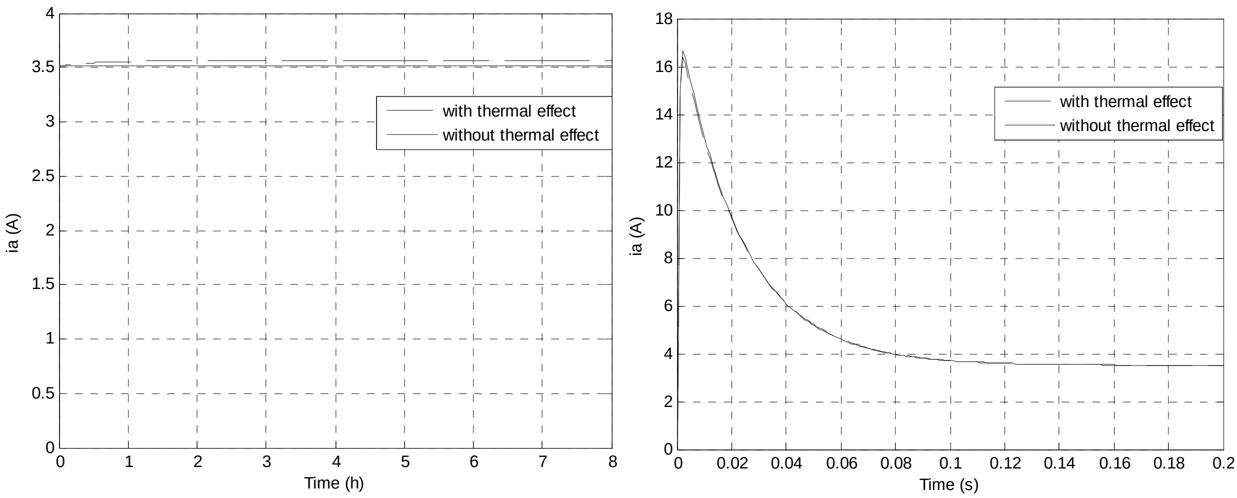
Fig. 4 K_e and K_t with the thermal effect

Fig. 5 described the speed of the DC motor without a controller following with a step input $V_t = 15$ V, and the load torque of 0.0565 Nm. The response curves in Fig. 5(a) indicate that the response with the thermal effect caused the speed reduction down to 641.6 rad/s. In terms of the rise-time, the response obtained with the thermal effect was slower. Fig. 6 illustrates the current of the DC motor. The response curves in Fig. 6(a) indicate that the current response i_a with the thermal effect was increased from 3.51 A to 3.56 A. In terms of the rise-time, both responses with and without the thermal effect are similar.



(a) (b)

Fig. 5 Speed of the DC motor without controller



(a) (b)

Fig. 6 Current of the DC motor without controller

3. PI SERVO WITH STATE D FEEDBACK CONTROL

A DC motor system having single input and single output can be described by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{9}$$

$$y = \mathbf{C}\mathbf{x} \tag{10}$$

The system states are fed back through the gain matrix $\mathbf{K_d}$, and the error signal, the difference between the reference input (*r*) and the output (*y*), is fed forward to the proportional-integral (PI) controller. This error signal is denoted as $\dot{\xi}$. The block diagram can be shown in Fig. 7 to represent the control system. Therefore, Eq. (11) and Eq. (12) express the control signal and the error signal, respectively.

$$u = -\mathbf{K_d}\dot{\mathbf{x}} + k_p\dot{\xi} + k_i\xi \tag{10}$$

$$\dot{\xi} = r - y = r - \mathbf{C}\mathbf{x} \tag{11}$$

where ξ is the output of the integrator, k_p and k_i are controller parameters. The design problem is to find the gains k_p, k_i and the matrix $\mathbf{K_d}$.

Recall the DC motor state matrix as in Eq. (8). Let states $\mathbf{x} = \begin{bmatrix} \omega & i_a \end{bmatrix}^T$. The general form of the state equation as described in Eq. (9) can be written in detail as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2,278.3 \\ -25.968 & -1,404.2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1,612.9 \end{bmatrix}u \tag{13}$$

Since, the speed of the motor is defined as the single output, Eq. (14) give the output equation.

$$\mathbf{y} = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x} \tag{14}$$

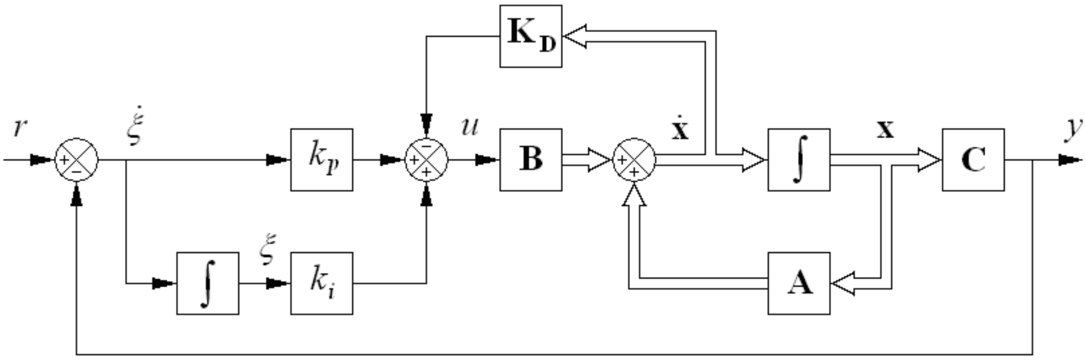


Fig. 7 Block diagram representing the PI-servo and state-D feedback control

Employing the Frobenius canonical form designated by

$$\mathbf{F} = \mathbf{Tx}\,,\, \mathbf{x} = \mathbf{T}^{-1}\mathbf{F}$$

$\mathbf{A_F}$ and $\mathbf{B_F}$ can be simply obtained from Eq. (15).

$$\mathbf{A_F} = \mathbf{TAT}^{-1}\,,\ \mathbf{B_F} = \mathbf{TB} \tag{15}$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{q} & \mathbf{qA} & \cdots & \mathbf{qA}^2 \end{bmatrix}^T \tag{16}$$

$$\mathbf{q} = \mathbf{h}^T\mathbf{M}^{-1} \tag{17}$$

$\mathbf{q}$ is a $(1\times n)$ vector. $\mathbf{M}$ is the controllability matrix expressed by

$$\mathbf{M} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} \tag{18}$$

One can obtain the Frobenius form of the system, Eq. (13) as $\dot{\mathbf{F}} = \mathbf{A_F}\mathbf{F} + \mathbf{B_F}u$. Transformation of the system results in the Frobenius form of

$$\dot{\mathbf{F}} = \begin{bmatrix} 0 & 1 \\ -59{,}162 & -1{,}403.2 \end{bmatrix}\mathbf{F} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \tag{19}$$

The control signal for the state-D feedback can be written as

$$u = -\mathbf{K_F}\dot{\mathbf{F}} \tag{20}$$

In which $\mathbf{K_F} = \begin{bmatrix} k_{f1} & k_{f2} \end{bmatrix}$ and $\mathbf{K_d} = \mathbf{K_F}\mathbf{T}$. Therefore, $\dot{\mathbf{F}} = \mathbf{A_F}\mathbf{F} - \mathbf{B_F}\mathbf{K_F}\dot{\mathbf{F}}$ represents the system with the inner feedback loop. It is characterized by $\Delta_d(s) = \det\left[s\left(\mathbf{I} + \mathbf{B_F}\mathbf{K_F}\right) - \mathbf{A_F}\right]$, which is desired to be $\Delta_i(s) = \alpha_n s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0$. Assign the desired poles located at $-1653, -147.01$. The desired characteristic polynomial can be expressed as

$$\Delta_i(s) = 0.24347s^2 + 438.24s + 59{,}162 \tag{21}$$

For the inner loop control system, the characteristic polynomial is given by

$$\Delta_d(s) = a_2 s^2 + a_1 s + a_0\,,\quad \Delta_d(s) = \left(1 + k_{f2}\right)s^2 + \left(1{,}403.2 + k_{f1}\right)s + 59{,}162\,. \tag{22}$$

For comparison, $\Delta_d(s) = \Delta_i(s)$, where $\alpha_0 = a_0$. Calculate the gain matrix $\mathbf{K_d}$ by

$$\mathbf{K_d} = \begin{bmatrix} \alpha_1 - a_1 & \alpha_2 - 1 \end{bmatrix}\mathbf{T}$$

$$\mathbf{K_d} = \begin{bmatrix} -993.96 & -0.75653 \end{bmatrix} \mathbf{T}$$

$$\mathbf{K_d} = \begin{bmatrix} -0.0002626 & -0.00046905 \end{bmatrix}$$

We can write the equation describing the error dynamic as

$$\dot{\mathbf{e}} = \mathbf{A_1 e} + \mathbf{B_1} u_e \tag{23}$$

where $\mathbf{A_1} = \begin{bmatrix} \hat{\mathbf{A}} & 0 \\ -\mathbf{C} & 0 \end{bmatrix}$, $\mathbf{B_1} = \begin{bmatrix} \hat{\mathbf{B}} \\ 0 \end{bmatrix}$, $\hat{\mathbf{A}} = \left(\mathbf{I} + \mathbf{BK_d}\right)^{-1} \mathbf{A}$, $\hat{\mathbf{B}} = \left(\mathbf{I} + \mathbf{BK_d}\right)^{-1} \mathbf{B}$ and the control signal

$$u_e = \mathbf{K_{pi} e} \tag{24}$$

where $\mathbf{K_{pi}} = \begin{bmatrix} -k_p \mathbf{C} & k_i \end{bmatrix}$. The error dynamic represented by Eq. (23) is characterized by the characteristic polynomial $\Delta_o(s) = \beta_m s^m + \beta_{m-1} s^{m-1} + \cdots + \beta_1 s + \beta_0$, in which $\beta_m = 1$. We can write the error dynamic by

$$\dot{\mathbf{e}} = \begin{bmatrix} 0 & 2,278.3 & 0 \\ -106.66 & -1,800 & 0 \\ -1 & 0 & 0 \end{bmatrix} \mathbf{e} + \begin{bmatrix} 0 \\ 6,624.7 \\ 0 \end{bmatrix} u_e . \tag{25}$$

Assign the desired poles at $-102.16 \pm j129.99$, $-1,595.7$ for the PI-servo control part, and $\Delta_o(s) = s^3 + 1800s^2 + 3.5336 \times 10^5 s + 4.3617 \times 10^7$ is the characteristic polynomial. Calculate the gain matrix $\mathbf{K_{pi}}$ by

$$\mathbf{K_{pi}} = -\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{B_1} & \mathbf{A_1 B_1} & \mathbf{A_1^2 B_1} \end{bmatrix}^{-1} \phi(\mathbf{A_1})$$

For the error dynamic, $\Delta_o(s) = \beta_m s^m + \beta_{m-1} s^{m-1} + \cdots + \beta_1 s + \beta_0$ represents the desired characteristic polynomial. From the Cayley-Hamilton theorem, we can write

$$\phi(\mathbf{A}_1) = \mathbf{A}_1^3 + \beta_1 \mathbf{A}_1^2 + \beta_2 \mathbf{A}_1 + \beta_3 \mathbf{I} = 0 \tag{26}$$

Resulting in $k_p = -0.0073119$ and $k_i = 02.8899$.

4. SIMULATION RESULTS

To be compared with the PI controller designed by Ziegler-Nichol's method to achieve the same pole locations as given above, one can obtain the $k_p = 0.0618$ and $k_i = 9.27$. The step reference input at 742 rad/s, and the torque disturbance of 0.0565

Nm are applied $t = 0$ s. Fig. 8 illustrates the current of the DC motor. The response curves in Fig. 8(a) indicates that the response of the PI controller and the PI servo with state-D feedback gave the same result. The response i_a of the thermal effect case is increased from 3.51 A to 3.56 A similarly to that of the uncontrolled case. In terms of the rise-time, the response of the PI controller has overshoot larger than that of the PI servo with state-D feedback.

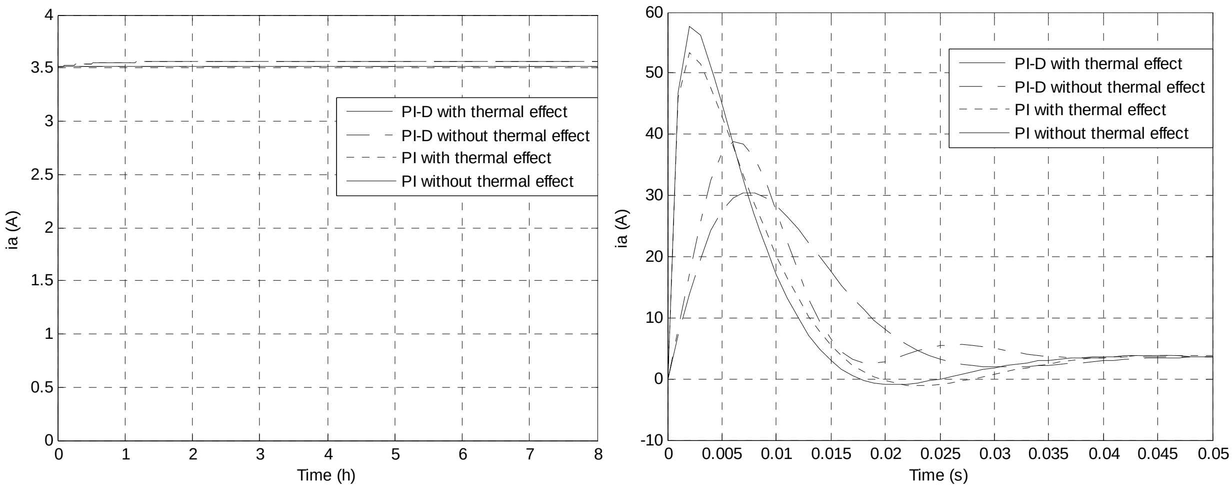


Fig. 8 Current response of the DC motor with controllers

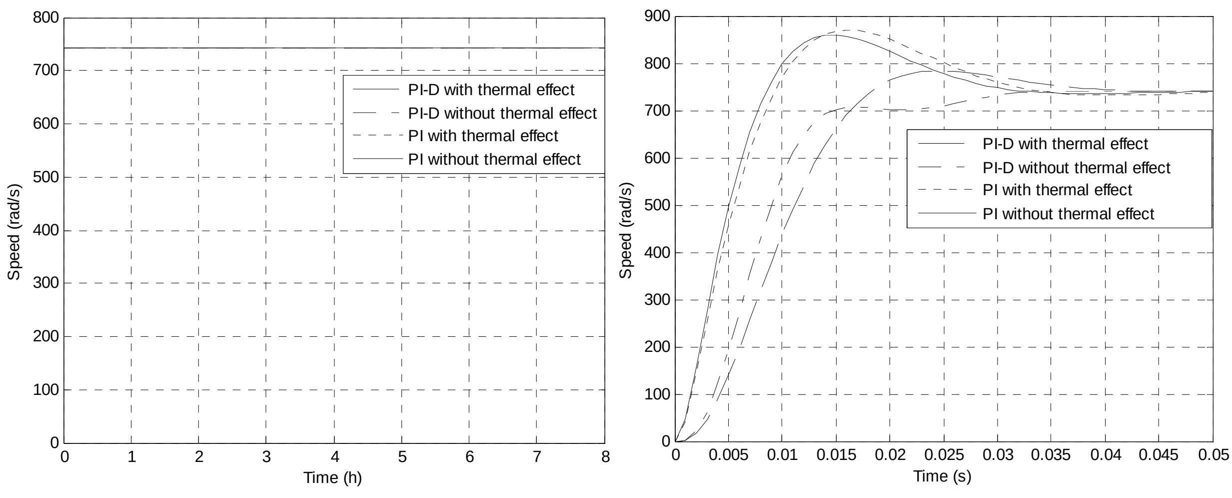


Fig. 9 Speed of the DC motor with controllers

Fig. 9 illustrates the speed of the DC motor. The response curves in Fig. 9(a) indicate that the response with the thermal effect, without the thermal effect, the PI controller and the PI servo with state-D feedback give the same result. In terms of the rise-time, the speed response with the PI controller experiences overshoot larger than that of the PI servo with state-D feedback. The overshoot of the responses is also similar. On the other hand, the rise-time of the thermal effect case is longer than that without the thermal effect. For PI servo with state-D feedback, the rise-time without the thermal effect is faster while the overshoot of the response with the thermal effect can be observed.

5. CONCLUSIONS

An integrator in the PI controller exhibits no steady-state errors in the response to the step input. The state-D feedback has an advantage over the conventional state feedback. It can reduce the zero effect. This paper has presented a control design method via the pole-placement approach for the DC motor speed control system with consideration of the thermal effect. The PI-servo with state-D feedback control and the PI control to achieve control objectives are similar. Furthermore, the controller design by using the proposed procedures is very simple and effective.

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