

The FDTD method for lightning surge propagation in 115-kV power transmission systems of PEA's Thailand

* Kokiatt Aodsup¹⁾ and Thanatchai Kulworawanichpong²⁾

^{1), 2)} *Power System Research Unit, School of Electrical Engineering Institute of Engineering, Suranaree University of Technology, Nakhon Ratchasima, THAILAND*

¹⁾ kokiatt_a@hotmail.co.th and ²⁾ Thanatchai@gmail.com

ABSTRACT

This paper describes a simulation of lightning surge propagation in power transmission lines of provincial electricity authority (PEA) Thailand by using finite-difference time-domain (FDTD) method. Numerical computation of solving the Telegraphist's equations is determined and investigated. A source of lightning surge wave on the line is modeled by using Heidler's surge model. The proposed method was tested against 115-kV power transmission systems in comparison with the solution obtained by using Bewley lattice diagram. As a result, the calculation showed that the FDTD method is one of accurate methods to analyze transient lightning surge wave in power transmission lines.

1. INTRODUCTION

Lightning and switching surge are any disturbance on a transmission of steady-state condition. These phenomena can produce high voltage level in a very short time that can damage insulation or can cause server flashover described in (Bewley 1951), (Hileman 1990).

Lightning or surge protection of electrical and electronic systems from disturbances has been increasingly important. Because the induced lightning surges can cause significant damages to electric power components, telecommunication equipment, computer networks, etc. These result in severe damages of equipment, interruption of services, increased operation and maintenance cost. For insulation design of the power transmission system, it is vital to exhibit the induced voltage behaviors propagated along the transmission lines. Consequently, both theoretical and experimental studies of lightning induced electromagnetic fields have been conducted continuously (Kokiatt 2012).

In theory, characteristics of lightning surge propagation in transmission lines can be described mathematically in forms of partial differential equations (PDEs) as the well-known Telegraphist's equations (Benesova 2006). These equations are linear second-order partial differential equations with constant coefficients. These equations fall into three basic categories: parabolic, elliptic and hyperbolic. These equations are

¹⁾ Graduate Student

²⁾ Professor

hyperbolic. In case of lossless lines where series resistance of lines and shunt conductance representing insulation losses are neglected, the system equations can be simplified into the wave equations which the lightning surge can propagate along the line without any line attenuation. Although the wave equations as hyperbolic PDEs have an exact equations in some circumstances, further investigation such as appearance of lightning surge arresters somewhere in transmission lines can raise complexity and nonlinearity in the governing system equations. Solutions of these equations were obtained several decades ago by Heaviside in England and Poincare in France (Bewley 1951). The FDTD method is basically a numerical tool and can be adapted in associating with surge arrester models in the future work.

2. POWER TRANSMISSION LINE

2.1 Mathematical model of a power transmission line

The study of transmission line surges regardless of their origin is very complex. Although the long-line model is recommended for lines more than approximately 150 mi long (Granger 1994), the lightning surge wave propagation is a very short-time impulse wave-shape therefore the long-line model is a good representation of power lines for a high-frequency impulse of lightning surges.

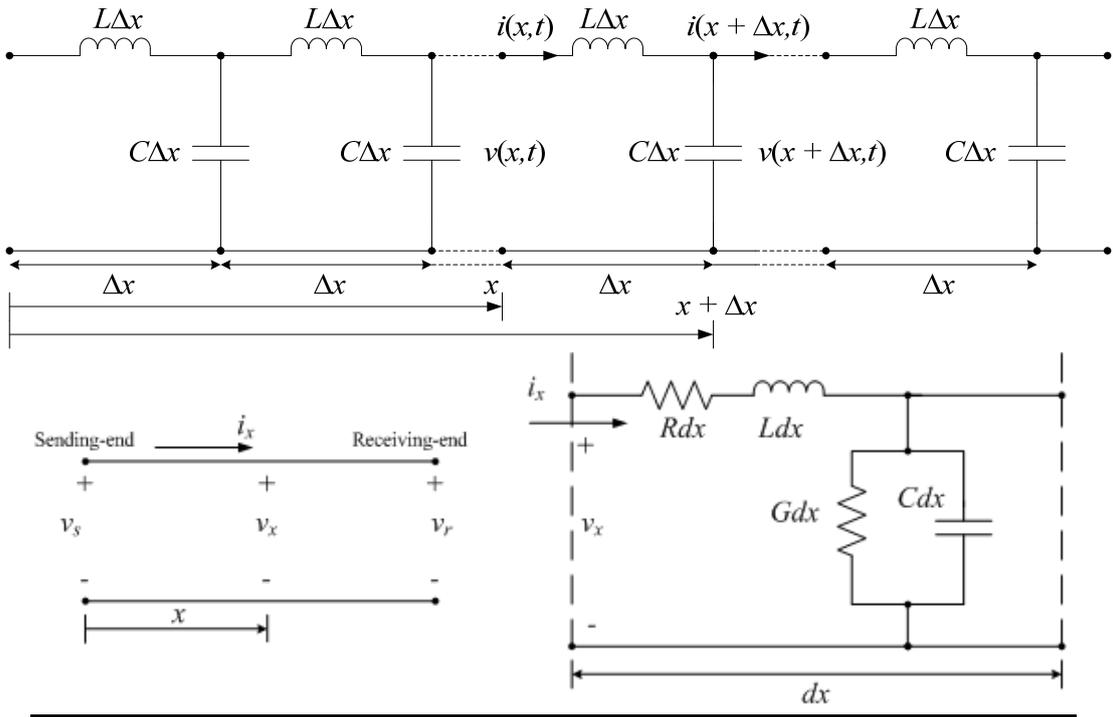


Fig. 1 Distributed line model for power transmission line wave propagation

Fig. 1 shows the frame and the equivalent circuit of a very small section of a single-phase power transmission line. Assuming that the line conductors are parallel to the ground and uniformly distributed, the time-domain characteristics in form of partial differential equations of the single-conductor line can be expressed as follows.

$$\frac{d}{dx} v(x, t) = -Ri(x, t) - L \frac{d}{dt} i(x, t) \quad (1)$$

$$\frac{d}{dx} i(x, t) = -Gv(x, t) - C \frac{d}{dt} v(x, t) \quad (2)$$

Where $i(x, t)$ is a current surge wave function
 $v(x, t)$ is a voltage surge wave function
R, L, C and G are per-unit length line parameters

Consider the distance x along the transmission line from the sending end (rather than the receiving end) to the very small different element of length dx shown in the above figure. The voltage $v(x, t)$ and current $i(x, t)$ are both a function of space and time so that they are in form of partial derivatives. Since it assumes that the transmission line is a lossless line, R and G will be equal to zero to give the following expressions.

$$\frac{d}{dx} v(x, t) = -L \frac{d}{dt} i(x, t) \quad (3)$$

$$\frac{d}{dx} i(x, t) = -C \frac{d}{dt} v(x, t) \quad (4)$$

Now either current $i(x, t)$ can be eliminated by taking the partial derivatives of both terms in Eq.(3) with respect to x and in Eq. (4) with respect to t , or voltage $v(x, t)$ can be eliminated by taking the partial derivatives of both terms in Eq. (3) with respect to t and in Eq. (4) with respect to x . This will produce a linear second-order partial differential equation in form of hyperbolic PDEs as shown in Eq. (5) for the voltage wave equation or called “travelling wave equation”.

$$\frac{1}{LC} \frac{d^2}{dx^2} v(x, t) = \frac{d^2}{dt^2} v(x, t) \quad (5)$$

2.2 Heidler's Surge Function

The model based on the travelling-wave source was introduced by Heidler (Bewley 1951). in which the surge wave propagates at infinitely large speed while the return-stroke speed (front speed) is still finite. The equation for surge function introduced by Heidler satisfies the two basic requirements needed for the lightning surge simulation, i.e. the current does not have discontinuity at $t = 0$ s and the current derivative also does not have a discontinuity at $t = 0$ s provided that $k > 1$. At present time, Heidler representation of the lightning surge wave is one of the most widely-used surge model for the lightning surge propagation in transmission line. The Heidler's surge function can be described by the following equation.

$$f(t) = \left(\frac{F_0}{\eta} \right) \left(1 - e^{-t/\tau_1} \right)^k e^{-t/\tau_2} \quad (6)$$

3. FINITE-DIFFERENCE TIME-DOMAIN (FDTD) METHOD

The standard example of a hyperbolic PDE is the one-dimensional wave equation as described follows.

$$c^2 \frac{d^2}{dx^2} v(x, t) = \frac{d^2}{dt^2} v(x, t) \quad (10)$$

Initial conditions are given for $v(x,0)$ and also its derivative. The boundary conditions are given at $x = 0$ and $x = L$ where L is the maximum limit of x . According to the explicit method of solving the wave equation, replacing the space derivative in the wave equation by finite difference formula at the λ^{th} time step, Eq. (11) is obtained. In the same manner, replacing the time derivative by the finite difference formula at the λ^{th} space step, Eq. (12) is formed. By substituting Eq. (11) and Eq. (12) into Eq. (10), it gives the updated voltage wave solution as summarized in Eq. (13).

$$\frac{d^2}{dx^2} v(x, t) = \frac{v(\lambda + 1, \tau) - 2v(\lambda, \tau) + v(\lambda - 1, \tau)}{\Delta x} \quad (11)$$

$$\frac{d^2}{dt^2} v(x, t) = \frac{v(\lambda, \tau + 1) - 2v(\lambda, \tau) + v(\lambda, \tau - 1)}{\Delta t} \quad (12)$$

$$v(\lambda, \tau + 1) = \phi^2 v(\lambda - 1, \tau) + 2(1 - \phi^2)v(\lambda, \tau) + \phi^2 v(\lambda, \tau) - v(\lambda, \tau - 1) \quad (13)$$

Where $\phi = c \frac{\Delta t}{\Delta x} = \frac{\Delta t}{\Delta x \sqrt{LC}}$ is the aspect ratio

4. REFLECTION OF TRAVELING WAVES

When a travelling wave on a transmission line reaches a transition point at which there is an abrupt change of line parameters a part of the wave is reflected back on the incoming line and the rest may pass through other line section. The travelling wave before reaching the transition point is called the incident wave. The incident wave may be decomposed into two component waves called the reflected wave and the transmitted wave. This relation is a voltage-wave solution of Eq. (5) and it can be expressed as in Eq. (7). The transmitted wave, $v''(x,t)$, is a wave portion travelling toward the next line section while the reflected wave, $v'(x,t)$, is a wave portion travelling backward to the source. These waves can be illustrated by the equivalent circuit shown in Fig. 2.

$$v(x, t) = v''(x, t) + v'(x, t) \quad (7)$$

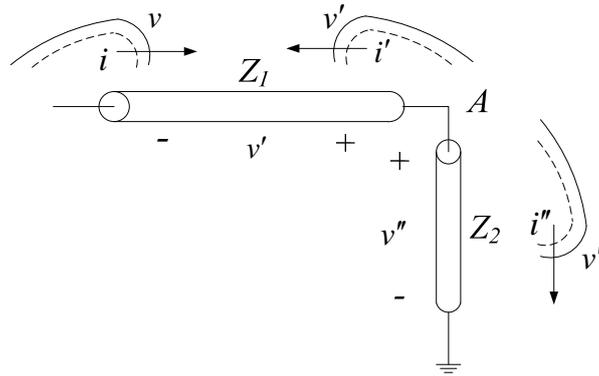


Fig. 2 Waves reflected and transmitted at the junction

If the line section # 1 has the surge impedance of Z_1 and the line section # 2 has the surge impedance of Z_2 , the transmitted and the reflected portions of the travelling wave can be represented in terms of the refraction coefficient (β) and the reflection coefficient (α), respectively, as given in Eq. (8) and Eq. (9).

$$\beta = \frac{2Z_2}{Z_1 + Z_2} \quad (8)$$

$$\alpha = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad (9)$$

Where $Z_1 = \sqrt{\frac{L_1}{C_1}}$ and $Z_2 = \sqrt{\frac{L_2}{C_2}}$
 $u_1 = \frac{1}{\sqrt{L_1 C_1}}$ is the wave speed of line 1
 $u_2 = \frac{1}{\sqrt{L_2 C_2}}$ is the wave speed of line 2

L_1 is per-unit inductance of line section 1
 L_2 is per-unit inductance of line section 2
 C_1 is per-unit capacitance of line section 1
 C_2 is per-unit capacitance of line section 2

In practical power network, many line sections are typical. This leads to multiple reflections among line junctions to exhibit complicated resulting waves. However, in a lateral line case, both the reflection and the refraction occur from the left to the right or from the right to the left, coefficients of reflection and refraction can be pre-calculated and then used repeatedly when any incident wave has arrived. The component waves calculated at any time and any position by using this pre-calculation of all coefficients at every junction can be drawn as the so-called “Bewley lattice diagram” (Bewley 1951) as illustrated in Fig. 3.

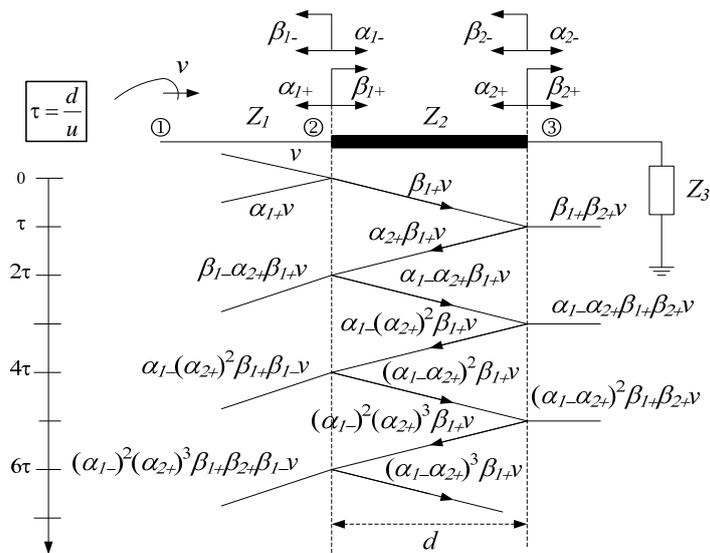


Fig. 3 Example of Bewley lattice diagram

4. SIMULATION RESULT AND DISCUSSION

The study of successive reflection of travelling waves caused by either direct or indirect lightning stroke can be investigated through a test in power transmission lines of provincial electricity authority (PEA) Thailand as shown in Fig. 4. This example consists of two transmission line sections with the open and short circuit line termination. The line parameters of each section are given as follows.

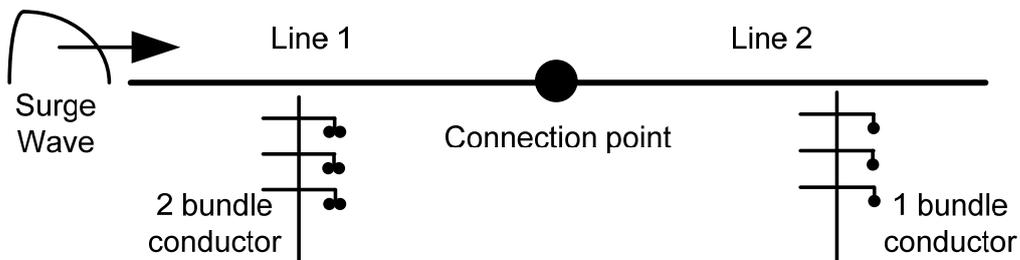


Fig. 4 The transmission line systems

The parameter of transmission line systems is :

Line 1 Transmission line from substations of Uthaitani to a connection point.

Line length: 2 km, $L_1 = 0.0012$ H/m, $C_1 = 0.0566$ F/m

Line 2 Transmissions line from a connection point to substations of Chainat.

Line length: 2 km, $L_2 = 0.00085$ μ H/m, $C_2 = 0.0412$ F/m

The Heidler's surge model of the lightning induced voltage can be characterized by the waveform in Fig. 5. The Heidler's surge wave has the 10-kV peak and 12/30- μ s of the rise and decay time constants.

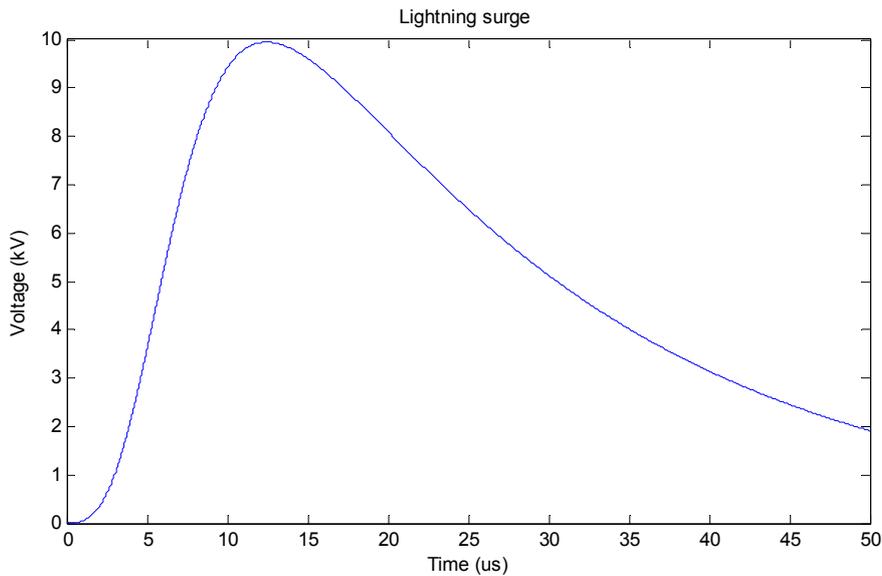


Fig. 5 Heilder’s surge wave for test the transmission line systems

With the help from MATLAB programming, lightning surge waves propagation on transmission lines can be simulated numerically. In this paper, this simulation used the time step of 0.1 μs and the step length of 5 m.

Assume that the lightning surge was induced at the sending end of the transmission line. The incident wave can travel along the line section 1 with a constant speed and without attenuation approach line junction A as shown in Fig. 6. After the incident wave hitting the junction, the incident wave of 10-kV peak was separated into the reflected wave of -1.527-kV peak and the transmitted wave of 8.442-kV peak. These two wave components can be depicted as shown in Fig. 7

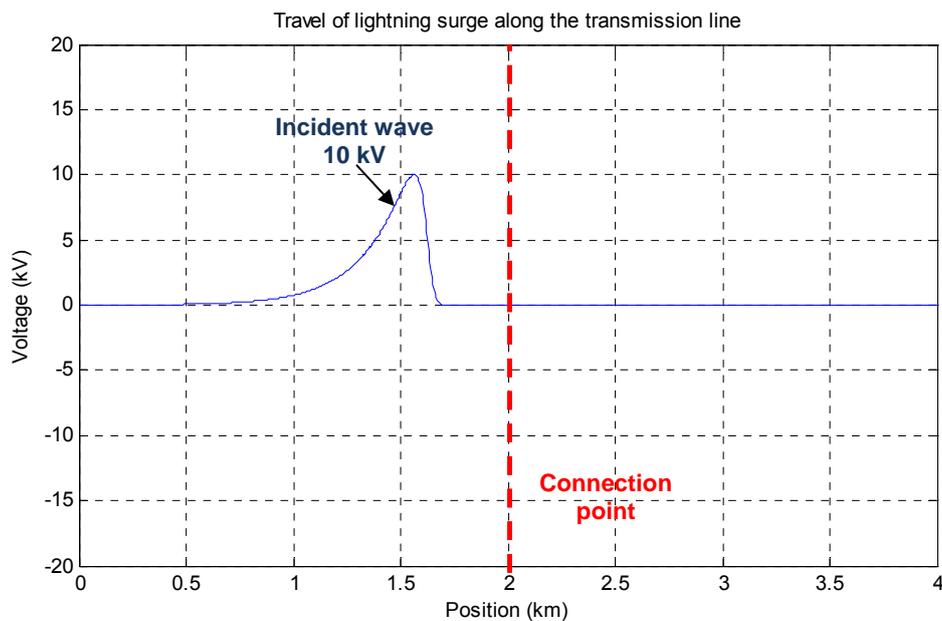


Fig. 6 Incident wave before arriving connection point.

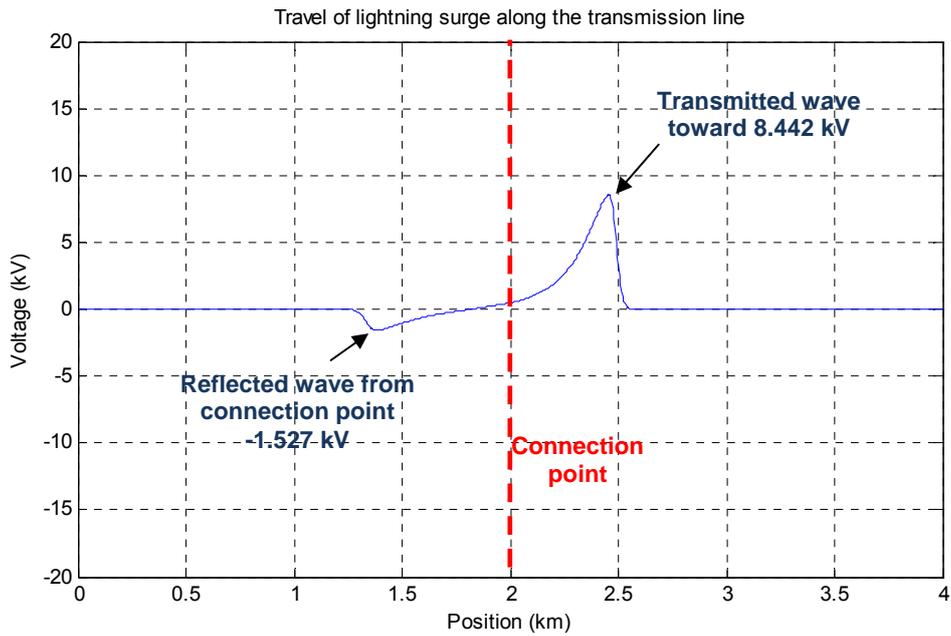


Fig. 7 Transmitted and reflected waves at connection point

When transmitted wave on a transmission line reaches a line terminal at which there is open and short circuit. Transmitted wave components can be depicted as shown in Fig. 8 and Fig.9 for a line terminal is open and short circuit respectively

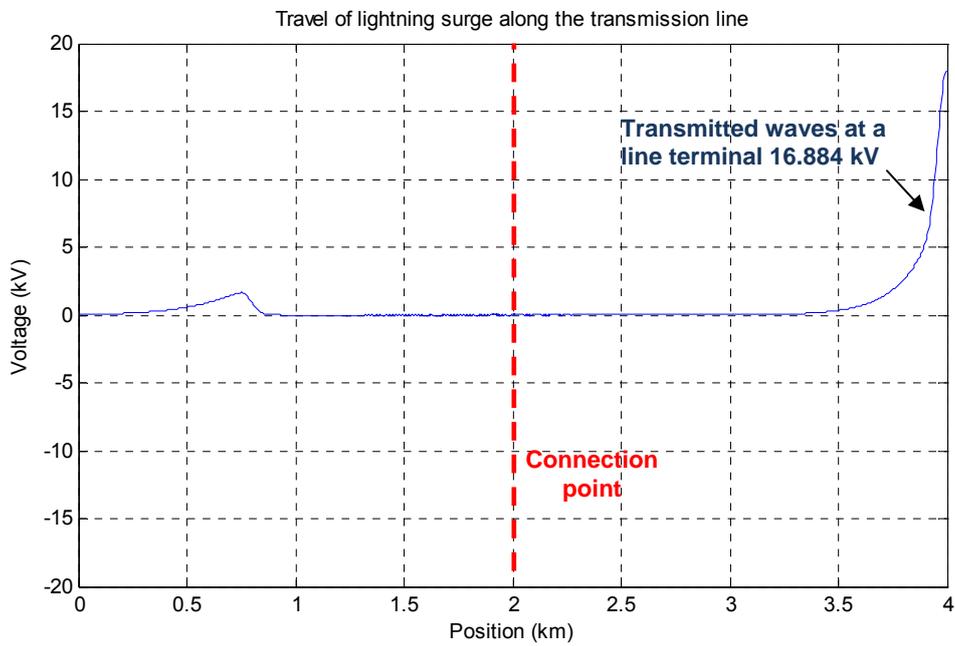


Fig. 8 Transmitted waves at a line terminal is open circuit

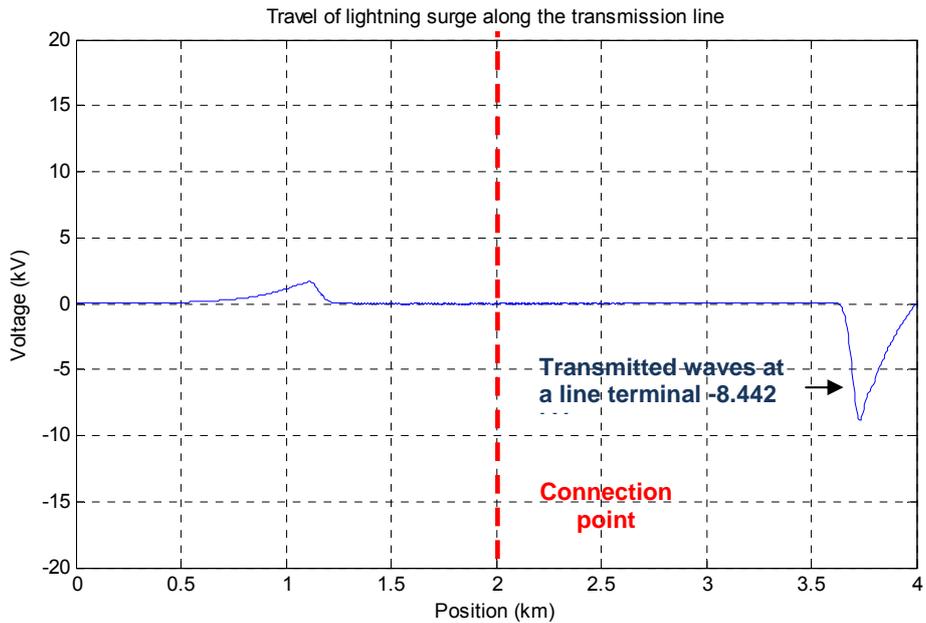


Fig. 9 Transmitted waves at a line terminal is short circuit

In addition, the full simulation of the whole system which consists of two line sections having a total of 4-km line length and the total time span of 2500 μs can be plotted in 3D surface for open and short circuit at terminal line as shown in Fig. 10 and Fig. 11 respectively.

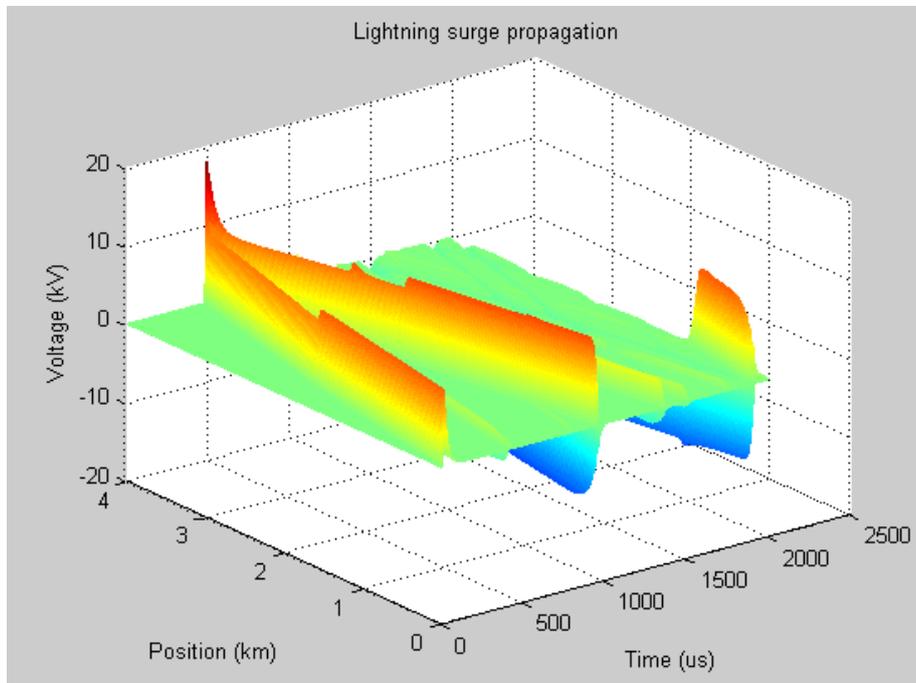


Fig. 10 Lightning surge propagation along the transmission lines of the test systems at open circuit line terminal after 2.5 ms

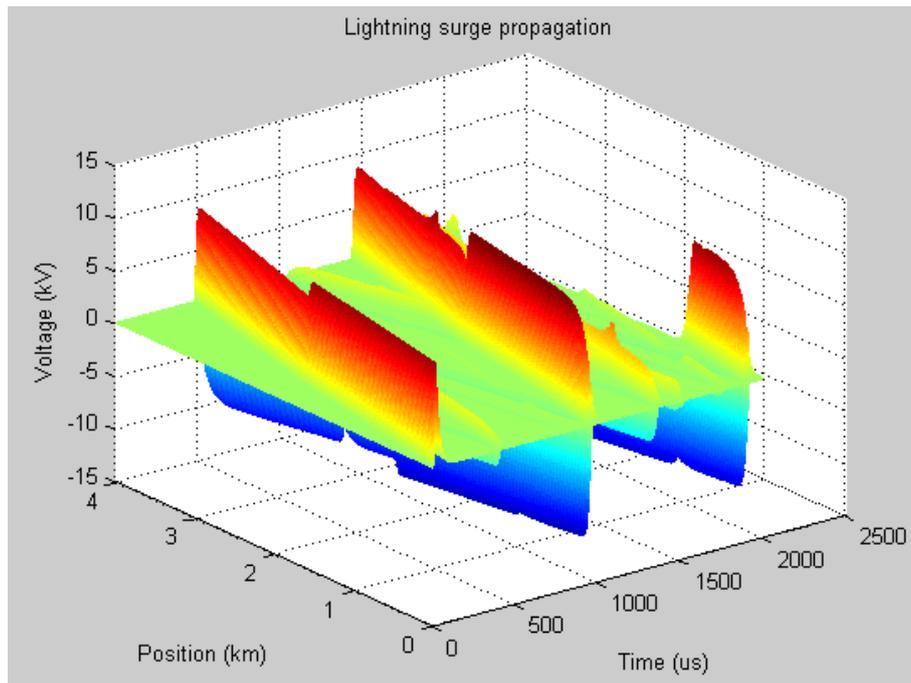


Fig. 11 Lightning surge propagation along the transmission lines of the test systems at short circuit line terminal after 2.5 ms

The FDTD method is comparison with the solution obtained by using Bewley lattice diagram as shown in Table 1.

Table 1 The FDTD method comparison with the solution by using Bewley lattice diagram.

	Reflection	Refraction
Bewley lattice diagram	-1.142 kV	8.858 kV
FDTD method	-1.527 kV	8.442 kV

6. CONCLUSIONS

In this paper, the finite-difference time-domain (FDTD) method to analyze lightning surge propagation in power transmission lines of provincial electricity authority (PEA) Thailand. Numerical computation of solving the Telegraphist's equations is determined and investigated. A source of lightning surge wave on the line is modeled by using Heidler's surge model. The proposed method was tested against 115-kV power transmission systems in comparison with the solution obtained by using Bewley lattice diagram. As a result, the calculation showed that the effectiveness and the accuracy of the solutions obtained by the FDTD method are confirmed

REFERENCES

- Bewley Cadappa, L. V. (1951), "Travelling Waves on Transmission Systems", Dover Publication.
- Hileman, A. R. (1999), "Insulation Coordination for Power Systems", Marcel Dekker.
- Granger, J. J. and Stephenson, W. D. (1994), "Power System Analysis", McGraw-Hill.
- Benesova, Z. and Kotlan, V. (2006), "Propagation of surge waves on non-homogeneous transmission lines induced by lightning stroke", Advances in Electrical and Electronic Engineering, Vol. 5, no. 1 – 2, 198 – 203.
- Fausett, L. V. (1999), "Applied Numerical Analysis using MATLAB". Prentice-Hall.
- Aodsup, K. and Kulworawanichpong, T. (2012), "Simulation of Lightning Surge Propagation in Transmission Lines Using the FDTD Method", World Academy of Science, Engineering and Technology, Issue 71, 427 – 432.