

Large-Eddy Simulation of Flows over Idealized Urban Roughness in Unstable Thermal Stratification

Ming-Chung Chan¹⁾ and Chun-Ho Liu²⁾

^{1), 2)} *Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong, China*

²⁾ *Tel: +852 2859-7901, Fax: +852 28585415, E-mail: liuchunho@graduate.hku.hk*

ABSTRACT

Large-eddy simulation (LES) equipped with the one-equation subgrid-scale (SGS) model is employed to investigate the mean flows and turbulence over idealized two-dimensional (2D) street canyons in various unstable thermal stratifications. The building-height-to-street-width (aspect) ratio is equal to unity so the flows inside a street canyon are in the skimming flow regime. Periodic boundaries are assigned to the domain inlet and outlet, and the spanwise extent, simulating the infinite horizontally homogeneous urban roughness. The buoyancy force is modeled by Boussinesq approximation.

In isothermal conditions, the flow exhibits a logarithmic mean velocity profile (law of the wall) in the region above the urban roughness elements. On the other hand, in thermal stratification with the presence of buoyancy force, the mean velocity profile deviates from its isothermal counterpart. The current LES results show that the deviation depends on the Richardson number, which is a measure of the relative contributions from natural convection and forced convection. In unstable stratification, the conventional law of the wall is modified by multiplying an empirical constant A to the logarithmic term to account for the contributions from thermal stratification. It is found that, with increasing buoyancy force, the empirical constant A and the aerodynamic roughness length scale z_0 decrease while the displacement height d increases. We therefore hypothesize that the aerodynamic roughness length z_0 and the displacement height d are functions of thermal stratification in addition to physical roughness geometry.

1. INTRODUCTION

Air pollution caused by road traffic has long been a serious problem to the residents of large cities, especially of those surrounded by dense high-rise buildings. Severe air pollution can lead to health problems, such as cardiovascular and respiratory diseases,

damages to residential properties due to acid rain, and visibility impairment. With the rapid increases of world population, industrialization, and urbanization, air pollution problem has attracted overwhelming public concern. Hence, it is necessary to enrich our fundamental understanding of the problem so as to formulate efficient rectification in urban planning and building construction, as well as more accurate prediction of micro-scale meteorology.

In modern cities, buildings collectively construct an array of obstacles that modifies the flow and turbulent transport behaviors in contrast to those in rural areas, which in turn affects the mechanism of pollutant removal from ground level. To gain a fundamental understanding, it is necessary to study the wind fields comprehensively, including mean wind velocity and fluctuations, above and within the street canyons of a hypothetical urban area with simplified configurations, such as the two-dimensional (2D) flows over idealized homogenous rectangular building structures (or urban roughness) to be reported in this paper. With the advantage of controlled environment, both computer simulation on a numerical model and experiment on a reduced-scale physical model can be utilized to investigate the problem. Thanks to the ever-increasing computing power, numerical simulation, based on computational fluid dynamics (CFD), is now able to provide a fast and rather complete numerical solution to flow problem under idealized conditions. On the other hand, physical modeling (e.g. wind tunnel or water channel experiments) is often taken as a conventional and standard research tool to validate a numerical model.

Many researchers have studied the factors governing the wind flow behaviors within and above street canyons, which include building dimensions and geometry, street width, free-stream wind speed and direction, thermal stratification induced by solar radiation and anthropogenic heating, etc. For example, in 2D isothermal turbulent flows, Oke (1988) has classified the street canyon flow into three regimes depending on the building-height-to-street-width (aspect) ratio h/b : isolated roughness flow ($h/b < 0.3$), wake interference flow ($0.3 \leq h/b \leq 0.7$) and skimming flow ($h/b > 0.7$), in which the mean flow pattern and turbulent intensity fields of each regime are totally different from the others. Just above the street canyons is the roughness sub-layer, where the mean wind profile is spatially inhomogeneous and the turbulence intensity is strong because of the shear of street canyon flows and free-stream flows (Raupach et al., 1980). Above the roughness sub-layer right over the building roof level is the inertial sub-layer where the mean wind U profile can be described by the semi-logarithmic profile equation (log-law of the wall):

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{z-d}{z_0} \right) \quad (1)$$

where u_τ is the friction velocity, κ (~0.4) the von Kármán constant, d the displacement height, z_0 the aerodynamic roughness length, and z the wall-normal coordinate (Tennekes and Lumley, 1972). The aerodynamic roughness length z_0 has long been regarded as a function of urban roughness dimensions and geometry. The research about the effect of roughness dimensions and geometry on the mean flow profile above roughness elements is plentiful in the literature (e.g., Macdonald, 2000; Coceal, et al., 2007).

However, when the thermal stratification is strong enough, the buoyancy force modifies the flow within and above street canyons. As a result, the conventional log-law Eq.(1) cannot be applied directly as in the isothermal condition. Thermal stratification arises when the surface of buildings (or urban roughness) is in different temperatures with the free-stream wind, which is noticeable as the solar radiation intensity is changing throughout the diurnal cycle. Unfortunately, the papers of turbulent flow within and over street canyons under thermal stratification are scarce in the literature. Uehara et al. (2000) conducted wind tunnel experiments with discrete rectangular blocks representing the urban roughness in different stratification levels. It was found that the recirculation and turbulence intensity in a street canyon are weakened when the stratification is stable (street canyon ground is cooler than ambient flows) and strong when unstable (ground is hotter). Similar to Uehara et al. (2000), Cheng and Liu (2011) utilized large-eddy simulation (LES) with infinitely long and homogenous 2D idealized street canyons and arrived at a similar conclusion.

In this paper, the mean flows over idealized 2D urban roughness in unstable thermal stratification are studied by LES and validated using the wind tunnel measurements from Uehara et al. (2000). The mean velocity is examined using vertical profile in the wall-normal direction that is described in a modified form of logarithmic equation Eq. (1).

2. Methodology

The LES used in the current study is performed by the open-source CFD code, OpenFOAM, version 2.1.0. In the LES, the flow variables are decomposed into resolved-scale components and subgrid-scale (SGS) components by filtering operation. The one-equation SGS turbulence model is employed to calculate the unresolved SGS stress.

2.1. Governing Equations

The filtered momentum equation for incompressible flows

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} \bar{u}_i \bar{u}_j = -\Delta P \delta_{i1} - \frac{\partial \bar{p}}{\partial x_i} + (\nu + \nu_{SGS}) \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + \alpha g \bar{\theta} \delta_{i3} \quad (2)$$

and the filtered continuity equation

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (3)$$

are solved numerically, where over-bar denotes resolved-scale variables. Tensor notation and summation convention on repeated indices are used ($i = 1, 2, \text{and } 3$ denote streamwise, spanwise, and vertical directions, respectively). \bar{u}_i are the resolved-scale velocity vectors, \bar{p} the resolved-scale kinematic pressure, ν the kinematic viscosity, $\Delta P (= 0.0005)$ the background kinematic pressure gradient (driving force for the flows in the free-stream domain which is switched off inside street canyons), α the thermal expansion coefficient, g gravitational acceleration, $\bar{\theta}$ the temperature, and δ_{ij} the Kronecker delta. Boussinesq approximation for the buoyancy force induced by vertical temperature gradient is adopted in Eq. (2). The eddy viscosity is calculated by

$$\nu_{SGS} = C_k k_{SGS}^{1/2} \Delta \quad (4)$$

where $C_k (= 0.07)$ is a modeling constant and $\Delta (= \Delta_1 \Delta_2 \Delta_3)^{1/3}$ the filter width. The SGS turbulent kinetic energy (TKE), k_{SGS} , is calculated by its transport equation

$$\frac{\partial k_{SGS}}{\partial t} + \frac{\partial}{\partial x_i} (k_{SGS} \bar{u}_i) = 2\nu_{SGS} \bar{S}_{ij} \bar{S}_{ij} - C_\epsilon \frac{k_{SGS}^{3/2}}{\Delta} + (\nu + \nu_{SGS}) \frac{\partial^2 k_{SGS}}{\partial x_i \partial x_i} + \frac{\alpha g \nu_{SGS}}{Pr} \frac{\partial \bar{\theta}}{\partial x_i} \bar{\theta} \delta_{i3} \quad (5)$$

where $\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$, $Pr (= 0.72)$ is the Prandtl number (air is assumed to be the working fluid) and $C_\epsilon (= 1.05)$ is another modeling constant. The resolved-scale temperature $\bar{\theta}$ is calculated by the energy transport equation

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x_i} \bar{\theta} \bar{u}_i = \frac{\nu + \nu_{SGS}}{Pr} \frac{\partial^2 \bar{\theta}}{\partial x_i \partial x_i} \quad (6)$$

2.2. Computational Domain and Boundary Conditions

The prevailing wind, which is driven by a background pressure gradient above building roughness, flows over twelve 2D idealized street canyons in the direction perpendicular to the street axis. This configuration represents the worst scenario of street canyon ventilation.

The domain dimensions and boundary conditions are shown in Fig.1. The aspect ratio h/b is kept unity so that the flows fall into the skimming flow regime. The discretization of the free-stream region (domain above building rooflevel) and each street canyon is summarized in Table 1 in which 4,546,560 brick elements are used in the computational domain. The grid is refined near all solid walls at the domain bottom so as to capture the rapidly changing flow variables. The smallest element size in free-stream region is $\Delta x \Delta y \Delta z = 0.0431 \times 0.0625 \times 0.0274 h^3$, located at the building block corners, while the smallest element size in street canyons is $\Delta x \Delta y \Delta z = 0.0431 \times 0.0625 \times 0.0431 h^3$, located at each corner of the cavity. No-slip boundary is prescribed at the building roughness surface and symmetry boundary is prescribed at the domain top where the flows are free-slip. Periodic boundaries are assigned to the domain inlet and outlet, and also the spanwise extent, such that the infinitely long and wide homogeneous urban roughness is simulated. The temperatures at the top θ_H and the bottom solid walls θ_w are set to -1 and 1, respectively, to model the unstable thermal stratification (upward buoyancy force). The magnitude of the gravitational acceleration is varied to control the buoyancy force. The data are collected at steady-state with a period of time of ten channel flows ($\sim 10L/U_H$), where U_H is the free-stream mean wind velocity (streamwise mean wind velocity at domain top). The data are then averaged in time and spanwise direction.

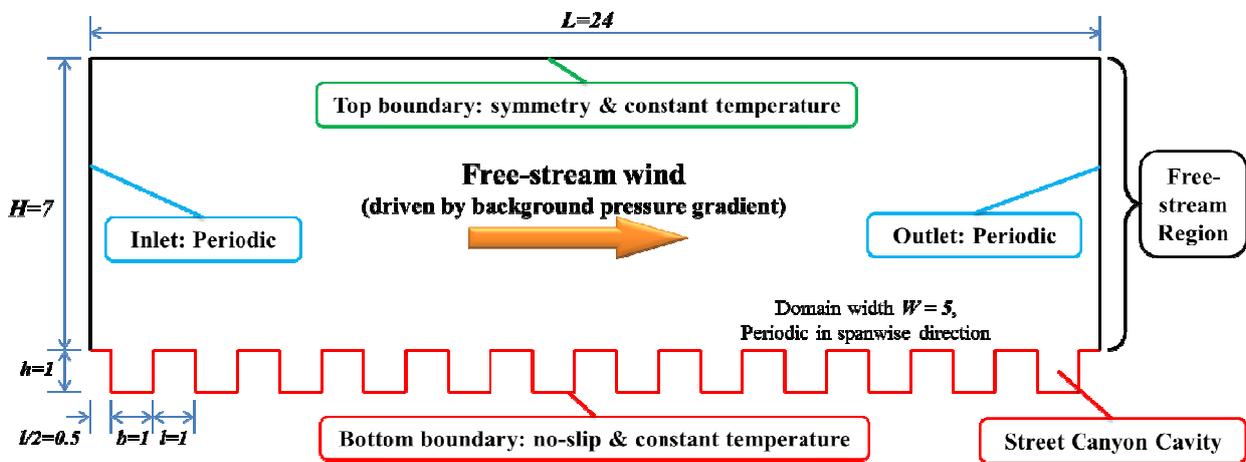


Fig. 1 Computational Domain and Boundary Conditions

Table 1 Domain discretization

	Dimensions	No. of grid points
Street canyon cavity	$b \times W \times h = 1 \times 5 \times 1$	$16 \times 80 \times 16$
Free-stream region	$L \times W \times H = 24 \times 5 \times 7$	$384 \times 80 \times 140$

3.3. Simulation Parameters

In this study, the flows are characterized by different dimensionless numbers which are summarized in **Table 2**. Based on the free-stream region height H ($=7h$) and free-stream mean wind velocity U_H , the channel flow Reynolds numbers

$$Re_H = \frac{U_H H}{\nu} \quad (7)$$

are between 48,000 and 86,000. Based on the building height h ($=1$) and mean wind velocity at canyon top center U_h , the building scale Reynolds numbers of the channel flows

$$Re_h = \frac{U_h h}{\nu} \quad (8)$$

are about 1,900. Based on friction velocity u_τ ($=(\tau_w/\rho)^{1/2}$), the Reynolds numbers

$$Re_\tau = \frac{u_\tau H}{\nu} \quad (9)$$

is about 4,100 for all flows, where τ_w is the wall shear stress that is calculated by the force balance with background pressure gradient. The effect of unstable thermal stratification is characterized by the bulk Richardson number

$$Rb_H = \frac{\alpha g H (\theta_H - \theta_w)}{U_H^2} \quad (10)$$

ranging from -1.17 to 0. While the building-scale Richardson numbers

$$Rb_h = \frac{\alpha g h (\theta_h - \theta_w)}{U_h^2} \quad (11)$$

are between -0.26 and 0, where θ_H is the temperature at the street canyon top center.

Table 2 Summary of dimensionless numbers

Gravitational acceleration	Channel flow Reynolds No.	Building-scale Reynolds No.	Bulk Richardson No.	Building scale Richardson No.
g	Re_H	Re_h	Rb_H	Rb_h
0	86,444	1,926	0	0
0.01	74,240	1,970	-0.124	-0.041
0.02	62,719	1,927	-0.349	-0.097
0.03	58,469	1,939	-0.602	-0.143
0.04	48,430	1,846	-1.170	-0.260

4. Model Validation

The streamwise mean wind velocity along the vertical centerline of street canyon of the current LES study is compared with the wind tunnel data from Uehara et al. (2000) as shown in Fig. 2.

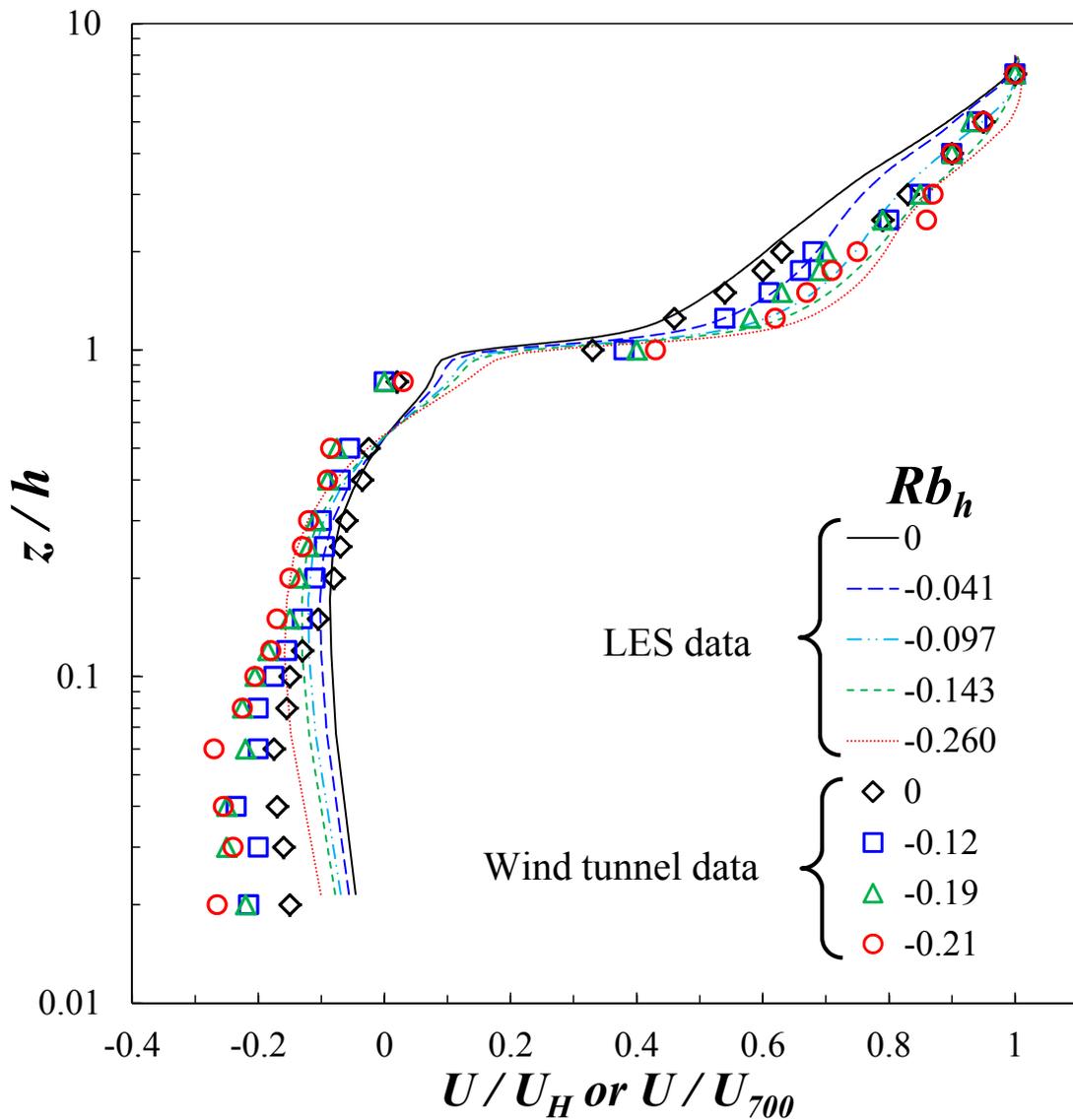


Fig. 2 Comparison of profiles of streamwisemean wind velocity along the centerline of street canyon

In the wind tunnel experiment, the test section was 2m high and 3m wide, and the wind speed was ranged from 0.2 to 10m/s. An array of cubical building blocks with dimensions 100mmx100mmx100mm was used and each was separated by 100mm in streamwise direction and 50mm in spanwise direction. The wind tunnel floor or inflow air was heated at different temperatures so that unstable or stable stratification was developed. The range of Richardson numbers was varied from -0.21 (unstable) to 0.79 (stable), where building-scale Richardson number Re_h was used, and only isothermal and unstable cases are considered here. The measurements were conducted in the street between the fifth and the sixth building rows. The centerline streamwise mean wind velocity profile was measured in both below and above the roof level of building blocks and was normalized by mean wind speed U_{700} at height 700mm. For the LES model, vertical profiles are obtained by averaging the centerline streamwise resolved-scale mean velocity in time, spanwise direction and twelve canyon sections. They are then normalized by the free-stream mean velocity U_H .

In Fig. 2, it shows that the streamwise mean wind velocity profiles are similar in the free-stream region ($z/h > 1$) for wind tunnel experiments and LES models at different unstable stratifications. However, the profiles do not match very well near the ground floor ($0 < z/h < 0.2$). It is because only the ground was heated in the wind tunnel experiments while all the solid walls (building roofs, street canyon floors, leeward and windward building walls) are heated in the LES models. The discrepancy could also be caused by the different building configurations, where discrete cubical blocks were used in the wind tunnel experiments while infinitely long rectangular bars are used in the current LES models. Nevertheless, the trends of the changes in profiles against different unstable stratifications are similar in both studies, in which the mean winds are stronger above and within street canyons in more unstable stratification.

5. Results and Discussion

5.1. Vertical Profile of Mean Flow in Linear Plot

For different unstable stratifications, the resolved-scale streamwise mean wind velocities U in free-stream region ($x/h > 1$) are averaged in time, spanwise and streamwise directions and are normalized by their respective free-stream wind speeds U_H to obtain a set of vertical mean wind profiles (Fig. 3). It is shown that the profile tends to shift outwards with increasing bulk Richardson numbers Rb_H . This means that the mean flow field is more vertically uniform in the free-stream region and the gradient is steeper near the building roughness with the enhancement of unstable stratification. This finding is caused by the

enhanced turbulent momentum transport as a result of the enhanced buoyancy force. The vertical profiles of the turbulence quantities, including the resolved-scale streamwise, spanwise and vertical velocity fluctuations $\langle u'u' \rangle^{1/2}$, $\langle v'v' \rangle^{1/2}$ & $\langle w'w' \rangle^{1/2}$ and vertical momentum flux $-\langle u'w' \rangle$, which are normalized by the free-stream wind speed U_H , are shown in Fig. 4. The plots collectively suggest that the turbulence is enhanced in the free-stream region when the stratification is more unstable.

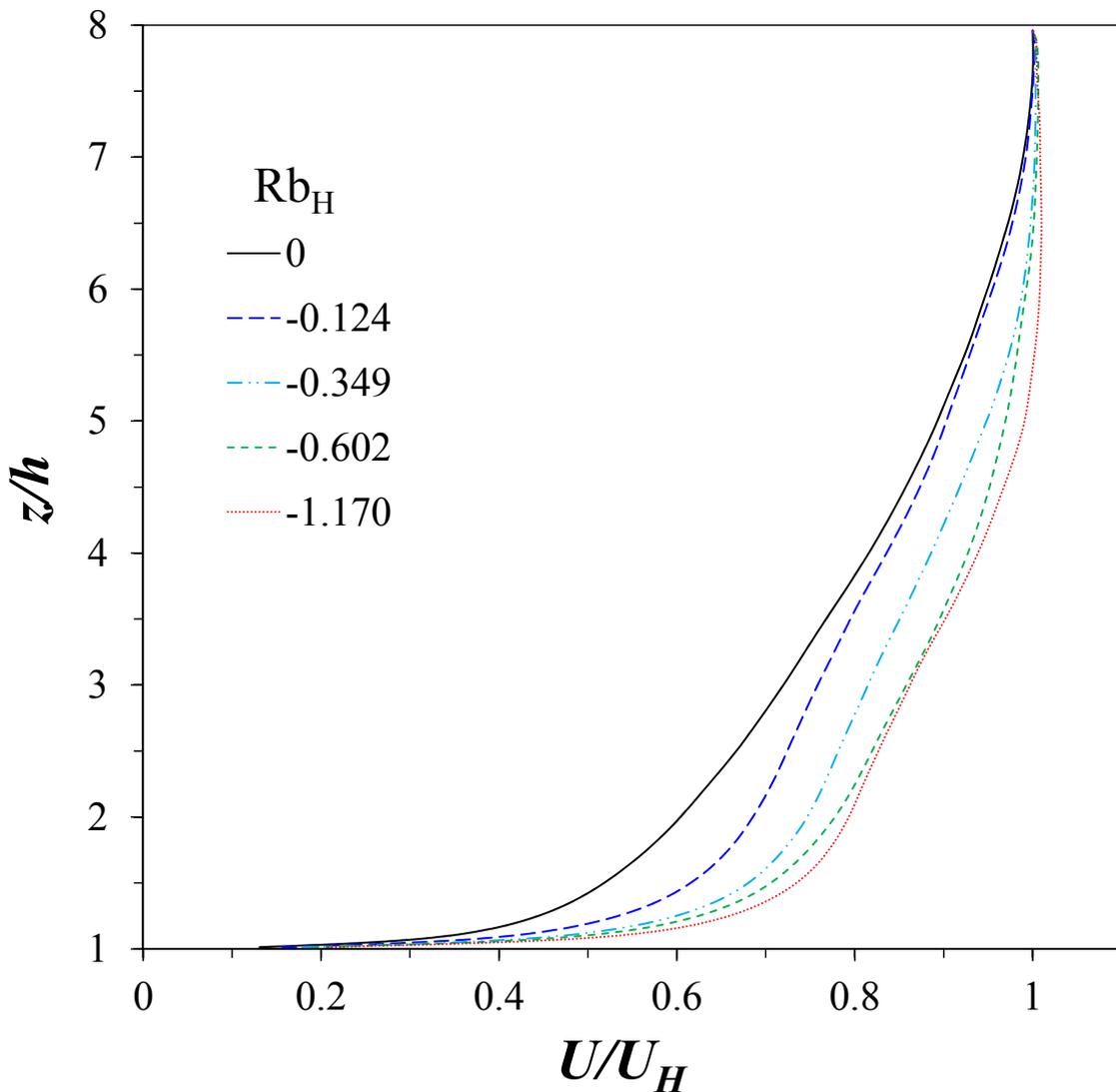


Fig. 3 Vertical profiles of streamwise mean wind velocity in free-stream region ($z/h > 1$)

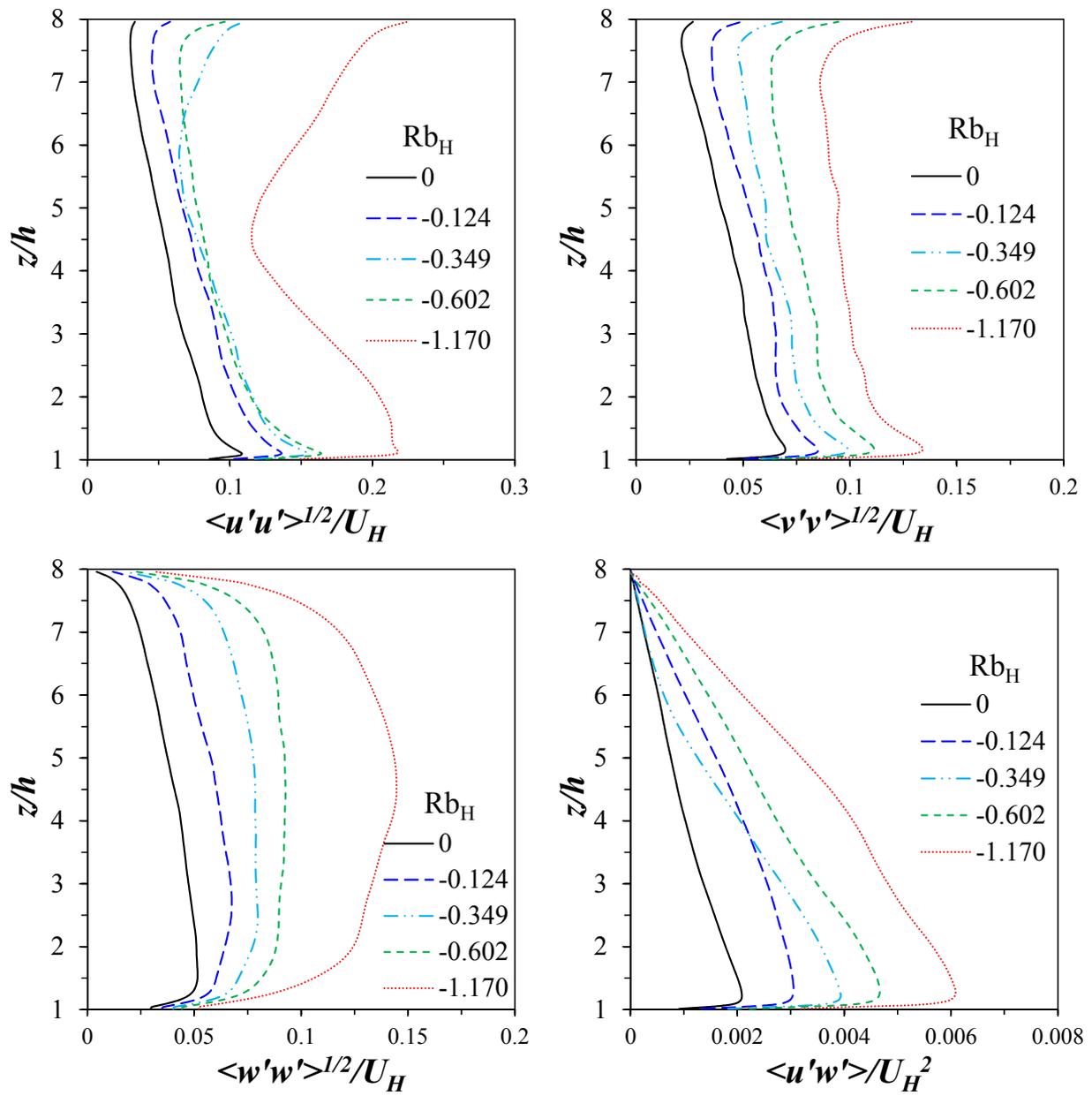


Fig. 4 Vertical profiles of streamwise, spanwise and vertical velocity fluctuations and vertical momentum flux in free-stream region ($z/h > 1$)

5.2. Vertical Profile of Mean Flow in Semi-Logarithmic Plot Using Viscous Length and Velocity Scales

The resolved-scale streamwise mean velocities at different unstable stratifications are plotted in semi-logarithmic scale (Fig. 5). In this semi-logarithmic plot, u^+ , the mean wind velocity normalized by the viscous velocity scale (friction velocity u_τ in shear flows) is used on the y-axis in linear scale. On the other hand, $(z-d)^+$, the vertical coordinate relative to the building roof level ($z = h = l$) normalized by viscous length scale $l_v (= \nu/u_\tau)$, is used on the x-axis in logarithmic scale. The superscript + denotes that the variables are normalized by their respective viscous scales that are commonly used in turbulent shear flow analysis. From the literature (e.g. Pope, 2000), the turbulent flow over smooth surface (i.e. $h = 0$) in isothermal condition can be basically divided into several characteristic layers, (1) the viscous sub-layer in the region $z^+ < 5$, in which the profile can be described by

$$u^+ = y^+ \quad (12),$$

(2) the log-law region in the region $z^+ > 30$ and far enough from the boundary layer top (the domain top surface in this study), in which the profile can be described by the logarithmic law of the wall

$$u^+ = \frac{1}{\kappa} \ln z^+ + B \quad (13),$$

where κ (~0.4) is the von Kármán constant and B (~5) is another empirical modeling constant, (3) the buffer layer in the region in-between the viscous sub-layer and log-law region ($5 < z^+ < 30$), in which the profile is in transition from Eq. 12 to Eq. 13. The profiles in viscous sub-layer and log-law region are also compared in Fig. 5. It shows that the profile for the isothermal turbulent flow ($Rb_H = 0$) over urban roughness in our case is approximately linear in the semi-log plot for region far enough from the domain top surface, implying that there exists a semi-log profile. It is also shown that this linear section of the curve is approximately at the same slope with and below the log-law of the wall for smooth surface. For different unstable stratifications, the curves deviate from linearity and curvature increases with bulk Richardson number (Rb_H) for the same range of $(z-h)^+$ as in isothermal case.

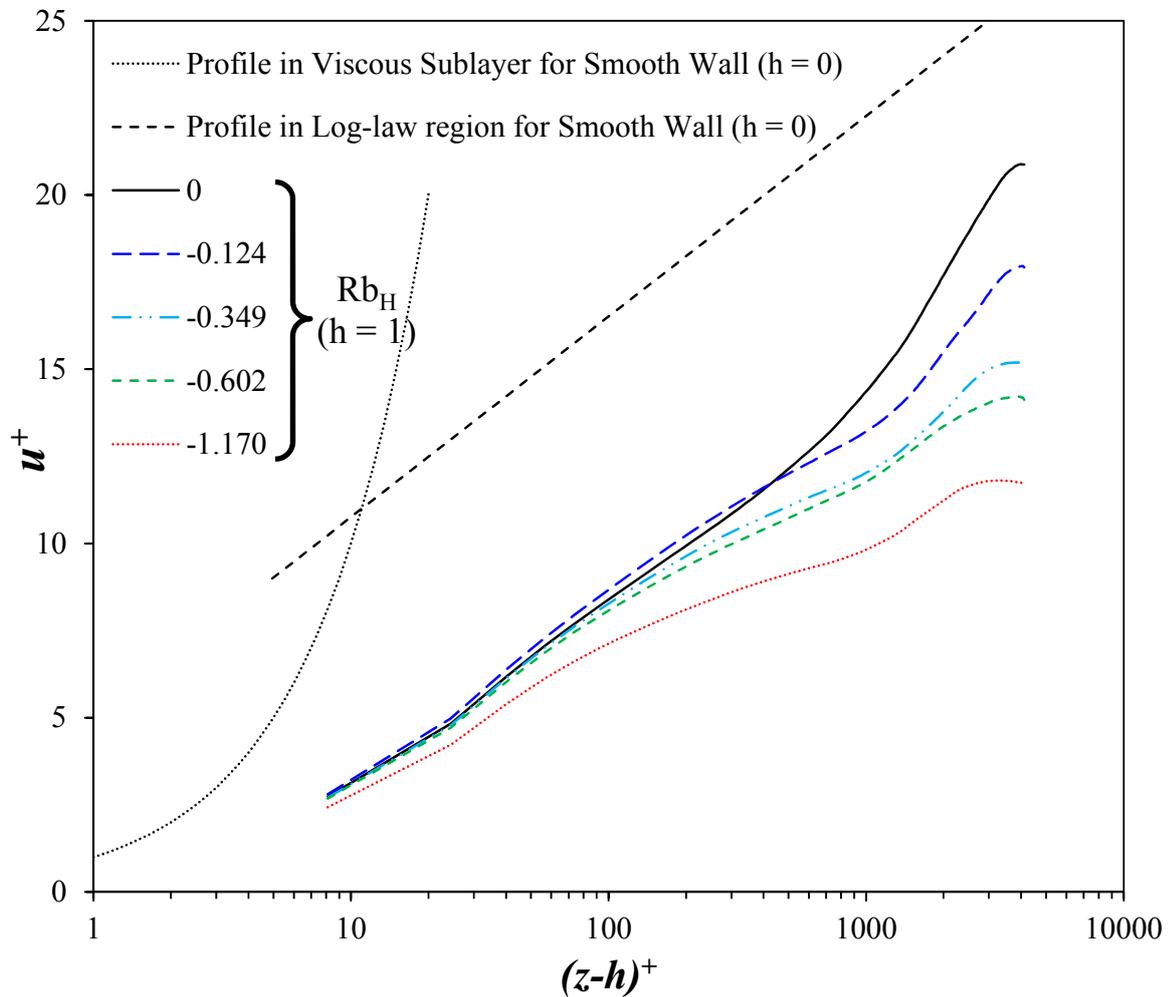


Fig. 5 Vertical profiles of streamwise mean wind velocity in Semi-logarithmic scale (in the form of wall variables)

5.3. Modified Semi-logarithmic Profile Equation

Next, the semi-logarithmic profile Eq. 1, that is commonly used to describe the mean flow profile in atmospheric boundary layer in isothermal condition, is modified to account for the effect of unstable thermal stratification. Similar to Eq. 1, the modified semi-logarithmic profile equation

$$\frac{U}{u_\tau} = \frac{A}{\kappa} \ln\left(\frac{z-d}{z_0}\right) \quad (14)$$

expresses the mean flow in which a semi-logarithmic profile exists by the von Kármán constant κ , displacement height d and aerodynamic roughness length z_0 . The difference from Eq. 1 is that the logarithmic term in Eq. 14 is multiplied by an empirical constant A , which is used to characterize the effect of unstable stratification, and thus A is expected to be a function of bulk Richardson number Rb_H . In our study, the three empirical modelling constants d , z_0 and A for each unstably stratified flow are obtained by data regression in the range of vertical coordinate z in which the semi-logarithmic region exists in the isothermal case, while the von Kármán constant $\kappa \sim 0.417$ is obtained by setting $A = 1$ for the isothermal case. The variations of d , z_0 and A with bulk Richardson number Rb_H are shown in Fig. 6. It is found that d increases with the magnitude of Rb_H and is slightly larger than 1 (the building roof level) for the unstable cases, and z_0 and A decrease with the magnitude of Rb_H . These results show that d and z_0 depend on the strength of unstable stratification, in addition to the roughness geometry and dimensions, the decrease of z_0 with the magnitude of Rb_H indicates that the bottom wall with roughness seems to be “smoother” with the enhancement of unstable stratification. Finally, Fig. 7 shows the plot of U/u_τ against $(z-d)/z_0$, indicating that the slope decreases with the magnitude of Rb_H for the straight line sections. This is consistent with the decrease of A with the magnitude of Rb_H .

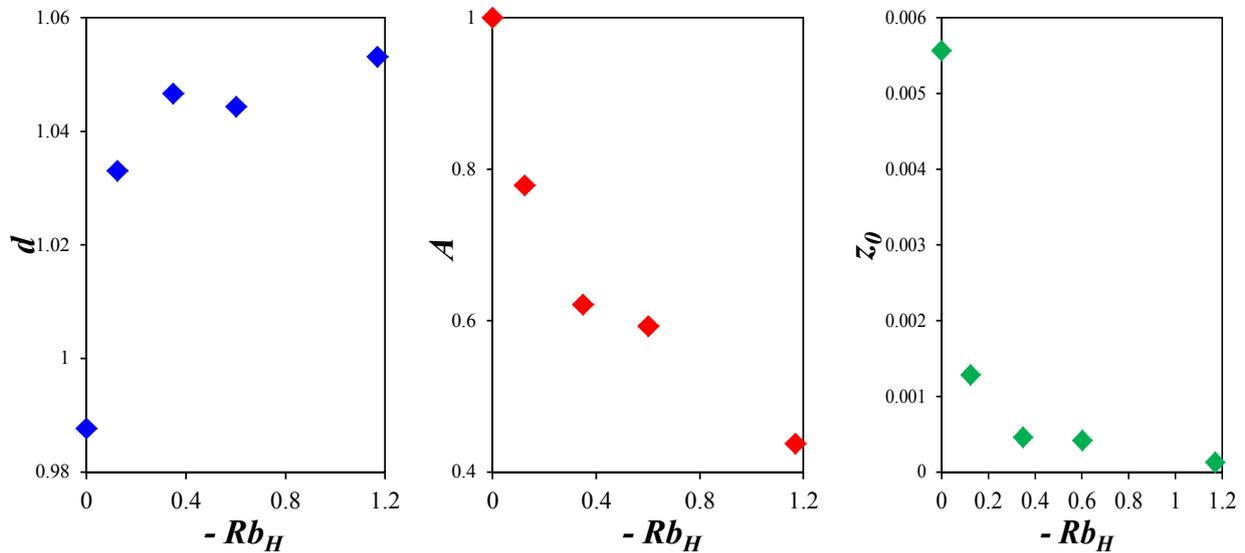


Fig. 6 Variations of displacement height d , aerodynamic roughness length z_0 and empirical constant A with bulk Richardson number Rb_H

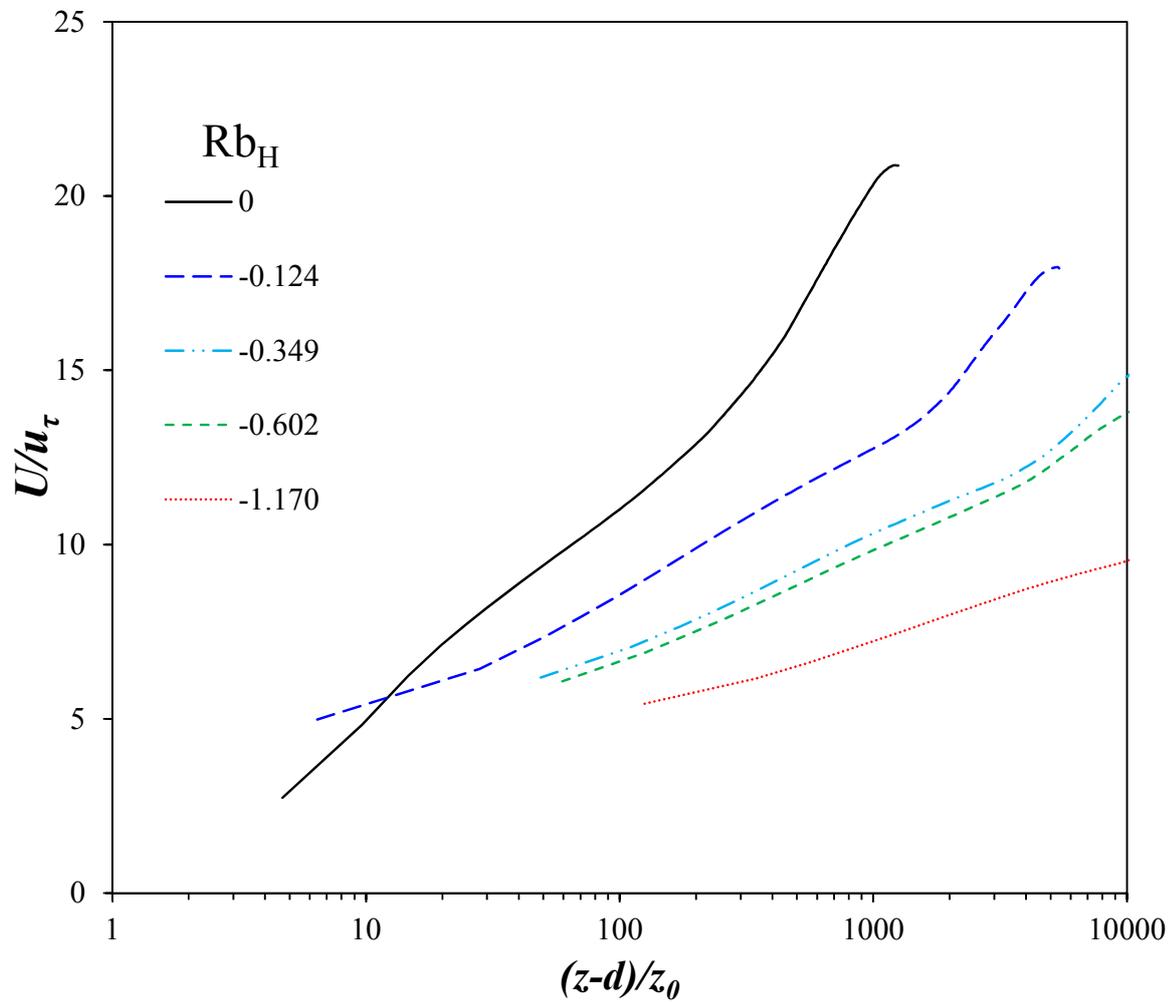


Fig. 7 Plot of U/u_τ against $(z-d)/z_0$ in semi-logarithmic scale at different unstable stratifications

6. Conclusion

In this study, five sets of mean flow data generated by LES with one in isothermal condition and four in unstable thermal stratifications are analyzed and validated by the wind tunnel data of [Uehara et al. \(2000\)](#). It is found that the mean wind velocity profile tends to shift outwards (more vertically uniform) with the enhancement of unstable stratifications. It is also found that the variation of u^+ with $(z-d)^+$ in semi-logarithmic scale plot

is linear in isothermal condition, but it deviates from linearity with the enhancement of stratification. With the use of modified semi-logarithmic profile equation, different sets of empirical constants d , z_0 and A are obtained for different unstable stratifications, where d increases while z_0 and A decrease with the enhancement of stratification. It reveals that d and z_0 depends on thermal stratification, in addition to roughness geometry and dimension.

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