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Assessment of wind-induced structural fatigue based on joint probability density function of wind speed and direction

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ABSTRACT

This paper investigates assessment method for wind-induced structural fatigue damage with consideration of joint probability density function of wind speed and wind direction (JPDF). By introducing the JPDF into the traditional equivalent narrow-band method, a new formula for calculating fatigue life is derived. A double integral operation of JPDF is introduced into the calculation procedure to replace the original sectional-accumulation of fatigue damage. Detailed derivation procedure for the formula is presented, followed by a brief description of the selection of JPDF. Applicability of the suggested method is verified by applying to a steel antenna, which is installed on the top of a high-rise building, to predict its fatigue life.

1. INTRODUCTION

"Higher, lighter and longer" is the word-wide trend for construction of civil structures. As a result, wind-induced fatigue damage becomes a major concern in the structural design of slender structures as antennas, TV towers, lamp posts, masts etc. For assessment of wind-induced fatigue of slender steel structures, the effect of wind directionality, though very important, has either been completely ignored or has been considered in a sectional manner that fatigue damage is first assessed in several wind direction sectors and accumulated damage is then computed and taken as the final result. Methods for assessing wind-induced structural damage can be classified into two types as time domain and frequency domain. Time domain methods usually obtain the structural stress time history by dynamic analysis and count the numbers of stress cycles by rainflow technique. Frequency domain methods transform strain records into frequency-domain by the theory of random vibration, which includes equivalent narrow-band method (Wirsching 1980) and upper and lower-bound solutions (Holmes 2002). Equivalent narrow-band method assumes the stress process as a narrow-band process and then corrects the result with an empirical factor. In addition, upper and lower-bound solutions introduce empirical expression of standard deviation of stress and then derive the closed-form solutions of upper-bound and lower-bound. Gu (1999) proposed the so-

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called time-frequency domain methods by combining the higher accuracy of time domain methods with the higher computational efficiency of frequency domain methods. This method gains stress power spectrum of the key point by the theory of random vibration firstly. Then it simulates stress time history by using Monte Carlo method and calculates damage by rainflow counting method.

It is well known that wind directionality has a significant effect on the fatigue damage. One will have a very conservative assessment of fatigue life if the wind directionality hasn't been considered. On the other hand, researches on joint probability properties of wind speed and wind direction (JPDF) has achieved significant improvement in the past decade. In this connection, we attempt in this study to assess the structural fatigue life by introducing a JPDF model into the traditional wind-sector-based method. The calculation procedure is based on equivalent narrow-band method which assumes the stress process as a narrow-band process and then corrects the result with an empirical factor. A double integral operation of JPDF is introduced into the calculation procedure to replace the original sectional-accumulation of fatigue damage. Therefore, the fatigue assessment accuracy can be improved in theory. Finally, the suggested method has been applied to a steel antenna with round-section to assess its along-wind fatigue life.

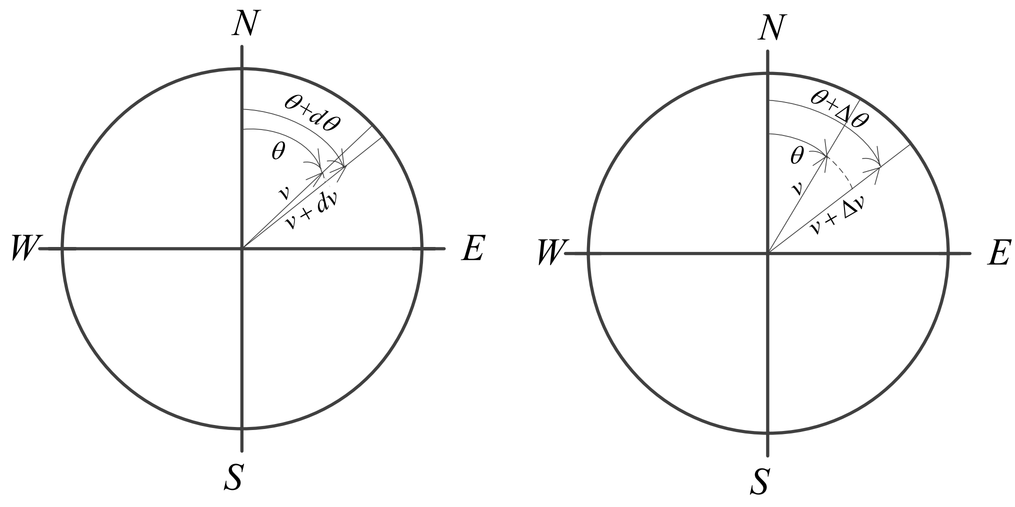
2. PROCEDURE FOR WIND-INDUCED FATIGUE ASSESSMENT USING JPDF

We derive in this section a new computation procedure by replacing the sectional damage calculation with a continuous JPDF function in the equivalent narrow-band method. The structural stress process is assumed as narrow-band process for which the damage D_{NB} calculation formula can be established. Then, an empirical factor λ is introduced to account for the broad band effect and the final damage will be $D_{NB} * \lambda$.

Note the polar coordinates express of joint probability distribution of wind speed (radial direction) and wind direction (angular) in Fig.1. Suppose the structural stress response due to the differential range ($v < V < v + dv, \theta < \Theta < \theta + d\theta$) in Fig.1 is a zero-mean, stationary and narrow-band Gauss process. In sector-based approach, the stress response is assumed due to a wind-speed-direction sector as shown in Fig.1b. Therefore, peaks of the stress process peaks, $P(t)$, will have a Rayleigh distribution as

$$f_{Pv,\theta}\left(p\right) = \frac{p}{\sigma_{Sv,\theta}^{2}} \exp\left(-\frac{p^{2}}{2\sigma_{Sv,\theta}^{2}}\right) \tag{1}$$

Where $f_{Pv,\theta}\left(p\right)$ is probability density function of the stress peaks, $\sigma_{Sv,\theta}^{2}$ is the standard deviation of the stress process.



(a) Differential range (b) sectional range

Fig. 1 Differential range of wind speed and direction plane

In order to calculate the probability density of stress process, we need to adopt the following two assumptions: (1) In stationary Gauss process, for any peak point, a minimum point which has the same absolute value with the maximum point (peak) point can be found, regardless of the position where the minimum point appear. (2) We believe only the positive stress range contributes to fatigue damage. However, the negative stress range should be neglected. Consequently, the relation of the peak random variable $P_{v,\theta}$ and the stress range random variable $S_{v,\theta}$ will be

$$S_{v,\theta} = 2P_{v,\theta} \tag{2}$$

According to the basic knowledge of probability theory, the probability density function of stress range, $f_{Sv,\theta}\left(S\right)$, can be expressed:

$$f_{Sv,\theta}\left(S\right) = f_{Pv,\theta}\left(\frac{S}{2}\right)\frac{dP_{v,\theta}}{dS_{v,\theta}} = \frac{1}{2}f_{Pv,\theta}\left(\frac{S}{2}\right) = \frac{S}{4\sigma_{Sv,\theta}^2}\exp(-\frac{S^2}{8\sigma_{Sv,\theta}^2}) \tag{3}$$

In the differential range ($v < V < v + dv, \theta < \Theta < \theta + d\theta$), the number of cycles within the stress range S to $S + dS$ is

$$n_{v,\theta,S}\left(S\right) = f_{v,\theta}^{+}Tf_{Sv,\theta}\left(S\right)dS \tag{4}$$

where *T* is time duration, $f_{v,\theta}^{+}$ is the rate of crossing in the range and can be obtained by

$$f_{v,\theta}^{+} = \frac{1}{2\pi}\sqrt{\frac{\alpha_{2v,\theta}}{\alpha_{0v,\theta}}} = \frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{+\infty}\omega^2 S_{Sv,\theta}(\omega)d\omega}{\int_{-\infty}^{+\infty}S_{Sv,\theta}(\omega)d\omega}} \tag{5}$$

where $S_{Sv,\theta}(\omega)$ is the power spectrum of the stress process.

Introducing Eq. (4) into the *S-N* curve function $NS^m = K$ (where *m*, *K* are material constants), we can calculate the fatigue damage in the differential wind-speed-direction range($v < V < v + dv, \theta < \Theta < \theta + d\theta$) and stress differential range (S to $S + dS$) as

$$D_{NB,v,\theta,S} = \frac{f_{v,\theta}^{+} T S^m f_{Sv,\theta}(S) dS}{K} \tag{6}$$

Therefore, the damage of narrow-band process within the total stress range will be

$$D_{NB,v,\theta} = \frac{f_{v,\theta}^{+} T \int_0^{+\infty} S^m f_{Sv,\theta}(S) dS}{K} \tag{7}$$

Introducing Eq. (3) into Eq.(7), we get

$$D_{NB,v,\theta} = \frac{T f_{v,\theta}^{+}}{K}\left(2\sqrt{2}\sigma_{Sv,\theta}\right)^m \Gamma\left(\frac{m}{2}+1\right) \tag{8}$$

where $\Gamma(*)$ is the Gamma function.

An empirical factor λ was suggested by Wirsching (1980) to count for the effect of wide-band process. The factor is defined as

$$\lambda_{v,\theta} = a + (1-a)\left(1-\varepsilon_{v,\theta}\right)^b \tag{9}$$

where material parameter a and b are determined by empirical function Eq. (10); $\varepsilon_{v,\theta}$ is parameter of spectral width and can be determined by Eq. (11)

$$a = 0.926 - 0.033m, b = 1.587m - 2.323 \tag{10}$$

$$\varepsilon_{v,\theta} = \sqrt{1 - \frac{\alpha_{2v,\theta}^2}{\alpha_{0v,\theta}\alpha_{4v,\theta}}} \tag{11}$$

In Eq. (11), $\alpha_{0v,\theta}$, $\alpha_{2v,\theta}$ and $\alpha_{4v,\theta}$ are the 0th, 2th and 4th moment of stress power spectrum.

Thus, the damage of wide-band process, $D_{WB,v,\theta}$ can be obtained by

$$D_{WB,v,\theta}=\frac{\lambda_{v,\theta}Tf_{v,\theta}^{+}}{K}\left(2\sqrt{2}\sigma_{Sv,\theta}\right)^{m}\Gamma\left(\frac{m}{2}+1\right) \tag{12}$$

The probability of wind occurrence in the range ($v<V<v+dv,\theta<\Theta<\theta+d\theta$) can be calculated from the JPDF $f_{V,\Theta}\left(v,\theta\right)$ by

$$P_{v,\theta}=f_{V,\Theta}\left(v,\theta\right)dvd\theta \tag{13}$$

Based on all the above equations, the total fatigue damage *D* in the entire range ($0<V<\infty,0<\Theta<2\pi$) will be

$$D=\int_0^\infty\int_0^{2\pi}P_{v,\theta}D_{WB,v,\theta}=\frac{T}{K}\Gamma\left(\frac{m}{2}+1\right)\int_0^\infty\int_0^{2\pi}f_{V,\Theta}\left(v,\theta\right)\lambda_{v,\theta}f_{v,\theta}^{+}\left(2\sqrt{2}\sigma_{Sv,\theta}\right)^{m}dvd\theta \tag{14}$$

Let *D*=1, the fatigue life of wind-induced with a double integral operation of JPDF is

$$T=\frac{K}{\Gamma\left(\frac{m}{2}+1\right)\int_0^\infty\int_0^{2\pi}f_{V,\Theta}\left(v,\theta\right)\lambda_{v,\theta}f_{v,\theta}^{+}\left(2\sqrt{2}\sigma_{Sv,\theta}\right)^{m}dvd\theta} \tag{15}$$

Eq. (14) and (15) are the formula for calculating wind-induced fatigue damage with consideration of joint probability density function of wind speed and direction.

3. SELECTION OF COMPUTATIONAL PARAMETERS

3.1 Calculation of stress power spectrum

Consider low damping case and ignore the coupling effect of various vibration modes of a slender structure with round section, the stress power spectrum of the key point can be expressed as (Wang 2008)

$$S_s(\omega) = \sum_{j=2k-1} \left[\frac{r\cos\alpha\omega_j^2 \int_{z_1}^{H} m(z)\phi_j(z)(z-z_1)\,dz}{I} \right]^2 S_{hj}(\omega) + \sum_{n=2k} \left[\frac{r\sin\alpha\omega_n^2 \int_{z_1}^{H} m(z)\phi_n(z)(z-z_1)\,dz}{I} \right]^2 S_{hn}(\omega) \tag{16}$$

where, *j=2k-1* is the mode of *X*-direction and *n=2k* is the mode of *Y*-direction; moreover, the max *j* is the numbers of the mode of *X*-direction and the max *k* is the numbers of the mode of *Y*-direction. *r* is outer radius of round-section. *α* is an angle with *jth*-direction. ω_j is jth frequency. *m*(*z*) is mass per unit length. $\phi_j(z)$ is *j*th mode. z_1 is height of key point. *I* is moment of inertia. $S_{hj}(\omega)$ is generalized displacement spectrum of *j*th mode.

3.2 JPDF

For JPDF model, this paper uses the Farlie-Gumbel-Morgensternmoxign (FGM) model suggested by Erdem (2011).

$$F_{V,\Theta}(v,\theta) = F_V(v)\cdot F_\Theta(\theta)\left[1+\delta\left(1-F_V(v)\right)\left(1-F_\Theta(\theta)\right)\right] \tag{17}$$

where $F_{V,\Theta}(v,\theta)$ is joint cumulative distribution function of wind speed and direction (JCDF); $F_V(v)$ is the marginal cumulative distribution function of wind speed; $F_\Theta(\theta)$ is marginal cumulative distribution function of wind direction, and $-1 \le \delta \le 1$ is a correlation coefficient. From Eq. (17), we can easily get the JPDF as

$$f_{V,\Theta}(v,\theta) = f_V(v)\cdot f_\Theta(\theta)\left[1+\delta\left(1-2F_V(v)\right)\left(1-2F_\Theta(\theta)\right)\right] \tag{18}$$

and the marginal PDF of wind speed and direction $f_V(v)$ and $f_\Theta(\theta)$ can be further expressed as Normal Weibull distribution and mixture von Mises model of order 5 (Zhang 2011; Erdem 2011). The coefficient δ can be obtained by the fitting the Eq.(18) to the measured JPDF. In this case, we use MATHCAD as a platform for the fitting process (Zhang 2011; Li 2005).

$$f_V(v) = \frac{\omega_0}{I(\phi_1,\phi_2)\phi_2} z(v,\phi_1,\phi_2) + (1-\omega_0)\frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right];\ -\infty \le v \le \infty \tag{19}$$

$$z(v,\phi_1,\phi_2)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(v-\phi_1)^2}{2\phi_2^{\ 2}}\right) \tag{20}$$

$$I(\phi_1,\phi_2)=\frac{1}{\phi_2}\int_0^{\infty} z(v,\phi_1,\phi_2)\,dv \tag{21}$$

where, β is scale parameter. α is shape parameter. ω_0 is weight. ϕ_1 and ϕ_2 are parameters which have the same units with *v*.

$$f_\Theta(\theta)=\sum_{j=1}^{N}\frac{\omega_j}{2\pi I_0(k_j)}\exp\left[k_j\cos(\theta-u_j)\right];\ 0\le\theta<2\pi \tag{22}$$

$$I_0(k_j)=\frac{1}{\sqrt{2\pi}}\int_0^{2\pi}\exp\left[k_j\cos\theta\right]d\theta=\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{k_j}{2}\right)^{2k} \tag{23}$$

where, *N* represents numbers of the model peaks. ω_j is a weight, $0\le\omega_j\le 1$, $\sum_j^N\omega_j=1$, (*j*=1,2 …*N*). $k_j\ge 0$ and $0\le u_j<2\pi$ are parameters. $I_0(k_j)$ is the modified Bessel function of the first kind.

4. NUMERICAL EXAMPLE

4.1 Steel antenna

The example used is a 87m high steel antenna with round-section that is installed on the top (the height is 246m) of a high-rise building. For numerical calculation purpose, the antenna is simplified as three sections whose lengths are 34m, 27m and 26m respectively. The outer diameter /thickness of each section is 2.4m/3cm, 1.75m/2.5cm and 1.0m/2.0cm, respectively, see Fig. 2 For dynamic response calculation, the mass density is set as 7.85×10^3kg/m^3, and a lumped mass model is used (see Fig. 2) to calculated the natural frequency and vibrational models. A Rayleigh damping model is used with the damping ratio being 0.005. Kaimal spectrum model is selected to simulate the along-wind turbulent component, and Shiotani model is used for the spatial correlation. The natural frequencies of the first six modes of the antenna are summarized in Table 1 and the mode shapes are presented in Fig. 3.

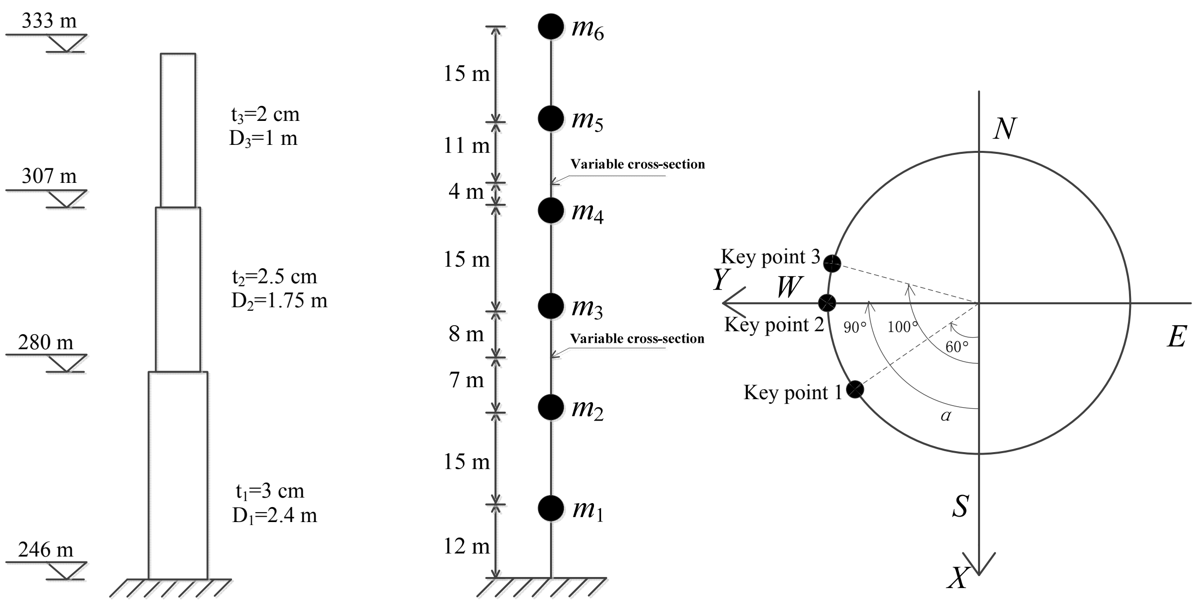


Fig. 2 A steel antenna

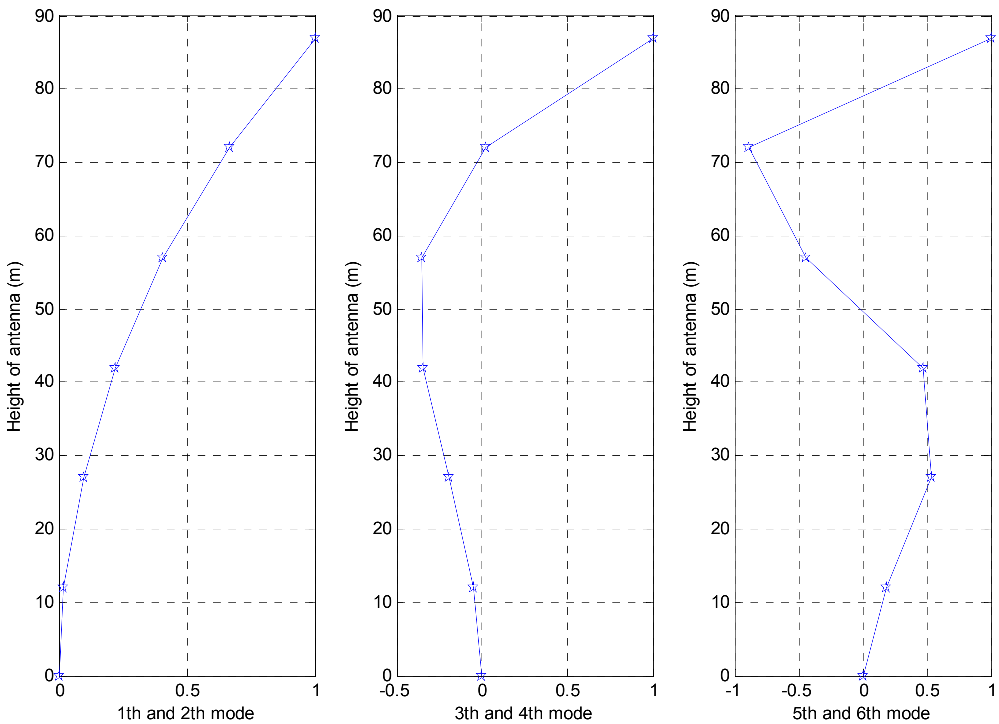


Fig. 3 Vibration modes of the antenna

Table 1 The first six natural frequencies of the antenna

*j*th	1	2	3	4	5	6
Frequency(Hz)	0.4610	0.4610	1.4104	1.4104	3.2581	3.2581

4.2 JPDF

Field measured wind data (Yang 2002) from weather station near the antenna have been analyzed and the parameters for the JPDF are calculated and summarized in Table 2 and 3. The correlation coefficient is calculated as 0.1015.

Fig. 3 and Fig4 show the measured JPDF and the calculated JPDF by the model defined by Eq. (18).

Table 2 Fitting results of wind-speed-PDF (Normal Weibull model)

Parameters	ω_0	ϕ_1	ϕ_2	α	β	R^2
Fitting results	0.35122	6.711	2.2446	4.2494	5.411	0.999499

Table 3 Fitting results of wind-direction-PDF (mixture von Mises model (N=5))

j(*N*=5)	1	2	3	4	5
ω_j	0.144	0.265	0.297	0.077	0.217
k_j	2.302	5.821	1.354	16.738	6.155
u_j	0.756	1.481	5.651	5.255	2.635

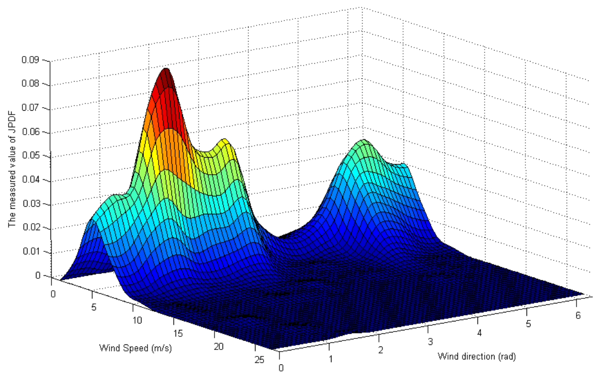


Fig. 4 Measured JPDF

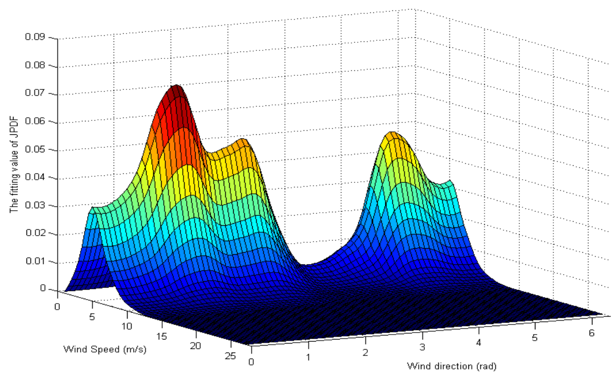


Fig. 5 Calculated JPDF

4.3 Assessment of wind-induced fatigue life

The three key points selected for the fatigue life assessment is demonstrated in Fig. 2. The points are on the bottom section with an angle of sixtieth, ninety and 100th degrees respectively. The wind-induced fatigue life under along-wind loading calculated as 536.2, 446.4 and 457.9 years respectively.

5. CONCLUDING REMARKS

A new wind-induced fatigue assessment approach has been suggested in this paper that has considered the joint probability density function of wind speed and direction (JPDF). This method uses a continuous JPDF rather than a sector based model and therefore is more accurate in theory compared with sectional-accumulation. Moreover, it is more flexible on getting the satisfying accuracy with smaller integral intervals of wind speed and direction. Fatigue assessment of a steel antenna is used to verify the application of this new method.

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