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Operational Response Analysis of a Tower Crane Structure Considering Crane-payload Interaction

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ABSTRACT

Tower cranes are widely used facilities for the construction of high-rise structures. In operation, the crane is required to lift the payloads of up to thousand tons from the ground and then transport them to the appointed place via lifting, rib rotation and lifting cart movement along the rib. These operations will induce the rigid body motion of the payload as well as the deformable vibration of the crane structure. Since the motion of the payload and the vibration of the crane are coupled, it is thus necessary to take the crane-payload interaction into account for crane operational response analysis. This paper presented a dynamic modeling and response analysis procedure. The motions of the payload are idealized to be the following three patterns, the straight line motion, the spherical pendulum motion, and the planar pendulum motions. The equations of motion of the crane-payload system for those three types of payload motions are setup. Iterative computational algorithms based on Newmark-β method are then programmed to solve the equation. Numerical study on a 232-meters-high tower crane structure which was used for the construction of Sutong Bridge is conducted. The results are compared with the static analyzing results and dynamic analyzing results not consider the crane-payload interaction. The comparison tells that crane dynamic responses are far bigger than the static computed responses. However, for the studied operational cases of the given crane, the difference between the peak response not considering the interaction and the value considering the interaction is not significant.

Keywords: crane payload interaction, vibration, operational response, coupled system

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1. INTRODUCTION

Tower cranes are widely used equipment for the construction of high-rise civil engineering structures. These cranes may rise hundreds of meters into the air with a required reach to cover the working range. Since it is a flexible spatial lattice structure, its dynamic responses due to payload motion during its operation are remarkable. However, the generally used engineering methods for the design of tower crane structure do not accurately assess the dynamic effect. Recent collapse accidents of some tower cranes in operation address the importance of the related researches once more (HSE 2004).

The critical problem for the operational response analysis of tower crane structure is to establish the equation of motion of the crane structure under operational loads. Studies related are generally based on some simplified crane models, such as the rigid body model with discrete springs (Ghigliazza 2002), a spherical pendulum and a rigid system model with two degrees-of-freedom (Chin 2001), or a beam model (Oguamanan 2001). These simplifications on crane structure modeling are acceptable for the analyzing the pendulum motion of the payload. However, if the main concern is to analyze the deformation and stress response of crane structures, a more detailed modeling of tower crane structure is required (Ju 2006).

For a crane in operation, it is ordered to lift the payloads to the appointed place via the lifting, rib rotation and lifting cart movement along the rib. These operations will induce the rigid body motion of the payload as well as the deformable vibration of the crane structure. The motions of the payload can then be idealized to be the following three patterns, the straight line motion, the spherical pendulum motion, and the planar pendulum motions. The equations of motion of the crane-payload system for those three types of payload motions are then setup. Iterative computational algorithms based on Newmark-β method are then programmed to solve the EOM. Numerical study on a 232-meters-high tower crane structure which was used for the construction of Sutong Bridge is conducted. By solving the established equation of motion, vibration response of the tower crane structure can be obtained. Numerical case studies on a practically used tower crane structure are conducted. Some discussions will then be made based on the analyzing results.

2. METHODOLOGY

2.1. Continuum Modeling of Tower Crane

The tower crane structure studied in this paper is a beam-like spatial lattice truss structure with repetitive segments (as shown in Fig.1). Following the idea of equivalent continuum modeling (Noor and Martin 1988), the typical repetitive lattice segment is isolated firstly. The stiffness and mass matrix of the equivalent continuum beam element are then developed according to strain energy equivalence under given deformation and kinetic energy equivalence under given vibration motion pattern.

For a lattice segment, its displacement field is uniquely defined as a function of nodal displacements of the continuum beam element as

$$u_n = f\left(u_i^0, v_i^0, w_i^0, \phi_{1,i}, \phi_{2,i}, \phi_{3,i}, u_j^0, v_j^0, w_j^0, \phi_{1,j}, \phi_{2,j}, \phi_{3,j}, \varepsilon_2^0, \varepsilon_3^0, \varepsilon_{23}^0, \bar{u}, \bar{v}, \bar{w}\right) \qquad n = 1,2,3 \tag{1}$$

where $u^1 = u, u^2 = v, u^3 = w$ are displacements of the original lattice segment in three directions; u_i^0, v_i^0, w_i^0 are displacements of node i of the continuum beam element in three direction; $\phi_{1,i}, \phi_{2,i}, \phi_{3,i}$ are rotation of node i of the continuum beam element in three direction; $\varepsilon_2^0, \varepsilon_3^0$ are longitudinal strain components in x_2 and x_3 direction; ε_{23}^0 shear strain in x_2-x_3 plane; $\bar{u}, \bar{v}, \bar{w}$ are warping and distortion components of the cross section. These six variables are assumed to be constant along x_1.

According to eq. (1), the strain components have a bilinear variation in x_2-x_3 plane and can be expressed as

$$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} \qquad i, j = 1,2,3 \tag{2}$$

Therefore, the axial strain of each bar components of the repetitive lattice truss structure can be expressed using the strain components of the continuum beam element as

$$\varepsilon^{(k)} = \sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon_{ij}^{(k)} l_i^{(k)} l_j^{(k)} \tag{3}$$

where $\varepsilon^{(k)}$ is the axial strain of the kth component of the repetitive lattice segment, $\varepsilon_{ij}^{(k)}$ is the strain components in three directions of the kth components of the continuum beam model, $l_i^{(k)}$ is direction cosines of the kth component. $\varepsilon_{ij}^{(k)}$ is expressed using the first two terms of Taylor expansion. The strain energy of a typical lattice segment is expressed to be

$$U_s = \frac{1}{2}\sum_{k=1}^{n} E^{(k)} A^{(k)} L^{(k)} \left(\varepsilon^{(k)}\right)^2 \tag{4}$$

where $E^{(k)}$ is the Young's modulus of the kth bar component, $A^{(k)}$ is the cross section area of the kth bar component, $L^{(k)}$ is the bar length of the kth component. Substituting eq. (3) into eq. (4), the strain energy of the repetitive lattice segment can be expressed as a function of nodal displacements of the equivalent continuum beam element. Draw an analogy between the equivalent continuum element and the classical beam element, the force associated with the strain components $\varepsilon_2^0, \varepsilon_3^0, \varepsilon_{23}^0, \bar{u}, \bar{v}$ and $\bar{w}$ are set equal to zero, which means

$$\frac{\partial U_s}{\partial \varepsilon_2^0}=\frac{\partial U_s}{\partial \varepsilon_3^0}=\frac{\partial U_s}{\partial \varepsilon_{23}^0}=\frac{\partial U_s}{\partial \overline{u}}=\frac{\partial U_s}{\partial \overline{v}}=\frac{\partial U_s}{\partial \overline{w}}=0 \tag{5}$$

Therefore, the strain energy can be condensly expressed as：

$$U_s=\frac{1}{2}\left(u^e\right)^T K^e u^e \tag{6}$$

where $(u^e)^T=\{u_i^0,v_i^0,w_i^0,\phi_{1,i},\phi_{2,i},\phi_{3,i},u_j^0,v_j^0,w_j^0,\phi_{1,j},\phi_{2,j},\phi_{3,j}\}$, K^e is an 12x12 elemental stiffness matrix of the continuum beam element.

Similarly, the kinetic energy of a typical lattice segment can be expressed to be

$$U_k=\frac{1}{2}\sum_{k=1}^{n}\rho^{(k)}A^{(k)}L^{(k)}\left(\dot{u}^{(k)}\right)^2 \tag{7}$$

where $\rho^{(k)}$ is the Young's modulus of the kth bar component. Substituting eq. (1) into eq. (7), the kinetic energy of the repetitive lattice segment can be expressed as the second order polynomial function of nodal velocities of the equivalent continuum beam element as:

$$U_k=\frac{1}{2}\left(\dot{u}^e\right)^T M^e\dot{u}^e \tag{8}$$

where M^e is an 12x12 elemental mass matrix of the continuum beam element..

The global stiffness and mass matrices of the lattice structure can then be assembled following the general finite element procedure for static and dynamic response analysis. The displacement and internal force of each component can then be computed from the solved nodal displacements of the continuum beam.

2.2. Crane Payload Coupled System Modeling and Response Analysis

During the payload lifting and rotation, the equation of motion of the mast structure can be established using Lagrange equation (Ju 2006). When the tower crane is lifting a payload of mass m_p, whose motion is expressed to be $u_p(t)$, the equation of motion of the tower crane structure can be expressed to be

$$\left(\left[M\right]+\begin{bmatrix}0 & 0\\ 0 & m_p\end{bmatrix}\right)\begin{Bmatrix}\ddot{u}_r\\ \ddot{u}_B\end{Bmatrix}+\left[C\right]\begin{Bmatrix}\dot{u}_r\\ \dot{u}_B\end{Bmatrix}+\left[K\right]\begin{Bmatrix}u_r\\ u_B\end{Bmatrix}=\begin{Bmatrix}0\\ m_p(g+\ddot{u}_p)\end{Bmatrix} \tag{9}$$

where *M*, *C*, and *K* matrices are structural global mass, damping and stiffness matrices, u_B are hanging point vertical displacements of the rib, u_r are displacements at the remaining degrees of freedom of the tower crane.

If the payload m_p makes a spherical pendulum motion, which means the angle between the lifting cable and the vertical axis $\theta = \theta_0,\ \dot{\theta} = \ddot{\theta} = 0$, the motion of the payload can be expressed to be $\varphi = \omega_0 t + \varphi_0$, where φ is the rotation angle of the payload around the vertical axis in the plane perpendicular to the vertical axis and ω_0 is the circular frequency expressed as $\omega_0 = \sqrt{g / L_p}$. The equation of motion of the tower crane structure can then be derived to be

$$[M]\begin{Bmatrix} \ddot{\Delta}_r \\ \ddot{u}_B \\ \ddot{v}_B \\ \ddot{w}_B \end{Bmatrix} + [C]\begin{Bmatrix} \dot{\Delta}_r \\ \dot{u}_B \\ \dot{v}_B \\ \dot{w}_B \end{Bmatrix} + [K]\begin{Bmatrix} \Delta_r \\ u_B \\ v_B \\ w_B \end{Bmatrix} = m_P\left(\ddot{w}_B + g\right)\begin{Bmatrix} \mathbf{0} \\ \theta_0 \cos(\omega_0 t + \varphi_0) \\ \theta_0 \sin(\omega_0 t + \varphi_0) \\ -1 \end{Bmatrix} \tag{10}$$

The displacement response of the mast structure can thus be solved and used to compute internal force and stress of each bar components.

If the payload m_p makes a planar pendulum motion, which means $\varphi = 0$, the motion of the payload can be expressed to be $\theta = \theta_0 \cos(\omega_0 t) + \varepsilon_\theta(t)$, where θ_0 is the initial planar pendulum motion angle of the payload to the vertical axis and $\varepsilon_\theta(t)$ is a small perturbation term. The equation of motion of the tower crane structure can then be derived as

$$\left([M] + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_P & 0 & 0 \\ \mathbf{0} & 0 & m_P & 0 \\ \mathbf{0} & 0 & 0 & m_P \end{bmatrix}\right)\begin{Bmatrix} \ddot{\Delta}_r \\ \ddot{u}_B \\ \ddot{v}_B \\ \ddot{w}_B \end{Bmatrix} + [D]\begin{Bmatrix} \dot{\Delta}_r \\ \dot{u}_B \\ \dot{v}_B \\ \dot{w}_B \end{Bmatrix} + [K]\begin{Bmatrix} \Delta_r \\ u_B \\ v_B \\ w_B \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ m_P \cos\varphi_0[-L_P\ddot{\varepsilon}_\theta - 2\omega_0\theta_0^{\,2} L_P \cos(\omega_0 t)\sin(\omega_0 t)\dot{\varepsilon}_\theta + 2g\theta_0^{\,2}\sin^2(\omega_0 t)\cos\varphi\varepsilon_\theta \\ +g\theta_0\cos(\omega_0 t) + 2g\theta_0^{\,3}\sin^2(\omega_0 t)\cos(\omega_0 t)] \\ m_P \sin\varphi_0[-L_P\ddot{\varepsilon}_\theta - 2\omega_0\theta_0^{\,2} L_P \cos(\omega_0 t)\sin(\omega_0 t)\dot{\varepsilon}_\theta + 2g\theta_0^{\,2}\sin^2(\omega_0 t)\cos\varphi\varepsilon_\theta \\ +g\theta_0\cos(\omega_0 t) + 2g\theta_0^{\,3}\sin^2(\omega_0 t)\cos(\omega_0 t)] \\ m_P[-\theta_0 L_P\cos(\omega_0 t)\ddot{\varepsilon}_\theta + 2\omega_0\theta_0 L_P\sin(\omega_0 t)\dot{\varepsilon}_\theta + g\theta_0\cos^2(\omega_0 t)\varepsilon_\theta + 2g\theta_0^{\,2}\cos(2\omega_0 t)] - m_P g \end{Bmatrix} \tag{11-1}$$

$$L_P\ddot{\varepsilon}_\theta + \left(\ddot{w}_B + g\right)\varepsilon_\theta = -\theta_0\ddot{w}_B\cos(\omega_0 t) - \ddot{u}_B\cos\varphi_0 - \ddot{v}_B\sin\varphi_0 \tag{11-2}$$

$$0 = \ddot{u}_B\sin\varphi_0 - \ddot{v}_B\cos\varphi_0 \tag{11-3}$$

It can be seen from the above equations that if the payload is in the planar pendulum motion, the dynamic response of the crane is fully coupled with the payload motion. The displacement response of the mast structure can then be solved using Newmark method and iterative approach.

3. CASE STUDY

3.1. Structure description

The tower crane structure studied herein is used for the construction of the cable pylons of Sutong Bridge. Figure 1 shows the layout of the cable pylon and tower crane system. The detailed layout of a typical lattice segment of the tower crane structure is shown in Figure 2. According to the design diagrams, the length, cross sectional area, Young's modulus, and mass density of the bar components (longitudinal bar denoted to be bar *l*; bracing straight bar denoted to be bar *b*; diagonal bracing bar denoted to be bar *d*; transverse bracing bar denoted to be bar *t* of the crane segment are l = 5.8m, b = 5.5m, $A_l = 3.14\times10^{-2}$ m^2, $A_d = 1.5\times10^{-2}$ m^2, $A_b = 1.5\times10^{-2}$ m^2, $A_t = 0.2878\times10^{-2}$ m^2, $E_l = E_d = E_b = E_t = 2.1\times10^{11}$N/m^2, ρ = 7900 Kg/m^3, respectively.

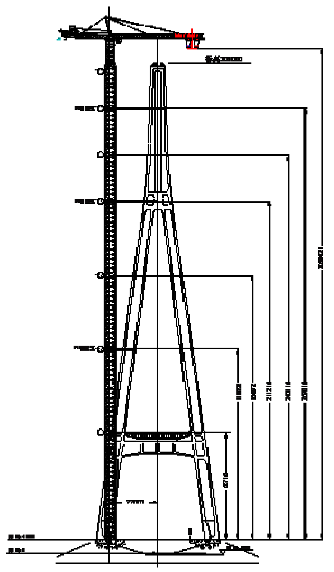


Fig. 1 Cable pylon - tower crane system of Sutong Bridge

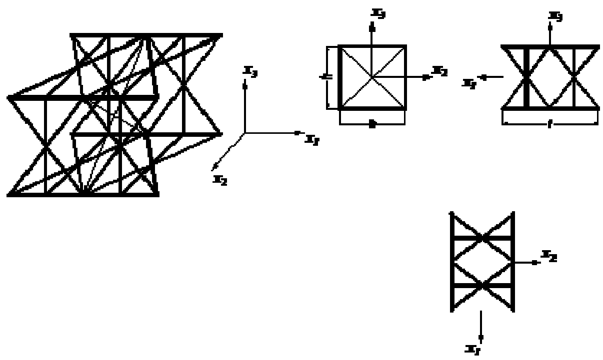
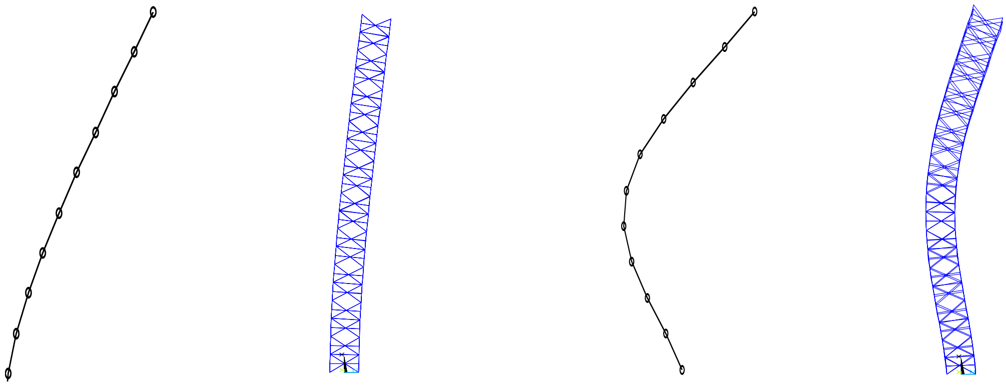


Fig. 2 The detailed layout of the typical lattice segment of tower crane

3.2. Crane Modeling and Eigenvalue Analysis

According to its layout, the elemental mass and stiffness matrices of the equivalent continuum beam element of the lattice segment are derived (Yan 2010). To verify its accuracy, a numerical model with 10 continuum beam elements (CBE) is setup. As a reference, a numerical model composed of bar elements (BE) is setup using commercial software. Eigen value analysis cases are conducted on both the CBE model and the BE model. Figure.3 shows a mode shape comparison of the first four bending modes. It can be seen from the figure that the mode shapes obtained from the CBE model match with the global mode shapes obtained from the BE model quite well.

Just some local vibrations are not caught by the CBE model.

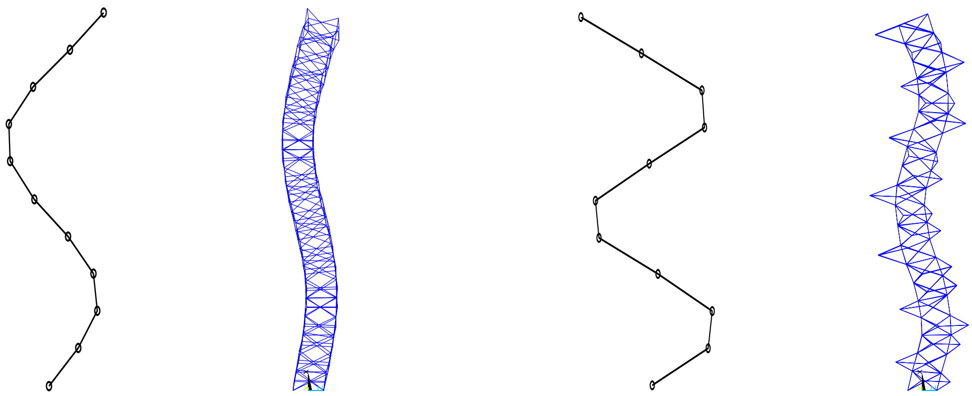


a) 1st bending mode using CBE model

b) 1st bending mode using BE model

c) 2nd bending mode using CBE model

d) 2nd bending mode using BE model



e) 3rd bending mode using CBE model

f) 3rd bending mode using BE model

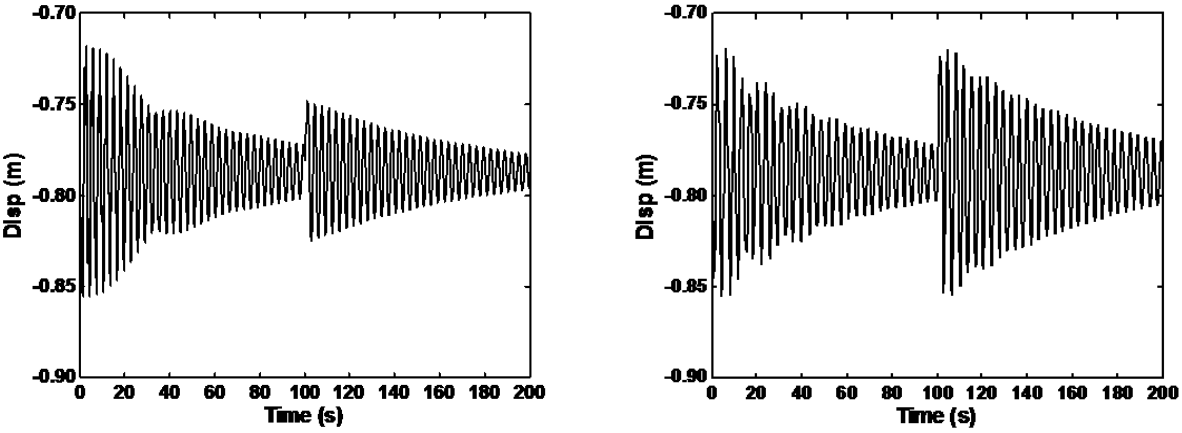
g) 4th bending mode using CBE model

h) 4th bending mode using BE model

Fig. 3 Mode shape comparison between the continuum-beam-element (CBE) model and the bar-element (BE) model of the tower crane

3.3. Operational Response Analysis

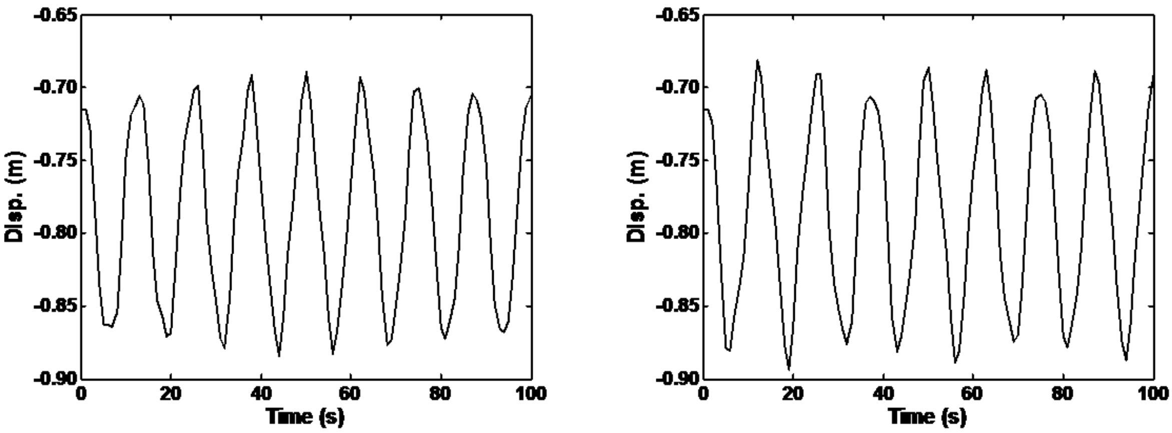
Three operational cases, one payload lifting case, one payload spherical pendulum motion case, and on payload planar pendulum motion case, are analyzed using the CBE model. During the response computation, Rayleigh damping is assumed for the direct integration procedure. The Rayleigh damping is determined according to the damping ratios of the 1st and the 2nd bending mode, which are assumed to be 1% for steel structures.



(a) (b)

Fig. 4 Tip displacement responses of the tower crane during payload lifting (a) without and (b) with considering the crane-payload interaction

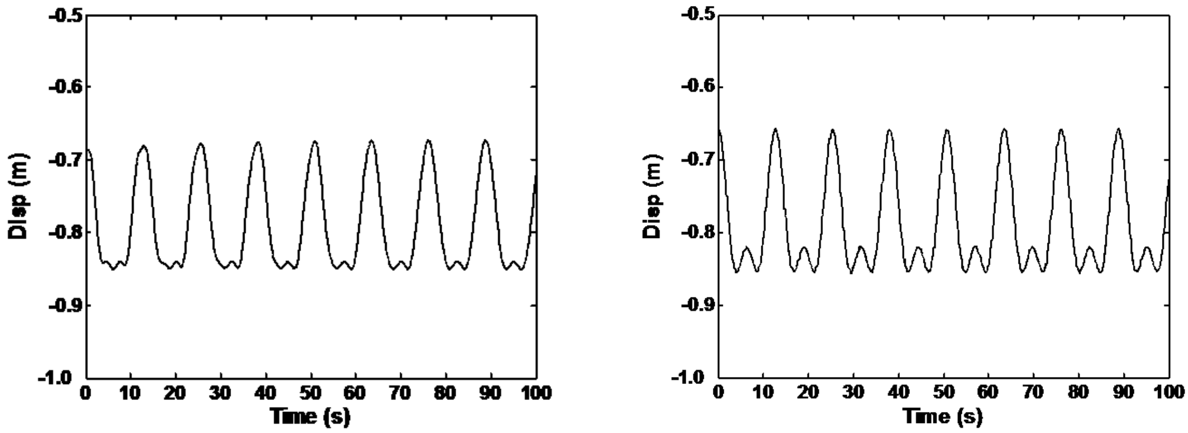
For the payload lifting case, an 80 ton payload is lifted. At the first 0.5 second, the payload is lifted from the static state to the constant velocity lifting state with a velocity of 0.5 m/s. After that, the payload moved upward in this velocity for 100 seconds. Finally, the payload is slowed down to the static state in one second. Figure 4 shows the tip displacement of the tower crane during above payload lifting process without and with considering the crane-payload interaction. As shown in the figure, the dynamic effect is quite remarkable in this process: structural maximum tip displacement without considering the interaction is 0.855m, which is 1.08 times of structural static displacements (0.787m). If the crane-payload interaction is considered, the maximum tip displacement is still 0.855m.



(a) (b)

Fig. 5 Tip displacement responses of the tower crane due to spherical pendulum motion of the payload (a) without and (b) with considering the crane-payload interaction

For the payload spherical pendulum motion case, m_P is set to be 80t, the length of the hanging cable L_P = 40m, the angle between the hanging cable and the vertical axis θ_o = 10^o. Figure 5 shows the horizontal displacement at the tip of the mast. As shown in the figures, spherical pendulum motion of the payload will induce harmonic vibration of the mast structure no matter the crane payload interaction is considered or not. If the interaction is not considered, the peak tip displacement is -0.8800 m. If the interaction is considered, the peak tip displacement is -0.8845 m, which is bigger than the value not considering the interaction. However, for this operational loading case, the difference is not big.



(a) (b)

Fig. 6 Tip displacement responses of the tower crane due to planar pendulum motion of the payload (a) without and (b) with considering the crane-payload interaction

For the payload planar pendulum motion case, m_P is set to be 80t, L_P = 40m, and $\theta_0 = 10^0$. Figure 6 shows the tip horizontal displacement of the mast. As shown in the figures, the effect of interaction is more significant than the operational case of spherical pendulum motion of payload: The peak response not considering the interaction effect is -0.8503 m; the value considering the interaction effect is -0.8566 m. The figures also tell the component response due to the horizontal force component (the ripples shown at each bottom of the response time history) during the pendulum motion of the payload is significant. It is thus required to control the initial vibration angle θ_o during the operation of the crane.

4. CONCLUDING REMARKS

This paper presents a method on dynamic modeling and response analysis of high-rise tower crane in operation. Numerical study on a 232-meters-high tower crane structure which was used for the construction of Sutong Bridge is conducted. The results are compared with the static analyzing results and dynamic analyzing results not consider the crane-payload interaction. The comparison tells that crane dynamic responses are far bigger than the static computed responses. The dynamic modeling and response analysis should thus be conducted during the design stage of the crane structure. The results also tell that for the studied operational cases of the given crane, the difference between the peak response not considering the interaction and the value considering the interaction is not significant. A simplified uncoupled dynamic system modeling and analyzing procedure will present results of enough accuracy for general operational cases. If some special operation cases are reserved, such as heavy payload, big initial pendulum motion angle, or hanging cable length corresponding to resonant driving frequency of the crane, the coupled system modeling and analysis is required to be conducted.

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